We are now considering the motion of a rigid body.

And we'd like to talk about kinetic energy of rotation and properties of that rigid body.

Let's consider a rigid body, and let's say that our rigid body is rotating about an axis passing through a point S.

If we look overhead at our rigid body, then what we're going to do is introduce a coordinate system.

So this is our overhead view.

And suppose that we have a small element of our rigid body here, which I'm going to write as delta m j.

And that's a distance r j from the center.

Our rigid body has an angle theta.

And what we'd like to do is describe a coordinate system, r hat, theta hat, and k hat pointing like that.

Now, with this rigid body that's rotating, we describe the angular velocity as the rate that the angle is changing with respect to k hat.

And so we can draw the vector omega, and this establishes our coordinate system for our rigid body and a small element of mass, delta m j.

Now, what we'd like to consider is what does this mass element do?

Imagine that it's over there.

Because every point in the rigid body has the same angular velocity, this object is going in a circle with angular velocity omega.

Recall that omega is perpendicular to the plane of motion.

But the object itself has a velocity, which I'm going to write v j.

And that velocity vector, v j, is in the tangential direction.

And it's related to the z component of our angular velocity by the following relationship-- it's how far we are from the center point, S, times the z component of the angular velocity.

And it's pointing in the tangential direction.

So remember that omega z is equal to d theta dt.

And this describes our coordinate system for the rigid body.

Now what we'd like to discuss is the kinetic energy.

And the way we're going to consider kinetic energy is, we're going to sum up the rotational kinetic energy of every single mass element.

So we begin by writing k j rotational.

And we know that that is just 1/2 times the mass element times the velocity of that element squared.

Now we can use our relationship for the tangential velocity element related to the angular velocity.

And we have 1/2 delta m j r j squared times omega z squared.

Now keep in mind that omega z is the same for every single mass element, but the distance of the mass elements are all different by r j.

So the total rotational kinetic energy is the sum over j from 1 to n of, let's put the 1/2 outside, times delta m j r j squared.

Now again, recall that every element has the same omega z.

So I can write parentheses omega z squared.

And that's our rotational kinetic energy.

So what we want to do now is look at the limit as our delta m j becomes very small, because we have a continuous body.

And we'll write a definition, which is going to be the moment of inertia passing through this point, S, about the axis passing through S is equal to the limit as delta m j goes to 0 or n goes to infinity of this sum-- delta m j r j squared, j goes from 1 to n.

Now because this is a limit for the continuous body, we'll define it as the integral over the body of a small mass element dm.

That's a distance r squared.

Now here, what is the meaning of the r?

For our continuous body here, if we call this dm and we define the distance from S to the body r-- and now I'm going to just put a little notation in here.

It's the distance from S, the axis we're calculating about, to where the body is dm.

So I'll write S dm.

This is what we call the moment of inertia of a continuous body.

Now again, what's very important to realize-- it's a moment about a particular axis.

So this is about an axis passing perpendicular to the plane of rotation and through S, point S.

So it's an axis that's passing perpendicular to the plane of rotation passing through the point S.

And this is what we call the moment of inertia of a body.

Now, we'll see that the moment can be expressed in terms of other physical quantities.

As the course develops, you'll see two or three more fundamental relations for moment of inertia.

But what we'd like to do now is summarize our results-- that the kinetic energy of rotation is 1/2 for rotation about this axis, I'm just to indicate passing through S, of I s times omega z squared.

Now keep in mind, because omega z is a component, it can be positive, 0, or negative.

But the square is always a positive definite quantity.

And that's our kinetic energy of rotation.

Let's contrast that with our translational kinetic energy.

And we remember there that was 1/2 times the total mass of the object times v squared cm, where we're looking at all of the objects at the center of mass.

And this is the total mass of the object.