MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

Receivers, Antennas, and Signals – 6.661

Problem Set No. 2 Issued: 02/11/03
Due: 02/20/03

Problem 2.1

The cosmic background radiation temperature is 2.7K and the average temperature of the terrestrial atmosphere (we approximate the atmosphere for this problem as a single homogeneous slab) is 250K. The zenith transmittance of the atmosphere at the frequency of interest here is $e^{-\tau} = 0.6$; τ is the "optical depth".

a) What is the brightness temperature T_{up} seen by a satellite ground station when it looks straight up? Hint: the physics of uniform plane wave propagation here resembles that of Figure 2.1-9 in the text for a TEM line--both cases involve only one propagating mode.

b) What is the brightness temperature T_{30} seen when the station looks up at a 30-degree elevation angle? Hint: the path length doubles.

Problem 2.2

A rectangular waveguide can propagate in many modes simultaneously. Since modes are orthogonal in space, their powers add to yield the total power P propagating through a waveguide. The cutoff frequency (below which no power can propagate) for the TE_{mn} or TM_{mn} mode in an air-filled waveguide is:

$$f_{mn} = c[(m/2a)^2 + (n/2b)^2]^{0.5} \quad [\text{Hz}]$$

where a and b are the interior dimensions of the waveguide. If the waveguide is matched at one end at temperature T_o, then each mode that propagates conveys kT watts/Hz.

a) Approximately how many modes can propagate below a frequency of 1 THz in a waveguide that has dimensions 1x2 cm? Hint: a 2-dimensional version of Figure 2.1-5 in the text might help.

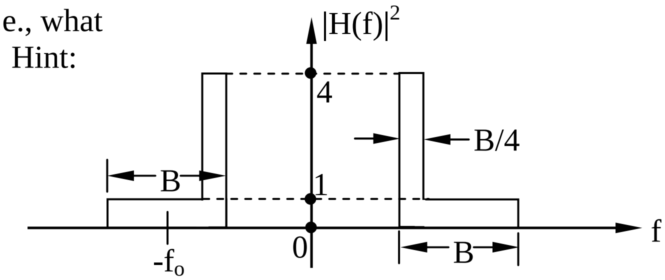
b) If a matched load at one end of this waveguide has temperature 300K, what is the approximate total thermal power propagating down this waveguide at frequencies below 1 THz ($\lambda \cong 0.3$ mm)? A numerical answer is desired. Hint: Waveguide modes can be characterized by a 2-dimensional array of quantum numbers mn, where each mode mn corresponds to both TE and TM modes (see Fig. 2.1-5). If we consider this distribution of modes as a continuum with so many modes per “square quantum number”, and note that each mode has a different propagating bandwidth, a simple integration gives an approximate answer to the total power radiated.

<u>Problem 2.3</u>

Show that Figure 2.1-14 in the text is essentially the same for an arbitrarily shaped brief current pulse i(t) of area e coulombs, where the average current $E[i] = \bar{n}e$. Note the typographical error in the figure; it should be: $\bar{n}e^2 = e\ \bar{i}$. Hint: $\Phi(f)$ is associated with the expected value of the square of the transform of an impulse having some area, as well as with the transform of an autocorrelation function; and the DC and AC portions of $\Phi(f)$ can be derived by separate methods.

<u>Problem 2.4</u>

A certain total power radiometer is similar to the one analyzed in Section 2.2.1 in the text, but it has the bandpass filter characteristic illustrated below. What is ΔT_{rms} for this system (i.e., what does Equation 2.2.14 become)? Hint: see Section 2.2.1 in the text.



<u>Problem 2.5</u>

A certain total power radiometer employs a detector characterized by $v_d(t) = v_i^4(t)$ instead of the standard square-law device. Show that ΔT_{rms} is degraded by a factor of $(4/3)^{0.5}$ relative to the ideal square-law result for a boxcar bandpass filter and a boxcar integrator. Hint: Analyze the sampled version of the total-power radiometer, where the sample period is $T = 1/2B$, as usual.