Interferometer Circuits

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Basic Aperture Synthesis Equation

Recall:

$$\underline{\overline{E}}(\overline{r}, t) \quad \leftrightarrow \quad \underline{\overline{E}}(\overline{\psi}, t)$$

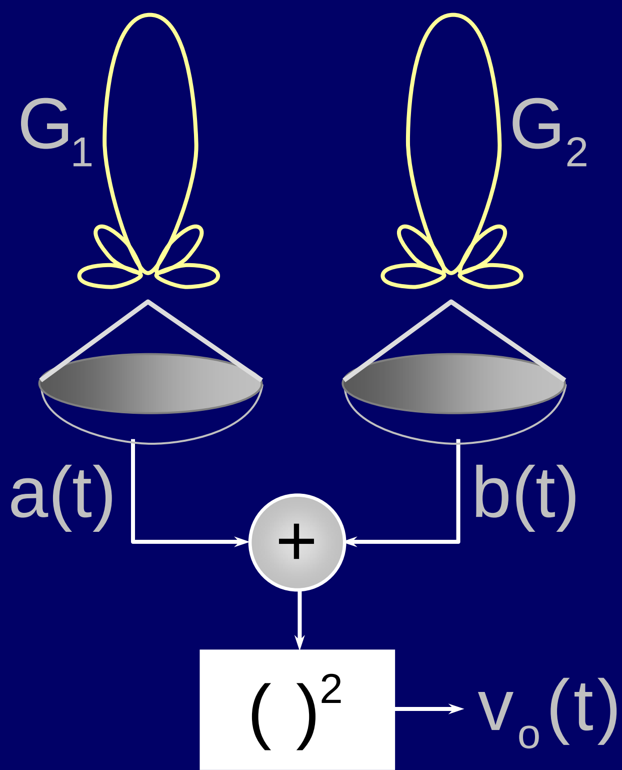
$$\downarrow\downarrow \qquad\qquad \downarrow\downarrow$$

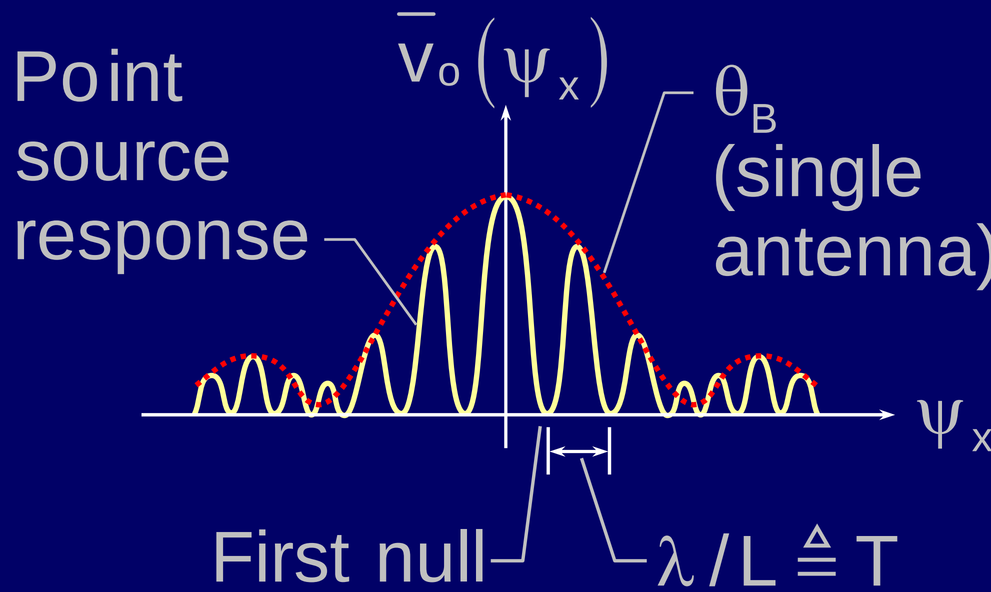
$$E\left[R_{\underline{E}}\left(\overline{\tau}_\lambda\right)\right] = \phi_{\underline{\overline{E}}}\left(\overline{\tau}_\lambda\right) \quad \leftrightarrow \quad E\left\{\left|\underline{\overline{E}}\left(\overline{\Psi}, t\right)\right|^2\right\} \propto I\left(\overline{\Psi}\right), T_A\left(\overline{\Psi}\right)$$

