I'd like to now show you two mathematical facts about how we integrate quantities of motion.

Suppose we have an object, let's call this the i hat direction, and it's moving, and it has an x component of velocity.

And suppose it starts at some initial position and goes at some final position.

And in the initial position, it was at time ti, and the final position was at t final.

Now, we have two very important integrals that we want to look at which govern how we describe, how we integrate acceleration with respect to time, and also how we'll integrate acceleration with respect to space.

Now, recall that our acceleration, which may be a function of time, is the derivative of the x component of velocity with respect to time.

So, let's first look at a simple integration of acceleration with respect to time and see what we get.

So, if we integrate acceleration, now I'm going to introduce an integration variable, dt prime.

And our integration variable goes from our initial time to some final time.

And if we use our fact that acceleration is the derivative of the velocity, then we can write this is dvx, dt prime, dt prime again.

After a while I'll drop the dummy variables and the endpoints of the integral.

And this simply becomes the integral from t prime, t initial.

t prime equals t final of dvx prime.

Now notice we've done a change of variables in the integration.

So instead of now talking about the endpoints of the integral from t prime, t initial to t final, now what we're doing is we changed our integration variable.

And so what we have is, we have the velocity integration variable is going from some initial value.

And that integration variable is going to some final value.

So again, our initial conditions may have some initial velocity and some final velocity-- three different ways of describing the initial and final states.

This integral is a very straightforward integral, the x final minus vx initial, which is the change in the x component of the velocity.

And that's our classic result that we've done that the integration of acceleration with respect to time is a change in velocity.

Now, let's see what happens as a comparison when we integrate our acceleration not with respect to time, but suppose that it's a function of space.

And so now we're integrating again.

We have a dummy variable.

We have to be careful, because this is the x component of acceleration, but x prime is our integration variable, and that's going from some initial position to some final position.

Now, we can write this as again, make the substitution, dx dt, dx prime, going from the initial to the final.

But now notice we're going to rewrite this integrand as dvx times dx prime, dt prime, dt.

And when we do that, we have this result that dx prime, dt prime is precisely what we mean by vx.

Now, I'll introduce some dummy variables there as well.

So what our integrand becomes is dvx prime, vx prime.

That's not a function, it's a product of a differential times vx prime.

And now our new integration variable is going from some initial value to some final value.

Once again, this is a straightforward integral.

For those who haven't seen integrals like this, it's just something like x dx is x squared over 2, but our integration variable is vx prime.

So what we get is 1/2 vx final squared minus vx initial squared.

And so what we see here is two fundamentally different facts that if you integrate acceleration with respect to time, you get the change in velocity.

But if you integrate acceleration with respect to space, you get 1/2 times the change not of velocity, but of the x

component of the velocity squared.

And both of these facts are central to how we'll analyze the concept of work and how we applied Newton's second law.