PROFESSOR: So for today's lecture we have Jason Ku guest lecturing. And he's the president of OrigaMIT, which you should all check out Sunday afternoons, origami club at MIT. He's an origami designer and a grad student in mechanical engineering, and he's going to talk about the more artistic perspective on how origami design works, in particular in the representational and tree method of origami design world. So take it away, Jason.

JASON KU: Hi. I'm Jason. Eric gave a bit of an introduction. I've been folding origami instance maybe I was five years old, and I've been designing origami for maybe the past 10 years. I'm a PhD student in mechanical engineering, working in folding things on the micro and nano scale. So that's how this is applying to my research.

I'm here to talk a little bit about origami art and how the concepts we've been talking about in class apply to origami in actually designing and folding artwork out of paper. These are all the websites that I'm going to be pulling pictures from. So if we can't use these pictures in future versions of this lecture, then you can still see some of the media.

I want first make the analogy of origami art to music. Many, many people make this analogy, and it's actually very apt analogy. In music you have composers, you have people who produce a work of music, design the structure, design what the main aspects of the piece are, in terms of a structural sense, but aren't necessarily performers themselves.

Now in origami, the performer and the composer are usually one and the same. But hopefully in the future, that won't always be the case. In music the composer usually makes a piece for multiple instruments or multiple voices or things like that, so most the time can't do all that performance. And some people are more gifted in the performance side. Some people are more gifted in the composition side. And I think it's a fairly apt analogy.

There's tons of mathematics in music. There's tons of mathematics in origami. But

there's also this level of artistic complexity, which we'll see later in this lecture.

I'm going to concentrate mostly on representational origami. Representational origami is traditionally representing living things in our world, but it's pretty much, you see something not necessarily living, but you see something and you want to make that form for the form's sake, to represent that form. And this is different than, say, patterning to create artistic patterns on a sheet of paper, tessellating, making geometric polyhedra, or making more abstract art that doesn't necessarily have a relation to a real world object.

So I'm going to start with a little bit about origami art. We've heard this. Eric mentioned this particular individual, Akira Yoshizawa. He is widely understood to be the father of modern origami. He was born in 1911. He was around for a very long time. Unfortunately, passed away in 2005. I was lucky enough to get to meet Akira Yoshizawa when he attended a convention in North Carolina when I was maybe around 10.

He was very powerful and influential in the world of origami, because he was one of the first people to start creating new models, be able to look at an object and create that object just from folding. He was one of the first people to actually try to make a large number of new models, as opposed to the past many centuries when only a few traditional models were known or pursued.

This is a picture of Yoshizawa right here. He's fairly happy in this picture. But he's holding the logo of the US organization in origami, OrigamiUSA, which is this sailboat.

But as you can see, different than the traditional origami crane or the frog or things like that, you see a lot of curves in his work, a lot of shaping. He uses a technique called wet-folding, in which he weakens the paper to some degree, weakens the paper fibers by applying water, shaping the paper, and letting it dry so that it holds that form. And you can see in this sparrow-- this is particularly one of my favorite works by Yoshizawa-- it really has the essence of this little bird, but is actually very simple and elegant.

Origami design isn't all about making the most complex thing in the world. It's really trying to represent a subject elegantly. And I think this model does a very good job with that. But you can see here, very clean surfaces, not a lot of extra creases that you can see. Traditionally, wet-folding uses thicker paper and is slightly more substantial. So here are some of his other works.

And I want to start out with Yoshizawa because he was represented as the father and the master, and many, many of the origami designers, if not all of the origami designers that I'm going to continue to talk about were heavily influenced by Yoshizawa.

So I'm going to first talk about the traditional style. And I'm going to compare it to, say, the crane or the frog. These types of models are characterized by straight, well-defined, polygons in the final form, typically folded flat. Little shaping is traditionally needed to go from the base of the model to the final form. It's very geometric, these models, characterized by very straight, precise creases.

So here is, I think, a very good example of this traditional style. While there are some curves here, everything is very well-defined, maybe just a slight shaping here. But even that is fairly well-defined. But you can see Komatsu, Hideo Komatsu, a Japanese folder, uses really clean, large polygons of open paper without creases on them to represent polygons on the model. The folded form.

His design process isn't really using tree theory. I mean, all origami design is subjected to the condition that no two points on the unfolded square can increase in distance in the folded form. That's a property called developability of the paper. The paper's not going to stretch, basically. So all origami design is subject to that condition, but you don't have to deal with necessarily these things called uniaxial bases. Pretty much all of these models are non-uniaxial.

His design process is kind of a trial and error process of folding along different 22.5 degree grids. 22.5 degrees is, I guess, 1/8 of pi. 1/16 of 360 degrees. And it's a particularly nice and useful discretization of angles in origami design.

All the traditional bases are based on this 22.5 degree grid system. And there's a certain elegance of that. Actually I think, the mouse is based on a 30 degree grid system, but is kind of an exception, but follows the same principles.

He keeps folding a piece of paper and tries to get these geometric shapes that really are able to by themselves capture the model. And I'm going to use some of that design technique later in a design example. He has a small but very distinguished repertoire because his process is less algorithmic-- I mean, he has algorithms, I'm sure, that are difficult to describe, but his process is actually very artistic. And while it's very exact, I think it's one of the most elegant examples of origami design.