Simple Adding Interferometers

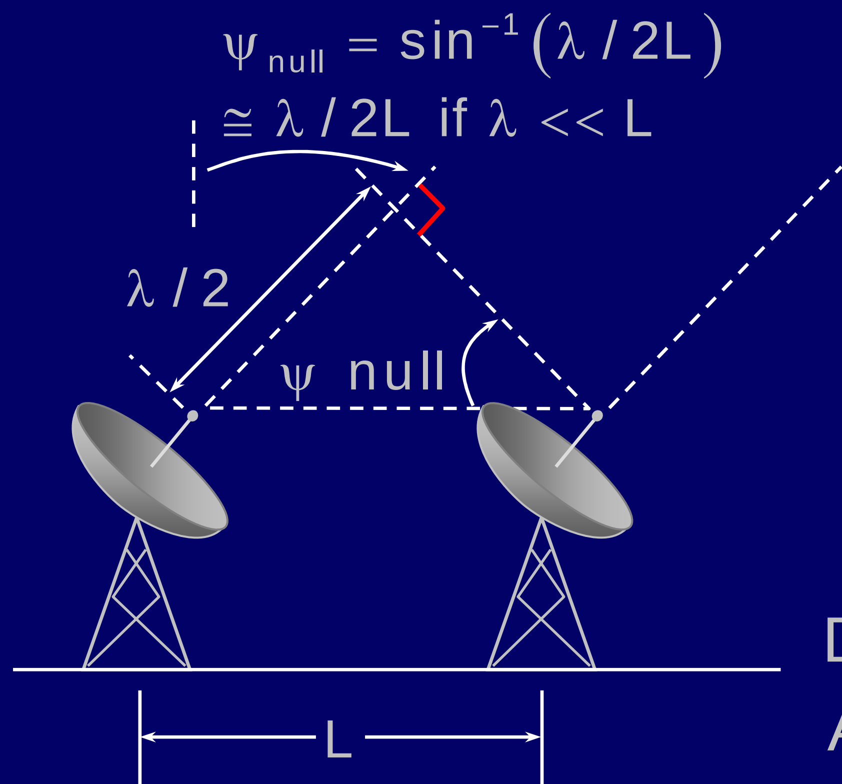
$$\overline{v_o}\left(\vec{\psi}\right) = \overline{a^2} + \overline{b^2} + 2\overline{ab}$$

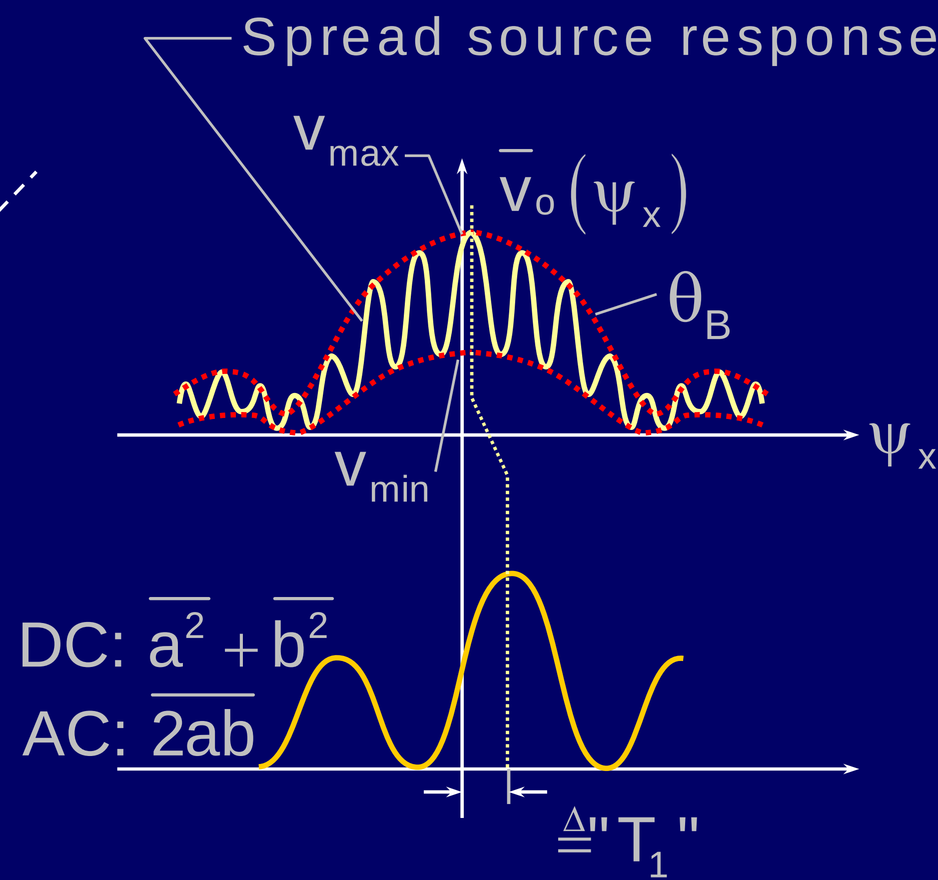
(overbars mean "time average" here)





