Receivers, Antennas, and Signals – 6.661

Issued: 4/08/03

Problem Set No. 9   Due: 4/17/03

Problem 9.1

Use the channel capacity theorem, $C = B \log_2(1 + S/N) \geq R$ in combination with the definitions: $S = RE_b$, $N = N_oB$, $N_o = kT$, and $Q = R/B$ to:

a) Show that the minimum achievable energy required to communicate one bit over any channel is $E_b \cong kT$ [J] in the Rayleigh-Jeans region. [Note, for interest, that when $hf > kT$ we have the option of timing the release of photons with great precision, so one photon could convey many bits, based on its time of arrival. To get the correct bound in this case we could replace $N_o = kT$ with the correct Planck expression and repeat the derivation above.]

b) Find the approximate range of values for Q that permits us to operate within one or two dB of the minimum E_b.

Problem 9.2

The 7-bit "perfect" Hamming code discussed in the notes corrects one bit in seven using three parity-check bits. It can be used in situations where $P_e \ll 1/7$. Suppose we are using 24-bit blocks and we are willing to allocate up to half of these bits to parity check purposes. Approximately how many errors per block can we correct with this bit budget? Hint: a calculator/computer that computes factorials up to 24 would help, but is not necessary.

Problem 9.3

The perfect rate-4/7 single-error-correcting Hamming code for 7-bit blocks is to be used for channel coding. Assume that the probability of channel bit error P_e is given by the figure below.

What is the approximate coding gain $G_c(dB)$ if the original $P_e = 10^{-3}$?
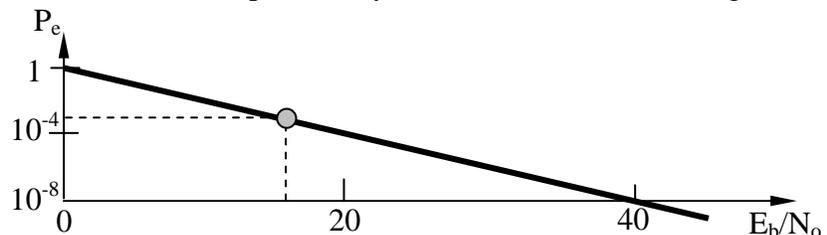
Problem 9.4

Assume Figure 4.6-10 in the text models the processing problem for a 1-Mpixel CCD digital camera, where h(t) is implemented in the user's PC after the pictures are downloaded. We wish to define h(t) for a commercial software package designed to work with this camera, but in order to solve for the optimum h(t) using some error-minimization procedure we shall need to identify quantitatively each of the other elements in the system.

a) Please sketch nominal shapes for f(t), g(t), i(t), and r(t) for this problem and align them on a single graph, noting that we are now in the spatial (or angular) domain, not the time domain. Indicate the physical origins of each of these functions.

b) What physical processes determine $n_1(t)$ and $n_2(t)$ here?