Here's another example of the traditional style. As you can see, there's slightly more curves and things in it, but it's fairly well characterized by these straight creases. Heavier paper for wet-folding.

This model on the left here is box pleated. So as opposed to the 22.5 degrees structure, box pleating is characterized by only multiples of 45 degrees. So pi over 4. And so you see the grid here, this model is based on a fairly large grid, so you can get the detail that it needs. These are not uniaxial bases, again, but they're still limited by this stretch ability constraint.

And those were styles that stemmed from the traditional crisp folding of say the crane and the crab and the frog and all these traditional designs. The non-traditional style is more an extension of Yoshizawa's work and shaping and curved folding and things like wet-folding. There is much shape that needs to be done to create the essence of the model. The model is encapsulated by not necessarily the structure as much, but of the final shaping, the undefined shaping that you kind of put into the model.

Here's an example of an English folder named David Brill, who is an investment banker, if my memory serves me. He now lives on a golf course. And I think he's retired now, but he likes to fold paper. But you can see here, a good example of this style, thick paper.

The character of the model is really defined by these curved tension folds, which is slightly different than at least the traditional style. And oftentimes, it's very, very difficult to replicate to any of these types of models, because it has so much to do with subjectivity as opposed to objectivity, as in the traditional style.

Here's another good example, Michael LaFosse. He's a paper folder who actually resides here in Massachusetts. He's in Haverhill, "Have-er-ill," something like that, Massachusetts.

He is unique in origami designers in the fact that he is also an avid paper maker. So he actually makes a lot of the media which he folds. And that gives this intimate relationship between the life cycle of the paper. He's able to make specialty paper that's really necessary to make some of the most complex works out there.

He's gone to culinary school. He was a chef for a while. And he was also a marine biologist for a while. So these origami artists have come from many different walks of life.

This next folder, Eric Joisel, he's a Frenchman, lives in Paris. He was a former clay sculptor. And actually, I think you can really see that in his work, the kind of solidness and really cohesiveness of his composition. All the detail and texturing are very well thought out in terms of the subject as a complete piece. Heavily influenced by Yoshizawa.

A lot of this use of texture, incorporating texture into his models, he was a big pioneer in that area. This texturing is fairly obviously non-uniaxial. He doesn't go through a tree method and represent each one of these points as a stick in a stick figure. These flaps don't lie along an axis. They don't hinge perpendicular to that axis. Yet he's able to create these amazing forms in paper.

He has stopped doing clay sculpture and does origami full time now. He's very well known for his depiction of the human form. This is taken from a collection of masks. He's done numerous, numerous masks that are really very expressive. He was one

of the first people to really, for me at least, evoke emotion and convey emotion in his work.

But you can see here, the structural crease pattern for this face is actually very, very simple. It's kind of represented by a few pleats. But the amount of work used to transform that very simple form into this very expressive, curved work of art is kind of astonishing.

Here's a more recent work of the entire human form. You see how this is starting to come as some sort of blend between the traditional and nontraditional forms. It lies somewhere along the spectrum. But it's a very complex model, so it had needs to have this structural complexity. But at the same time, he shapes it to an extent that very few people can do.

I'm paraphrasing a quote of his, but he's of the opinion that if you can reproduce exactly a piece of origami then it's not really art, because you're not putting anything more into the model, if it doesn't have something unique and original and something that can't be reproduced in the model.

Here's two fantastic subjects in terms of art, to me. Very Escher-like and it's self-referencing. This is called the *Self-made Man.* And I forget the title of this work, but he's basically emerging from the paper. You see that his arm and leg are not actually finished. I think this is called *Birth,* actually.

But really, using paper to express an artistic idea, very few people get to that stage of competency with the technical and being able to infuse that emotion into the subject. So Eric Joisel is a pioneer in that realm of origami art.

Here are three very, very complex-- These are very recent works, probably within the last year or two. Lots of use of texturing to make the armor here. Lots of planning, tree theory included. These are mostly box pleated models, but you can't really tell from here because of his impeccable ability to shape a model.

I'm going to remind you guys that everything I'm showing to you is a representational work. Each one of these is made from a single uncut square. Pretty

much, I believe everything I'm going to show you today has that property.

AUDIENCE: When people fold these, do they fold them by hand or do they need special tools? To me, this looks like it would be completely impossible to just fold it by hand.

JASON KU: These are actually fairly large works. Each one stands maybe about that tall. So the paper's very large to begin. But yeah, I believe he just uses his hands and this wet-folding technique to allow things to be held in place. I mean, many people, including Eric Joisel, use clips and braces and things like that to hold certain things in place while he's working on other areas of the model, but it's pretty much by hand. Some people use tweezers or things like that. But most of it is by hand.

AUDIENCE: Does he add color or shade or those things?

JASON KU: Sometimes. For example, that mask I think was speckled with paint after, before or after. There are different opinions on this idea of origami purity. I like Robert Lang's definition of origami, that it's any work whose primary structure is defined by folding. And that's a very broad definition of origami. But I think it works really well.

So if the subject matter is still heavily characterized by the folding and not some other thing that you do to the model, I think most people are OK with that, as long as you're not trying to pass it off as something it's not. There are many origami designers to do multi-sheet things and do very complex works and very beautiful pieces of art.