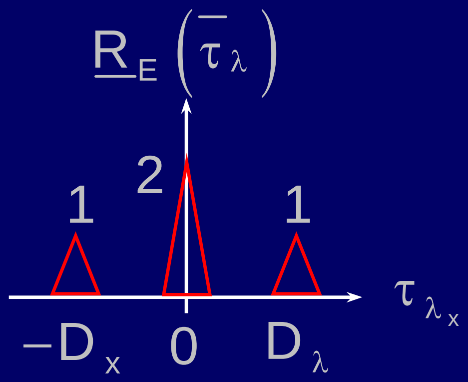
Simple Adding Interferometers





$$\text{Complex fringe visibility } \underline{v} \triangleq \frac{v_{max} - v_{min}}{v_{max} + v_{min}} e^{j2\pi T_1/T} \propto R_{\underline{E}}(\bar{\tau}_\lambda) \leftrightarrow T_A(\bar{\psi})$$

Interferometry as Fourier Analysis



Recall: $\phi_A(\bar{\tau}_\lambda) = \underline{R}_E(\bar{\tau}_\lambda) \bullet \phi_E(\bar{\tau}_\lambda)$

$$\updownarrow \qquad \sim\updownarrow \qquad \sim\updownarrow \qquad \sim\updownarrow$$

$$T_A(\bar{\psi}) = G(\bar{\psi}) * T_B(\bar{\psi})$$

$$\updownarrow \qquad \updownarrow\sim \qquad \updownarrow\sim \qquad \updownarrow\sim$$

$$\underline{T}_A(f_\psi) = \underline{G}(f_\psi) \bullet \underline{T}_B(f_\psi)$$

