We now would like to consider the motion of a rigid body that's undergoing angular acceleration about some axis.

So let's consider, for simplicity, a disk.

And we have an axis that's passing through some point, as here is the center, but that's not crucial.

And we want to consider the fact that this object-- we'll call this our Z-axis-- K hat unit vector in that direction.

So it's pointing up.

And this object is undergoing an angular acceleration where alpha Z is the Z component of the angular acceleration.

Now, what we'd like to consider is the torque on the subject.

So the way we'll do it is we'll divide the object into a bunch of pieces.

And let's identify a piece here as Delta MJ.

That's that piece.

And this piece has some force acting on it.

Now, this force can be a vector.

But the important thing to realize is that this force is only the external force, because we've already got to make the assumption that all internal torques cancel in pairs.

And we'll show that in the video a little bit later.

So we're only considering the external forces that are acting on this element.

And recall that we have the vector.

RS to J is the vector.

From point S to where this mass element is.

And now what we'd like to do is calculate the torque.

In general torque is given by the expression-- we'll sum over all the elements.

J goes from 1 to N. And it's the cross-product of our SJ cross FJ external.

Now, our goal here is only to calculate the Z component of the torque.

So we can simplify our understanding a little bit by writing out this vector F external J as a component in the plane.

And in order to describe that we'll choose some unit vectors.

R hat theta hat going into the plane.

We'll make another picture in a moment.

So our vector can have an R component.

And I'll keep the external in there.

It can have a theta hat component.

And it can also have a Z component.

But recall that a cross-product is always perpendicular to either of the elements.

So when I cross R with anything in the K direction then that component will give a component that's not in the Z direction, and so I can ignore that.

So my first simplification is to say that the Z component will only come from the cross product of RSJ, with these two pieces.

So that's cross FJR R hat plus FJ theta theta hat.

Now, again we can make another simplification, and perhaps here it's helpful to have another overhead view.

And here's our mas element, delta NJ.

That's our vector from F, from the center, to delta NJ.

We have unit vectors, R hat, and theta hat.

And so we see that this vector, RSJ, has some length in the R hat direction.

So we're picking S as the origin of our coordinate system.

This can generalize.

So we don't worry about if the object is not symmetric.

So when you take the cross product of R hat verse R hat, that also is zero.

And so we see our simplification is quite nice, that the Z component of the torque, is only arising from the sum of J goes from 1 to N of our RSJ.

Well, we've already-- we'll write this all as vectors.

R ht cross F J theta theta hat.

Now, again, even if our external force-- by the way, we're dropping external.

We can always say external, but I think, for simplicity, we'll now drop the external.

And what we're considering is just the component of the force.

We can write that FJ-- this is a little complicated-- theta.

Just the theta hat component of that force.

This is the only piece of the force that matters in contributing to the Z component of the torque.

And this cross-product is very direct because we've chosen a right handed system, where R hat cross theta hat is K hat.

So we see that this becomes J equals 1 to N of RSJ FJ theta in the K hat direction.

Now, that's just the calculation of the torque.

But, as always, it's crucial to understand where Newton's second law appears in these calculations.

And Newton's second law, for this mass element, delta MJ, So the second law is telling us that the tangential force is proportional to the mass element times the tangential acceleration of that mass element.

Now, for this type of rotation about the Z-axis.

So when we're rotating about that Z-axis we know that AJ theta, so the acceleration of this tangential element, is just proportional to RJ distance from-- well we've called that our RSJ-- so how far away from the center?

And here's the key thing.

It's also proportional to the Z component of the angular acceleration.

And every mass element in the body has the same alpha Z.

And so our sum, for the torque, can now be written in the following way.

The Z component of the torque-- now, I'm going to do something here, which is, I'll write J equals 1 to N, a parnetheses mark.

I have one of the RSJs.

I have another RSJ.

A delta MJ.

So I have delta MJ RSJ squared.

But every single component has the same alpha Z.

And we're in the K hat direction.

Now, because we have a continuous body we have to, again, consider a limit.

So let's call the limit as delta MJ goes to 0 of this sum, delta MJ RSJ squared.

Well that's an integral over the body of DMRS squared.

And we identified that before as the moment of inertia about the Z-axis.

So the quandary in parentheses is just a measure of the mass distribution about the axis.

You see the R squared, the delta MJ.

And so in conclusion we have that the Z component of the torque is equal to IS alpha Z K hat, which is now a vector, because that's the vector alpha.

And this is our crucial result for a body that's rotating about the Z-axis.

This result can generalize to not just a disk, but any body that's looking at the Z component, for this fixed axis rotation.