I think Joseph Wu is a great example of this, who I don't have pictures of his work. But he doesn't try to pass them off a single sheet origami. He is a very skilled designer. He could do it with a single sheet, but he finds that the solution is more elegant using multiple sheets. Any other questions?

Just one more picture of some of Joisel's work. He actually made an entire orchestra of these little guys. This is two sheets. The saxophone is a different sheet. But again, he's not trying to pass them off as being the same sheet here, whereas in here, the weapons actually are from the same sheet of paper.

And these multi-subject pieces, each one of these, it's not all three of them together as one sheet. Just to clarify. But these multi-subject, trying to represents clothes, and weapons, and the human, and all these types of things, is becoming more and more a way to a push the limits of origami design. So again, trying to breathe life into the paper is really what Yoshizawa's mantra was, and so is Eric Joisel's.

All right. So I'm going to move on to this independent concept of really the ability that we have right now to pretty much-- We have the algorithms to make anything we want and really trying to capture that is this idea that I call this modern realism. The style, like Eric Joisel's work, kind of follow along the spectrum of this rigid structure and this free-form shaping, but really try to capture this realism of the subject.

So I think Robert Lang is one of the foremost origami designers in this kind of area. He's a guy from California who is a pioneer of algorithmic origami design. You've heard his name a number of times. He has kind of codified tree theory, if not one of the pioneers of establishing that research himself. He wrote the program TreeMaker that you guys are all probably using to do your homework.

He was at Caltech Ph.D., and was a laser physicist for NASA. And he decided maybe less than 10 years ago to quit and do origami full time. So that's what he does now. So here's a number of his works. Very complex, very exact.

For example, he was a huge pioneer of what we call the Bug Wars. When we had these tools at our disposal to make very complex trees, we can represent very, very complex subjects. And that led to this Bug Wars of trying to one-up each other on how many legs you could make or things like that.

So here's a centipede, for example, with lots and lots of legs. And the exactness to which we can specify the tree is phenomenal. For example, the scorpion here is a design that Robert Lang has approached-- a subject he's approached-- many, many, many times.

This is a design that I particularly like. It's very clean in its folded and its structural

forms. But he actually used TreeMaker and designed each of these pairs of legs to actually be increasing in length as they go back. So really being very exact with the proportions of the model, the proportions in the tree. And tree theory really allows you to do that, to capture that.

Here's a slide for the mathematicians in here. This is not one square sheet of paper. This is probably the only model here that isn't. But it's what we call modular origami, making a single unit and sticking them all together in a very complex and elegant way. Here's a representation of some of the tessellation work that Robert Lang has been working on. This is a vase form.

And all these, or at least these three, were very much characterized by using mathematics to find these forms. And while they're very heavily rooted in mathematics. Mathematics, as I'm sure all of us can appreciate, is an elegant subject in and of itself. There are elegant solutions to problems. And in origami it's particularly nice, because these elegant solutions often are very elegant and pleasing to the eye, as well.

So this is also a Klein bottle. It's kind of a joke. But it topologically does intersect and things like that. So in an interesting work.

I want to move on to a guy named Brian Chan, who is an alumni of MIT. He got his bachelor's, his master's, and his Ph.D. at MIT. He defended his Ph.D. in 2009, but he's still around Cambridge. He is a big pioneer of pushing the limits of complex folding. He's picked up origami design very quickly. And so it is possible to do. So I encourage all of you to try it.

Here is an example of a very, very complex centipede that he designed kind of in response to Robert Lang's. There's a huge history of really trying to one-up each other in origami. And it really helps spur the creativity. And playful competition is very useful to any subject.

These multi-subject things, like this rose, the stem and the petals itself, all from one square sheet of paper. He uses color change. One side of the paper is red; one

side of the paper is green. There have been tons of people that design just the rose part of the rose, and then they make an additional stem and stick it on. This is the first one-piece model of that. And was somewhat influential in that respect.

Here's a very complex, textured character from an anime TV show. I forget which one it's called.

AUDIENCE: *Rozen Maiden*.

JASON KU: *Rozen Maiden* Thank you. That is correct. But you really can see his use of color change here. Again, being able to make this cross in the fabric here. The zigzags of lace, and this texturing of the dress, very, very complex, in its form. But these are all actually uniaxial bases, all come from this idea of tree theory, being able to map things on your subject to the sheet of paper in an algorithmic way.

Here's a very complex, another anime work, a Neko Bus. Neko is, I believe, Japanese for cat. And it's very, very complex. Again, similar to the centipede. Lots and lots of points. But this tree is actually kind of represented by-- There's a head region and there's many points sticking out on both sides. And then this flap kind of comes over and attaches up here, and you've got the tail.

Here's another example of a multi-subject model. Every year there's a design challenge in New York for origami. And this was the sailing ship category. And he kind of went another direction with it. He did make a sailing ship, but this is a kraken attacking the ship. He's got a little person in one of his tentacles. Part of the ship and the ship itself, and it's all one square sheet of paper without cutting.

And if that wasn't enough, then the MIT seal, as well. One square sheet of paper without cutting. The mens and manus, so the mind and hand. And I believe this isn't traditionally a crane, but yeah.

The last person I want to touch on is a guy named Satoshi Kamiya, who's represented as probably the foremost pioneer of super-complex origami. He is a little further on the spectrum on the traditional style than many of these other super-complex folders, characterized by kind of very exact, straight creases, this texturing

for example, a unique balance between making a very cleanly folded-- The Japanese traditionally make very clean subjects in terms of exactness and form.