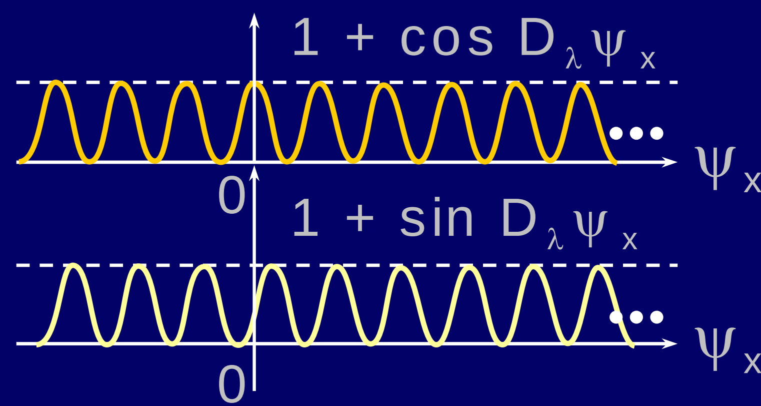
observed = antenna times signal

Example: 2 small duplicate antennas separated by D_λ in the x direction

2-element adding interferometer

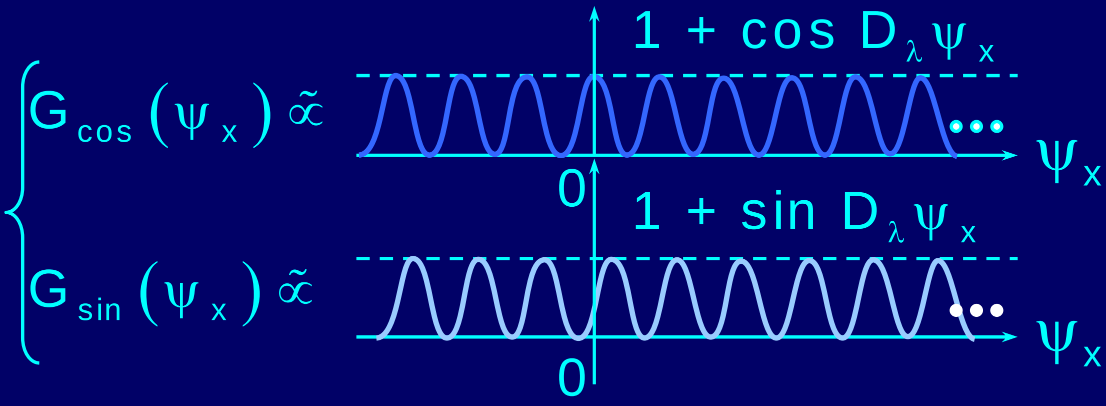
$G_{cos}(\psi_x) \ \tilde{\propto}$

$G_{sin}(\psi_x) \ \tilde{\propto}$



Interferometry as Fourier Analysis

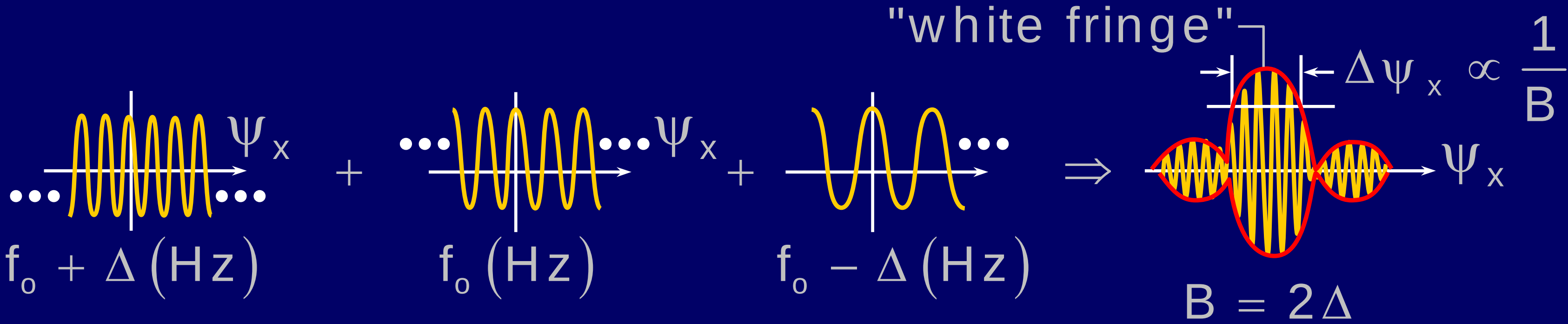
2-element adding interferometer



Interferometer directly measures Fourier components of source

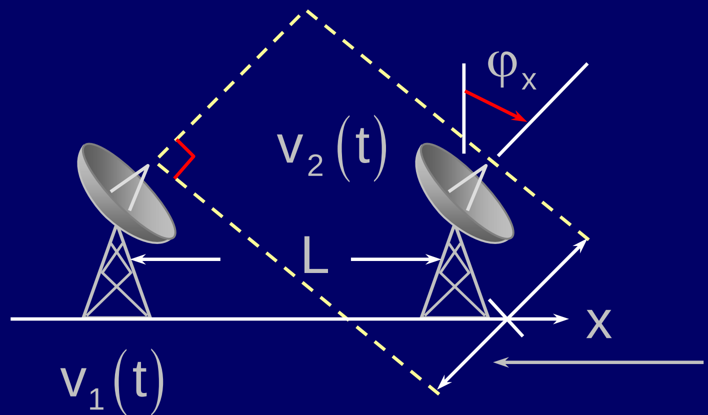
Effect of finite bandwidth:

Fringe patterns for all frequencies (colors) add in phase to create



A delay line in one interferometer arm can redirect the strong white fringe in other directions.

Broad-Bandwidth Effects in Interferometers



$\underline{E}(x, y, t)$ (the "slowly varying" part) may vary rapidly enough that the offset time $L \sin \varphi_x / c$ is significant

