We can consider a given coordinate system as a reference frame within which we can describe the kinematics of an object.

By "the kinematics," I mean the position, the velocity, and the acceleration as a function of time, basically a geometric description of the motion.

Some aspects of these kinematics will look different in different reference frames and I'd like to examine that now.

First, I want to define what I mean by an "inertial reference frame." An inertial reference frame is one in which an isolated body, one with no net force acting on it, moves at constant velocity, where that constant velocity might be zero.

Another way of saying this is that an inertial reference frame is one in which Newton's laws of motion apply.

Recall that Newton's first law of motion states that an isolated object with no forces acting on it moves at constant velocity.

So let's begin by considering an observer in a particular reference frame.

We'll call that reference frame S and denote it by coordinate axes x and y.

And let's consider an object that in that reference frame is at a position vector small r.

We can then consider a second reference frame, which I'll call the frame S prime, and I'll denote that with coordinate axes x prime and y prime.

In my example here, I'm going to assume that the coordinate axes in frame S prime are parallel to but displaced away from the coordinate axes in frame S. More generally, we could have the S prime coordinate axes rotated with respect to the frame S axes.

That's a complication I'm not going to add now but conceptually, it's not really different.

For simplicity, we'll stick to parallel axes in this example.

So imagine we have two observers, one in frame S at the origin and one at the origin of frame S prime, both looking at the same object.

Observer S will measure a position vector little r.

Observer S prime will measure a position vector little r prime.

The two observers have a relative position vector, capital R, which is the position of S prime relative to the origin of frame S.

So what we'd like to see is how are these different position vectors related.

Well, from the geometry of the diagram, we can see that the position measured by observer S, which is just little r, is equal to the position of observer S prime relative to observer S, which is capital R, plus the position vector measured by the observer at S prime, which is little r prime.

I can rewrite that if I'd like to write the position measured by the observer at S prime in terms of what's measured by the observer at S. I could just rearrange this and write that little r prime is equal to little r minus capital R.

Now let's add a further complication and assume that the observer S prime is not just at a different location from the observer of S but is moving at constant velocity relative to frame S.

So we'll assume that frame S prime is moving at constant velocity with respect to frame S at a constant velocity vector V.

So V vector is a constant.

And in that case, my offset of observer S prime relative to observer S, which is the vector capital R, is a function of time.

So capital R is a function of time and it's given by the offset at time 0, which I will call capital R0, plus the elapsed motion due to the constant velocity, which is capital V times time.

So since capital R is a function of time, that tells me that in this equation, little r prime, the position vector measured by the observer in frame S prime, is also going to be a function of time, even if capital-- sorry-- even if little r is a constant.

So notice what that means.

If the object is at rest in frame S, the object will appear to be moving.

Its position vector will be time-dependent in frame S prime because capital R, the location of S prime relative to S is changing.

So this relation tells us how the position vectors in the two frames are related.

What about the velocities?

Well, to compute how the velocities are related, we can just take the time derivative of the relation of the position vectors.

So in this particular case, we have that the time derivative of the S prime position, d little r prime dt, is equal to the time derivative of the position in frame S, which is d little r dt, minus the time derivative of the offset of S prime relative to S. So that's minus d capital R dt.

And I can rewrite that in terms of symbols for the velocity.

So I have here the velocity little v prime, which is the velocity measured by an observer in frame S prime, and that's equal to the velocity of little v, which is the velocity measured by frame S, minus d capital R dt, which we see is just capital V vector, which is the velocity of S prime relative to S. So this is how the velocities are related.

And again, notice that if the object is stationary in one frame-- so if it's 0 in one frame, it will be nonzero in the other frame.

So in general, you will measure different velocities in the different frames, even if one of those velocities is 0.

Now, how are the accelerations related?

Well, again, we can just take the time derivative of the velocities to figure out what the relationship of the accelerations is.

So differentiating this equation, I have that d little v prime dt is equal to d little v dt minus d capital V dt.

But here, something interesting happens because remember, we said that capital V is a constant vector.

And remember, capital V is the velocity that frame S prime has relative to frame S.

So since capital V is a constant, that means that this term goes to 0.

And so we see that the acceleration in frame S prime is equal to the acceleration in frame a.

So if I have two reference frames, one moving at a constant velocity relative to the other, in general, I will measure different positions and different velocities for an object as measured by the two frames.

However, the accelerations measured in both frames will be identical.

Because the accelerations are identical, we'll see that Newton's laws will look identical in the two frames.

And we can see that in the following way.

In frame S, we have that the force is equal to the mass times the acceleration.

In frame S prime, the force F prime is equal to the mass times a prime.

But a prime is equal to a, as we calculated here.

So we see that the forces in the two frames are identical, even though the positions and velocities in general will be different, as measured in the two frames for the same object.

But the accelerations will be identical and so the forces will be identical.

So if one of these reference frames is an inertial frame, one in which an isolated body moves at constant velocity, then any other frame moving at constant velocity with respect to the first frame will also be an inertial frame.

What this means is that you're always free to transform from one inertial frame to another.

And what that means is that you can always transform to another frame that is moving at constant velocity with respect to an original inertial frame.