Here's a little more shaping in the wet-folding. But again, this is one of my favorite works of his, another *Lord of the Rings* character. What's neat about this sea turtle actually, the diagrams for it were just published. I first all this work in 2001 or something like that. But it has these plates on the back, this texture, but it also has plates on the front of the model. So you can actually pick it up, and it looks very, very convincing.

Here are some more models by him. Again, you can see a lot of this texturing here in this wasp, very clean folding. A dog, multi-headed dog, a caribou with very complicated antler patterns, and this dragon. And again, you see the crisp, clean folding, but at the same time very well-planned and well-designed 3D structure to be shaped afterwards.

Here's another work that I particularly enjoy. Really lending this texturing he applies throughout the model, and it's a very cohesive subject, from an artistic sense. It's very complete. There's the same level of detail everywhere on the model, which is very useful.

And I'm going to kind of end this artistic side with a model which is widely regarded as the most complex single work in origami. This took Kamiya over the course of a year to fold. There's thousands of scales on this guy. And again, it's one square sheet of paper without cutting.

You see that it's a very long model. You'd think that this subject would be much better represented by a long rectangle or something like that. But actually it's very symmetric. This crease pattern, which we'll look at later, actually has an asymmetric crease pattern and is quite ingenious in how he decides to accomplish this form and structure.

If you're interested in learning more about the origami art side of things, there's this phenomenal document documentary which you can and purchase online. Or I've

believe OrigaMIT has a copy of this, and we'll probably be screening it some time this semester or next. It's called *Between the Folds.* And it features, among others, both Erik and Marty Demaine, Robert Lang, and many more. And there's a picture of Stata from the film.

Now we're going to move on a little bit to origami design. We've learned what the algorithms are behind a lot of origami design, but now we're going to see how that applies more directly to creating a representational work of art.

If you're really serious about wanting to get into origami design, this book, *Origami Design Secrets* by Robert Lang, is really the first major book on the methods of origami design. Most origami books are traditionally about diagrams, trying to fold specific models. This is the first book really to lay out some of the ground rules of how you create models. And it goes through a number of the things we've talked about.

So just to review a little bit about tree theory, the idea, the process is you start with a subject, like this picture I took at a Japanese museum of a little crab. You kind of draw a little stick figure of what that crab might look like in a one-dimensional form, characterized just by the lengths of these flaps and the connectedness. You go from here to here to an origami base, which has all of those flaps of the right length and connected in the right way. And then you shape it into an origami model.

Now this method, this step here might seem hard to you guys. With a little experience, it's actually very reasonable to assume that someone fairly well versed in the vocabulary of origami will be able to accomplish that step. This step, again, this kind of child's play, somewhat. It's actually not, to do it really well, to represent this model as a stick figure, and we'll see that when we try to go through an example.

This step is the one where algorithms and mathematics really help to do a lot of the work for us, and essentially is kind of the easy part from our perspective, because it's kind of methodical and there's algorithms involved to help us out.

The most artistic and free things we can do with origami design are kind of this step in the shaping and this step in defining the proportions. In this step really you define what the abstraction you choose to characterize in your model. Like here, we are choosing to represent all four legs on either side. You don't have too. But we also decided to model the eyes and the claws as is.

But an underbelly to a crab. We could have modeled that with the texture. We could have modeled the little mouth parts of the crab. There are many things we could choose to model on here that we don't choose to. So this is one level of abstraction.

And this comes with a lot of choice. Here, there's lots of algorithms and math to help us out. But as we'll see, there is actually a lot of choice going from here to here as well, artistic choice, and from here to here, again, probably the most blatant way that an artist can put his style in essence into an origami work.

AUDIENCE: What is the extra fringe?

JASON KU: Which one? This?

AUDIENCE: Diagonal from the top.

AUDIENCE: On the left.

JASON KU: On the left. Oh, this? OK. So I modeled here the body of this crab as a flap coming from here. I kind of wanted a flap to cover the rest of this. And so that's why I've added this leg of the tree there. While branch edges-- this is a branch edge, it doesn't terminate-- will provide paper in that region, as we'll see later, branch edges of the tree, rivers in the space allocation, really don't lend themselves to being shaped very easily. And so if I isolate that body segment as a leaf edge for itself, then I can actually do control a little more about how I'm able to shape it. Good question.

So we're going to review a little bit about uniaxial bases. This these are the definitions that Erik Demaine posed in the algorithm, I think in lecture four. Again, you have this uniaxial base. It has these characteristics that it's in the positive space

above the z equals zero plane. And that's kind of represented here.

The intersection with that plane is the projection. So if you shine the light above it, it would cast a shadow of a stick figure out, which is exactly kind of what we want. We want to make an origami base that associates itself with a stick figure. And then we partition the faces into flaps. So there's all these definitions.

I think to put these in kind of layman's terms from an origami designer's point of view, what do these really mean? Really, the important characteristics that we want are that the flaps lie along or straddle a single line. Because if they do that, then we could just fold it in half and it will have that property of everything being above an axis and everything lying along an axis, and that the flaps hinge perpendicular to that axis.