$$\frac{L \sin \phi_x}{c} \text{ seconds}$$

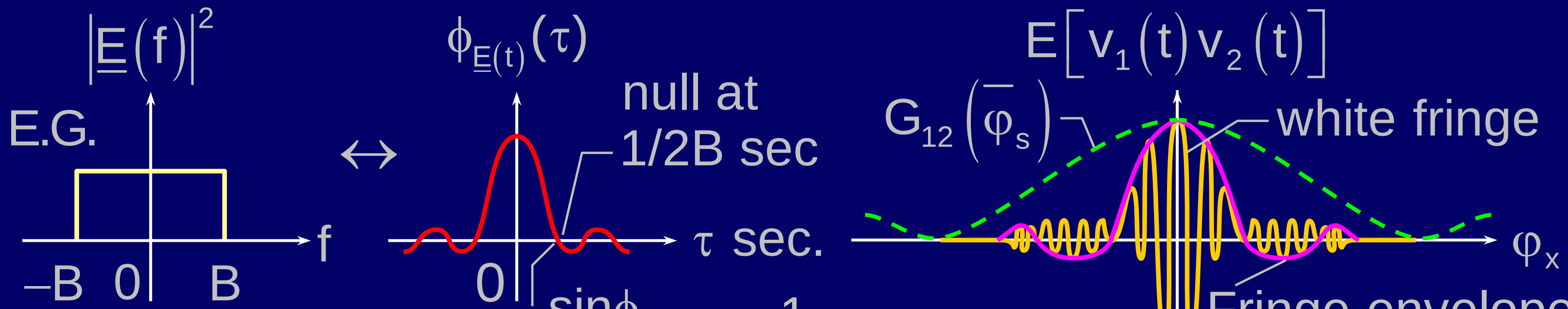
$$E[v_1(t) v_2(t)] = \frac{k_1 k_2}{2} G_{12}(\bar{\phi}_x) E\left[R_e \left\{ \underline{E}\left(x, y, t + \frac{L \sin \varphi_x}{2c} \right) \right. \right.$$

$$\left. \left. \underline{E}^*\left(x - \tau_x, y - \tau_y, t - \frac{L \sin \varphi_x}{2c} \right) \bullet \underbrace{e^{j\omega L \sin \varphi_x / c}} e^{-j\gamma} \right\} \right]$$

$$\left[e^{j\omega(t + L \sin \varphi_x / 2c)} e^{-j\omega(t - L \sin \varphi_x / 2c)} = e^{j\omega L \sin \varphi_x / c} \right]$$

$$E[v_1(t) v_2(t)] = \frac{k_1 k_2}{2} \underbrace{G_{12}(\bar{\varphi}_s)}_{\text{cross-gain}} R_e \left\{ \underbrace{e^{-j^{\gamma}} e^{j2\pi L \sin \varphi_x / \lambda}}_{\text{monochromatic fringe}} \bullet \underbrace{\phi_{\underline{E}(t)}\left(\frac{L \sin \varphi_x}{c} \right)}_{\text{fringe envelope}} \right\}$$

Bandwidth-Limited Angular Response



$$\text{Null @ } L\frac{\sin\phi_{x_{null}}}{c} = \frac{1}{2B}\text{ sec}$$

In broadband optical interferometer all colors contribute to central "white" fringe; sidelobe fringes appear colored.

$$\text{Therefore } \varphi_{x_{null}} = \sin^{-1}(c/2LB) \approx c/2LB \text{ for } \varphi_x \cong 1$$

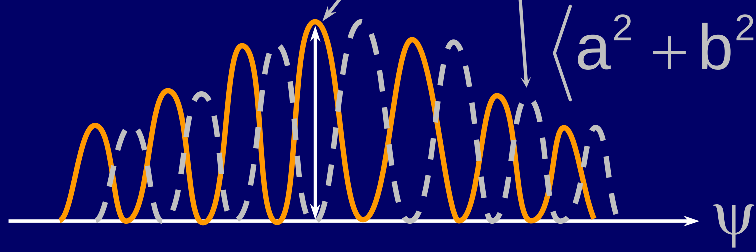
$$\text{e.g. } \varphi_{x_{null}} \cong \frac{3\times 10^8}{2\times \underset{\uparrow\,10\text{-m}}{10}\times \underbrace{10^7}_{10\text{ MHz}}} = 1.5 \text{ radians for } L = 10 \text{ m, } B = 10 \text{ MHz}$$

If $B = 1$ GHz, and $L = 100$ m, then $\varphi_{x_{null}} = 1.5$ mrad $\cong 5$ arc min

$B = 3\times 10^{14}$ Hz and $L = 100$ m, then $\varphi_{x_{null}} = 10^{-3}$ arc sec.

Dicke Adding Interferometer

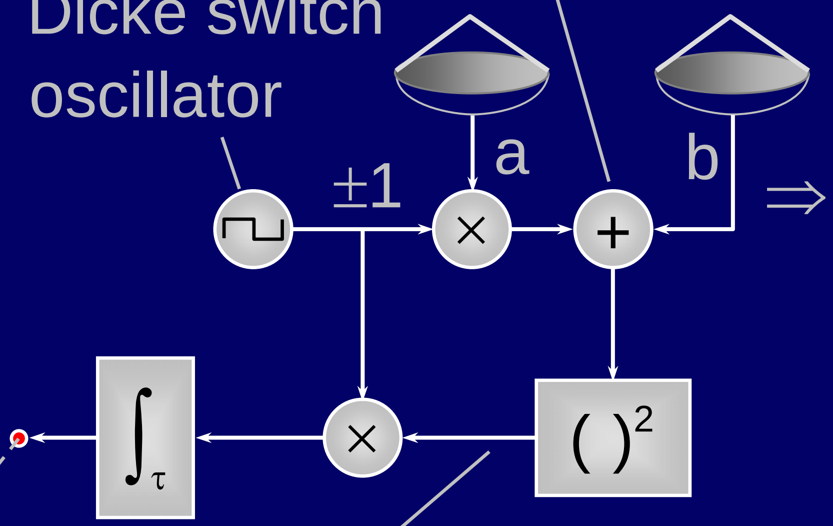
$y(\psi)$ point source response



(Also called "lobe-switching" interferometer)

"Adding" interferometer

Dicke switch oscillator



$$y(\varphi) = (\pm a + b)^2$$

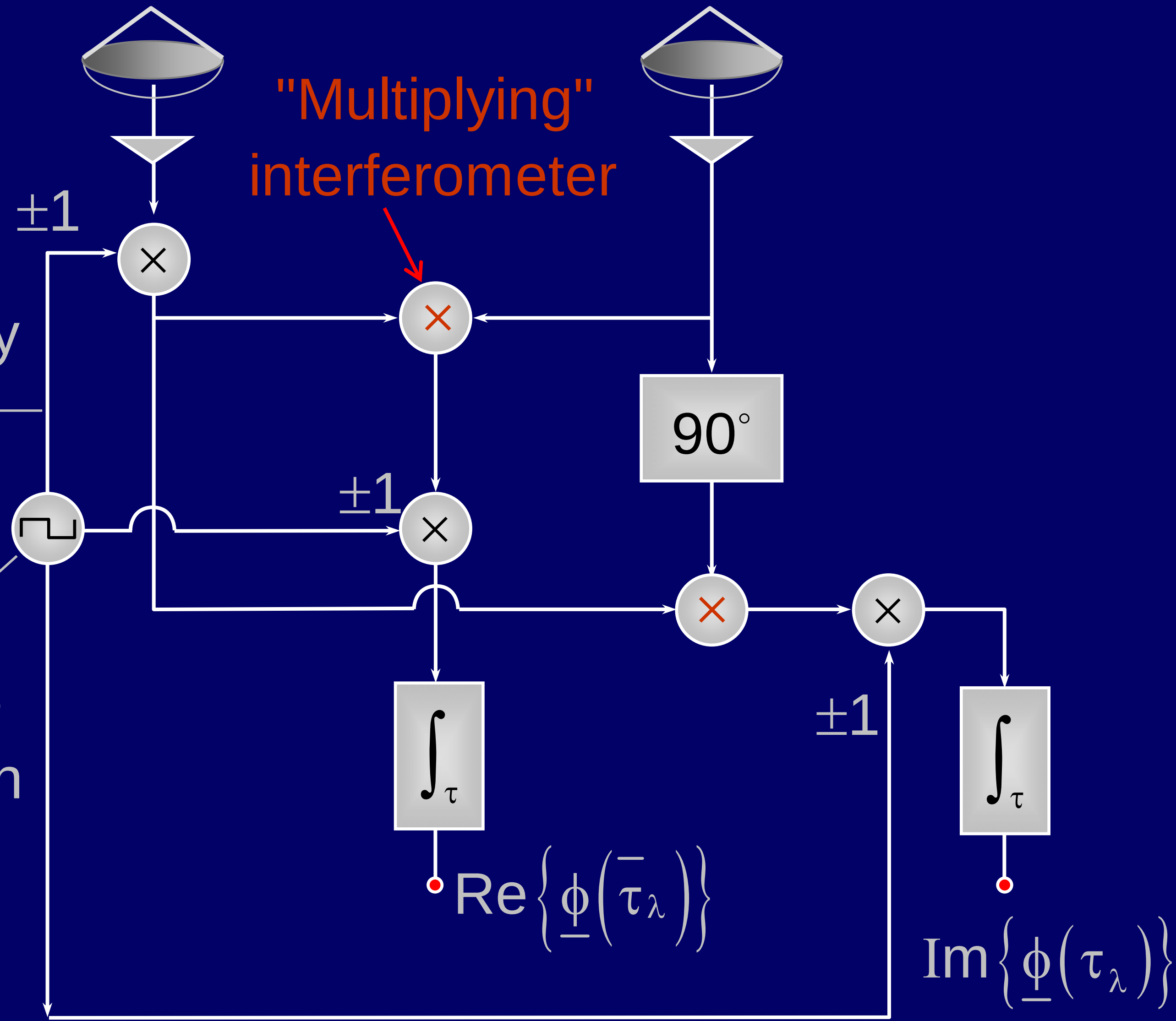
$$\left[\overline{a^2} + \overline{b^2} + 2\overline{ab}\right] - \left[\overline{a^2} - 2\overline{ab} + \overline{b^2}\right] = 4\overline{ab}$$