The reason why we need the flaps to hinge perpendicular to the axis is if they don't hinge perpendicular to the axis then you will not be able to create a projection to the plane that is a one-dimensional stick figure. If these hinges are tilted then that line will project to a line instead of a point, like we'd want. We'd want it to project to a single node on the tree.

In any of these uniaxial bases, think about the base being thinned in the limiting case, that we can create folds parallel to this axis and thin this model until it's right along the axis. And then in that limiting case it is a stick figure, essentially. And once it is a stick figure, layering and orientation of the flaps really don't matter, because it is the stick figure.

So this is kind of an informal definition, but we'll use these later in the lecture. So what is a flap? We kind of made this argument a couple lectures ago. So we want to model a flap, so that we can kind of stick it together. And this is kind of an intuitive sense of we take a sheet of paper, we thin it a little bit, we hinge it perpendicular to some axis. And when we do and we unfold the paper, we see that it takes up this kind of quarter octagon of paper.

Now, if we continue to thin this, if we make it really, really thin, you see how deep

the boundary, this fold that we make, will little closer and closer approximate a circle. Everyone see that? It's kind of like an umbrella. I like this analogy with an umbrella. That you have a single point that's the center the umbrella, and when you close the umbrella, all of the umbrella kind of maps to a single line.

And so it's neat to see on the paper. It's kind of what you could think of as a projection to this tree. Lines on the unfolded square, these lines at the edges of the circle, map to a single point on this flap, or infinitely thin flap, or essentially the tree. This is kind of a leaf edge of our tree. And so everything along this line maps to a single point, is compressed onto a single point.

You can do that with any point going up this flap. We can actually pick off a point here, and we see a line of constant elevation with respect to this flap. And so now we've created a very, very simple tree. Instead of one leaf edge extending off of the rest of the model, we have a branch edge, and then a leaf edge. And this branch edge is corresponding to this strip of paper here of constant width. That's what we call a river.

And we see that a circle is just a limiting case of a river. Rivers separates two parts of the model off from each other by a constant distance. That's what that constant thickness strip of paper means. And the circle is really just a limiting case off that river that separates only a single point away from the rest of the model. That's all I want to say about that.

And we can actually tile these rivers onto a plane to create arbitrary trees. So here's an example of the correspondence. I call these circle/river packings. That's the common term in origami design. This is a circle/river packing. It's kind of a space allocation.

It's an idealization. The model we make is actually not going to be infinitely thin. So each flap is going to take up more space than these circles. But it's a good idealization. This circle/river packing or this space allocation actually maps uniquely to as a tree.

So if we go through it, this point, this circle here might map to this line on the tree. It can actually also map to this line, this edge or this edge, as well. Because I don't really care how these flaps are oriented.

The tree is just supposed to preserve length and connectedness. It doesn't really have to do with where they're mapped. And so we'll see some examples of that later. But we can kind of go through this tree and see all the different aspects of it, how the edges correspond to circles and rivers on the packing.

And we're going to do a little bit of practice for that, because I think that was one of the day hardest parts for me starting out in origami design, was being able to be comfortable with going from a tree to a space allocation, from a space allocation to a tree. Getting that concept in my head was kind of difficult.

So how about we practice a little bit. We have this space allocation of maybe two circles, a river, and three more circles. I'll just give you a second to see which one of these trees is represented by this space allocation. Or should I say how many of these. Because some of these trees might be equivalent.

So we're going to start with the upper right one here. Does it correspond to this space allocation? Yes or no? No. Why?

AUDIENCE: [INAUDIBLE]

JASON KU: Yeah. So the topology's kind of wrong. You've got three equal length flaps up here, which is what we want. We want three equal length flaps separated off from the rest of the model by a river of the same length. That makes sense.

But instead of separating off two flaps, two leaf edges of maybe twice the length, it separates three of the same length, which doesn't quite work out. So the distances and the connectivity's kind of off here, just terms of the numbers. So this one's wrong.

How about this one? Yes. Right? It has the right topology. This one? No, again. The wrong typology. There is, again, three separated from two by a branch edge. But it

doesn't have the right lengths associated with this space allocation.

And how about this one? Yes. Right? I've transformed this tree from here to here. I just moved them around with respect to each other. They're equivalent, in terms of how we choose our tree.

And this will be important when we actually use TreeMaker. Because it doesn't matter how we orient things in our tree, we can manipulate where we put our circles on the paper to get the same tree. The mapping from here to a tree is unique. Mapping from a tree to a space allocation is not, which leads to interesting design choices that you can make in designing an origami model. Yeah, those two.

One more time. We'll go through this one a little quicker. I'll give you maybe five seconds or so. So we're going to start with this one, the first one. Does that map to this space allocation?

AUDIENCE: [INAUDIBLE]

JASON KU: Yes, it does. We have two equal length set off from the same length to equal, and then one twice as big.

And see how I've actually added a redundant node here. I've split this leaf edge into a branch edge and a leaf edge. That's kind of a redundant node that I don't really need. It doesn't really change the topology of this at all. It would just map to a line right here. Everyone see that?

How about this one? No. Right? For a number of reasons that I won't go into. How about this one? No. Again, the distances and the topology are wrong. This one? No. This is actually one of the trees from the slide before.

And this one? Yes. This is actually just a manipulation of that tree. So yay! We're awesome.