This circuit cancels D.C. term, leaving only $\langle 4ab \rangle$ as source traverses beam ⇒

Can add second adder and square-law device operating on a and –jb to yield sine terms in Fourier expansion

Dicke Adding Interferometer

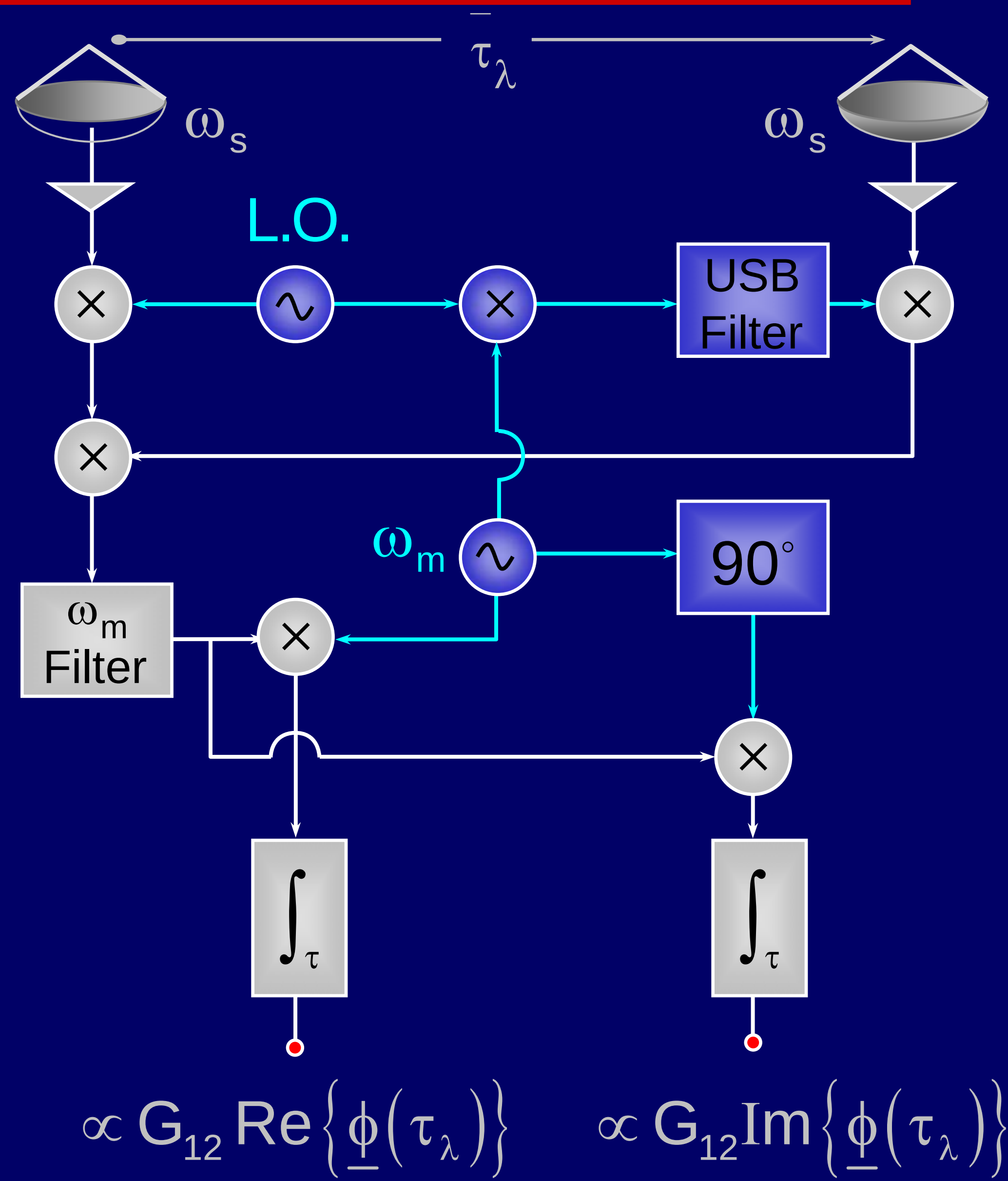
Yields $\underline{\phi}(\tau_\lambda)$ for even a stationary source