Now, going the other way is not necessarily unique. So there would be multiple answers here. Is this a correct representation of this tree? Yes or no?

AUDIENCE: No.

JASON KU: It has the correct topology, right? It has three equal length flaps separated off by a river from three equal length flaps. That's what we have here.

But this river is actually twice as long as any of these flaps. So this is actually a little bit shorter than the length of any of these flaps, not really working for us there.

How about this one? No, for pretty much the same reason. It actually has a very similar, if not identical, tree to this one. How about this one? No, topological problems there.

This one? Yes. So it's got three equal length flaps separated off by a large 2x river, I guess.

And this one? No, for the same reason here. And here we can actually see three different packings of a very similar, if not same, tree. And this goes to show you that there could be many different ways we could put these disks on a sheet of paper that could either improve efficiency or be more useful.

For example, this packing has a central flap. We may or may not want that central flap. You can see that a central flap will use more paper than a flap at the corner or the edge of the paper, because it has 360 degrees of paper that you have to fold as opposed to 180 or even just 90 degrees of paper.

So typically in origami design, if you have a flap you need a little bit of bulk in, a little more paper, you might want to consider making that a central flap. If you want to make a very thin, maybe an antenna or something, a corner flap might be a better choice. So correct, in that sense.

Now again, I want to stress the fact that this is an idealization. These are circles. They don't really account for all the paper in the square. This paper between the circles and the rivers is not really used. Pretty much everything in this no man's land here actually maps to a single point in the tree right. This is a bad example, so I'll use this correspondence. This kind of looks like a bikini or something like that.

That's neither here nor there.

But this space all maps to one of these branch nodes. Everyone see that? Because in the situation where we thin this model infinitely, this is kind of extra space that we kind of just don't even deal with. In reality, that extra space will have to go into either the rivers or the circles in the packing.

So the reason why uniaxial bases are nice in this model is because since all they hinge creases of the model, basically the boundary of the flap with the model, hinge 90 degrees to some axis, then its projection maps to a single point. So if we cut off all the flaps along the hinge creases, we should actually get a very similar mapping to what we have here.

And here's an example of a fairly complex model. But you can see, I've just highlighted the locus of possible hinge creases on this model. There's a unique way to do this. I won't go into it. But there is a unique way to add these hinge creases.

But as you can see, the idea is very similar. But instead of having these curves of constant width, you have these discrete angular curves of constant width. So for example here, you have a river of constant width that changes directions at a discrete corner, but it's still a strip of constant width. Everyone see that?

So maybe we could go ahead and see-- If we had this crease pattern and we didn't know what the model was, we could actually pick off the tree and figure out what this model is. So maybe we start with these two points down here. They're all points separated off the rest of the model by a certain distance.

And that distance here, all these lines that are connected must be at the same location, the same node on the tree, because all those hinge creases must map to a single point. So these two flaps connect with each other because they share this set of hinge creases. And so that's that point right there.

I'm going to ignore these two flaps at the bottom for now. We have this big long river. I'm just going to deal with the big points first. And that connects to two more big points. Everyone see that? I don't want to go too fast.

And actually, you can do that. You can just keep doing that, and methodically picking off distances on this hinge crease representation, this is discrete space allocation and fill in the whole tree. Anyone can think of what this might be? Maybe a four-legged animal with antlers, like maybe a moose. So this is a model I designed, I think, my freshman year as an undergrad.

AUDIENCE: What happens when you have the squares inside the squares?

JASON KU: That's an excellent question. First, I want to answer one other question before I get to that one. Here, these polygons, these squares, you could think of maybe putting a circle in them. And the square is taking up more paper than the circle, and so the flap that this represents would be the largest circle that would be fully contained in that square. And that would be the length of that flap.

What does it mean to have instead of this point separated off from the rest of the model have a line separated off from the rest of the model? Can anyone guess why would you want that as an origami designer?

Well, you might want that property if you want not just a point separated off from the rest of the model, but a line. You might want thickness. It's a qualification of thickness of that flap at the extreme distance away from the model.

And I don't have this example with me. I talked about it yesterday at the OrigaMIT lecture. But let's say you wanted to model a butterfly wing. It's not well characterized by a stick. Its thickness is kind of important. So how I designed a butterfly wing is I separated a line off from the rest of the model, something similar to this, so that I would have enough paper to kind of spread out that idealized single point. I could spread the end of that point to have some thickness and to make a full butterfly wing.

And what you're saying is what does it mean to have these points, these single leaf edges, separated off kind of surrounded by river? You see what that means? Yep, this is just a river. Rivers, again, don't have to go all the way across a model. They can also connect. You're separating these two points off from the rest of the model

by a certain constant distance. So excellent question.

And as I promised before, I want to take look a little bit about the structure of this model. It looks very symmetric, right? And you'd think that maybe it would be well represented by a rectangle of paper instead of a square. How do you fit this into a square by still having this detail?

How do you think this texture was made? Anybody? It's kind of just pleating the paper back and forth. If you've ever taken a sheet of paper and pleated it to form a texture, kind of a one-dimensional problem, but you're pleating it. But after you pleat it, it's smaller.

If we take a look at the crease pattern here-- This is actually a crease pattern to an earlier version of this model. This is slightly less detailed, if you can imagine, than this model right here. What do you think this is? Maybe the scales, right? This is the head region.

We can actually do a rough version, perform a rough version of this kind of hinge crease representation, and get an idea for the structure of this model. So here, we see the tail. I'll talk about this later. We have the two back feet separated off from the rest of the model by a distance. That's this distance here. Two more feet. This is kind of the neck region. And here's the head.

OK. So this looks kind of weird. I haven't really been specific about the details here. But what does that pleating do? Well, it shrinks the useful area of the paper, because I pleated it. So that's why here the length of this flap is this distance here. That's the length of the tail.

But when I make pleats, this thing shrinks. And it actually shrinks to this distance. This whole thing is cut in half. So we make these pleats, it shrinks, and then it can lie along this segment.

Then this area here also shrinks by half. So the length is here. And it is able to cover this aspect, this part of this middle river with texture. Please ask questions,

because this is complex.

AUDIENCE: [? What's the ?] distance between the front and back legs?

JASON KU: Yes. So this is the distance between the front and back legs. But we have to cover it with texture. There's no texture here. So what we do is create this extra flap here where the back legs are with a length of half of this, and cover it with texture. So that's what he's done here. And so the same goes for here. It's not quite half down here, but this covers up the rest of that section.

And there's actually some overlap so that they can mesh correctly. Then here, we have enough paper to provide texture to the neck region, and then there's the head. It's kind of an ingenious way of actually the top and bottom, this top texture and this bottom texture, folding up onto this line segment which represents the length of the dragon, and still having space for these toes and feet. It's an ingenious way to distribute the paper, in this case.

Here we can understand another reason why we might want to separate a line off from the rest of the model, because then that line has some thickness, you have a certain amount of space out there, and you can actually then create more points from that line being out at a certain distance. We can create a number of little points, which are then toes. So I thought that was pretty cool. One of my favorite examples of structure.

AUDIENCE: [INAUDIBLE]

JASON KU: These were all drawn by hand using a program very similar to Adobe Illustrator. So yeah, it's very tedious, and lots of copying and pasting.

But you should see the more complicated version of this pattern. Because as you can see on this model here, there are actually scales on the feet part itself. These claws actually are longer in proportion to everything else in the model. So we actually add some more things. There's also a strip of paper here that has spines on the back. This crease pattern doesn't represent those things. So this is a simplified version, if you will.

AUDIENCE: What was the starting size of the paper?

AUDIENCE: Yeah, how big is it?

JASON KU: It's actually an amazingly efficient use of paper. The length of the dragon is pretty much this length right here, which is actually quite impressive for the amount of detail there is. The shrinkage factor is something like to the length of the squared to the length of the dragon is not even a half.

The overall structure of this model is actually quite simple. The model itself is maybe about this big. So I'm guessing the size of the square was something like a meter, if not a little larger. It's a long time to work with a single sheet of paper.

All right. So we're going to very quickly, maybe for the next 10, 15 minutes, go through a design example of a crab. And so we're going to kind of go through it quickly. To help you do your homework, I just want to let you know about some details of TreeMaker that might be useful to you to be able to make a cleaner or nicer crease pattern.

So to go to a TreeMaker example, I'm going to open up TreeMaker. I need to bring TreeMaker over here. So we have TreeMaker. And let's say we want to make a crab. So how do you want to draw this tree? Maybe I'll just draw the tree that we had before.

First we have four legs all of equal length. We could have them all coming from the same spot. But traditionally, if we take a look at a crab-- That's a cartoony version of a crab, but we see that these maybe our axis of our model is here.

These legs actually don't need to split at the axis. We could actually model this as in the tree, maybe we have our body segment, and maybe we separate these four flaps off from the axis by a certain distance so that we actually can save paper. We don't have to make each one of these flaps this long. You see?

So I'm going to add a little line segment there. Repeat on the other side. You get the

idea. Then maybe you have some modeling of the thickness of the model. Then we have claws.

One nice thing about this is we could view just the tree. That might make things a little easier. There's lots of these view characteristics that we're going to take advantage of. And maybe we want to represent the eyes.

Now, the lengths of these edges in the program don't really mean anything. So take that into note first. You actually I've to click on each edge and specify its length relative to all the others. So maybe we want to make the claws half as long as the branch connecting them.

Bear with me. There's no good way of automating this process at this point.

And maybe we make the eyes-- they're pretty short-- so we maybe make them a quarter of the length of those. The body segment, I don't know. Also a quarter. This is really kind of arbitrary, but you can play around with these dimensions. And these guys, also a quarter. And the back legs can also be one. Something like that.

All right. When we've got that, we see that we actually have circles there. Now, these circles are kind of crossing. We don't want that, because paper can't go to two points at once. What we can do now is scale everything. So it tries to pack all the circles such that none of the conditions are being violated.

So this is a valid packing, except these points in the middle here, this whole polygon is constrained. The green line segments here are active paths. Basically, the distance between these points on the tree and these points on the paper are minimized, or they're equal. So there must be a crease there. That is a key statement of uniaxial bases, is that there must be a crease along active paths.

Now, these two points can actually stand to get larger. That's evident to the fact that we can move these around and it's not violating any conditions. Well, if I move it over here, it's violating a condition. Whenever a condition is violated then you have these red lines that yell at you.

But we can move this around in this area without violating any conditions. So it's not happy. It's not completely crystallized or well-constrained, so it's going to yell at us when we try to build the crease pattern. TreeMaker was not able to construct all polygons because a polygon was either non-convex or contained one or more nodes in its interiors. So these have nodes in its interior, so was not able to fill in this polygon.

What we can do about that is we don't mind if these points get a little bigger. Or we could add an extra point. So we never modeled a body segment here. So maybe we just add in a body segment. So scale everything here.

We still have this problem. This guy is unconstrained. So what I'm going to do is make this guy a little bigger by selecting the node and the edge. You have to do both. I can go here and scale just this selection. And it'll increase it by itself. Actually, nicely, this is somewhat of a symmetric crease pattern, which didn't occur before.

So you see these lighter edges of the tree are fully constrained edges. These darker ones are not fully constrained edges. So this guy can actually also increase a little bit. So I'm going to scale selection. Now everything should be good. I can build the crease pattern.

Guh! It built it, so whatever.

So this is a foldable crease pattern that will form what we want it to. We can also go to this creases view, and it will show the creases of the model. It was not able to find valid mountain-valleys. Anyway.

So to make this cleaner, you might want to deal with symmetry. So there's an ability to select diagonal symmetry and either add conditions to make a node fixed to the symmetry line, so add additional constraints to our system to make it cleaner. We can fix them to the symmetry line. We can fix it to the corner or the paper edge, fix to any arbitrary position. Or we can select two nodes and pair them about the symmetry line, which is a very useful thing to do.

I don't know if it will yell at me again. Yeah, it didn't do anything. What I'm going to

do is go in here. There's lots of things you could do here. We could perturb all the nodes, so they move if by a slight distance. And maybe if we try scaling it again, it'll find a valid solution.

This was unfortunate. Scale everything. Kill the strain on this. Yes?

AUDIENCE: I'm confused. Is it failing because the problem is over-constrained or under-constrained?

JASON KU: It's not failing because of either. It's failing because certain creases get very close together. Now it's failing because it can't find a correct valley-mountain assignment for the crease pattern. So it's able to build the creases fine. So build crease pattern, fine. It just wasn't able to construct a mountain-valley assignment, which in the creases view would usually give you mountain and valley assignments.

AUDIENCE: That means it's not possible?

JASON KU: It couldn't find it. It's not that it's not possible. It just couldn't find it. I want to say one other thing. Kill the crease pattern. We have a polygon bounded by active paths that's not triangular. But we can actually split it any of these up into triangles.

I'm just going to mention that you can do this. You click on one of these polygons, go here, stub, triangulate tree. It adds random points. And now all the polygons are triangles, and that's much easier to fold. Or not. That's how you would do it.

Anyway, I'm running out of time, so I'm going to go back to the presentation. But if you need any help with these, or the tutorial with this program, I'm around. You can contact me through the OrigaMIT website, or you can come to it an OrigaMIT workshop on Sunday and ask me questions then.

So I'm going to quickly just go right back to the presentation. Play slide show. Nope. Technical difficulties. Play slide show.

So that was the example of TreeMaker. Here's an example of a non-TreeMaker example that I designed this weekend of a crab. I actually designed this model it

after I had drawn this picture. And I wanted to incorporate some of the elements of this picture into my design process.

So one of the first I actually started was designing the back so that it would have this kind of structure with this polygon there, kind of a Komatsu-like design process, and trying to make the final form polygons and incorporate those into my crease pattern. So that's what this area is right here.

It has a very similar structure to the tree we drew already. We have the four points for the legs, the body segment. These are going to be the eyes. And so there are some extra points just to make things easier to fold. The claws. Here's the body segment.

There's these extra two things on either side. And I made those so that there could be an underbelly to the crab, and that I could add some texture in and things like that.

But you can see some of the constraints that I've put on it are that I want this to be 22.5 degree folding, which is hard to implement in TreeMaker. I've also shown some of the thinning to make these points thinner on the right.

So if we actually pick out the tree for this hinge representation, we get something that kind of looks like this. So we have our legs. Here's our body segment. Here are the flaps that I make into the underbelly, eyes, and the claws.

And here's the folded proof of concept version that I folded last night. It's actually a really crappy picture. I apologize. But it's up here. I fold it like 10:00 last night, because I thought it would be useful for you guys to see what a folded one might look like.

But it actually turns out to be somewhat non-uniaxial. And you can see some of the texturing on the underbelly, if you take a look at it and come up here. So that's kind of describing some of the design process of a real work that uses the concepts of uniaxial bases, that I can make this hinge crease representation, but then use some shaping to modify it.

If you're interested in learning about anything related to origami, there's an excellent online forum that you can ask questions or show off work that you do or anything like that. And if you want to do something slightly more local-- this is shameless self-promotion-- but the origami club at MIT welcomes you with open arms. We meet every Sunday in the Student Center from 2:00 to 4:00 PM. You can find all sorts of details on our website. So that's about it.

AUDIENCE: Are those origami letters?

JASON KU: Those are, yes. Each one of these letters was a model. They're all the same model I designed, in which you have a 3 by 4 grid of flippable squares of color change that you can flip to either be in the all white state or all black state. And you could also do some of these half pixeling.

But you can basically make any of these letters-- I have a whole alphabet of things-- from a single model. I was lazy. I didn't want to design 26 models. I just want to design one model. So that that's what those are.

So that concludes the lecture. We're just about at time.