Note: If no Dicke switch, have gain fluctuation vulnerability

"Lobe-Scanning" Interferometer

Lobes are scanned at ω_m, demodulated, and averaged to yield $\underline{\phi}(\tau_\lambda)$

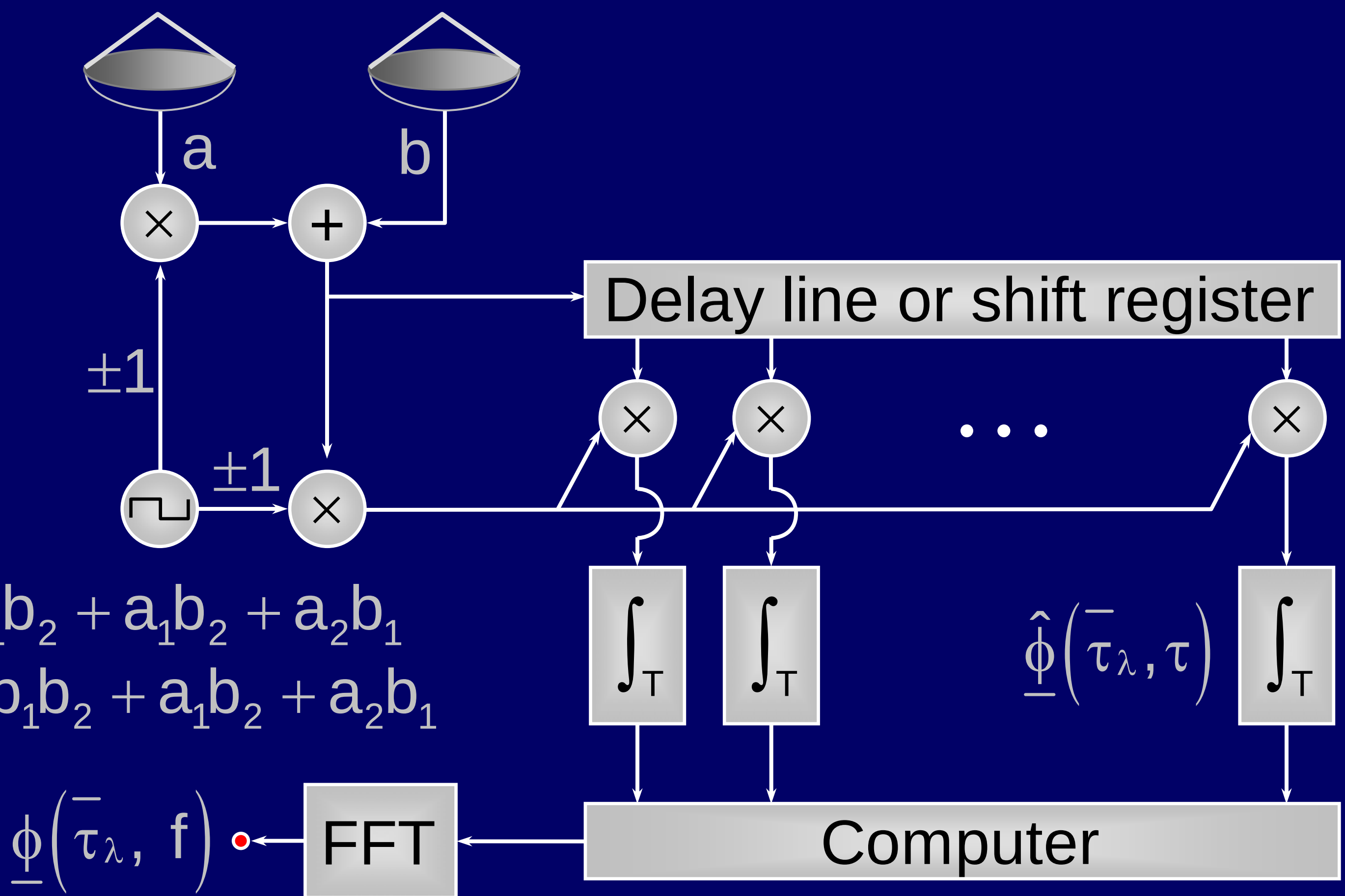


Because all bias and large-scale sources yield no fine-scale response, can integrate long times seeking fine structure, e.g. 10^{-3} Jansky point sources like stars (can measure stellar diameters at ~10^{-3} arc sec)

Cross-Correlation Interferometer Spectrometer

Let $a(t) \triangleq a_1$

$a(t-\tau) \triangleq a_2$



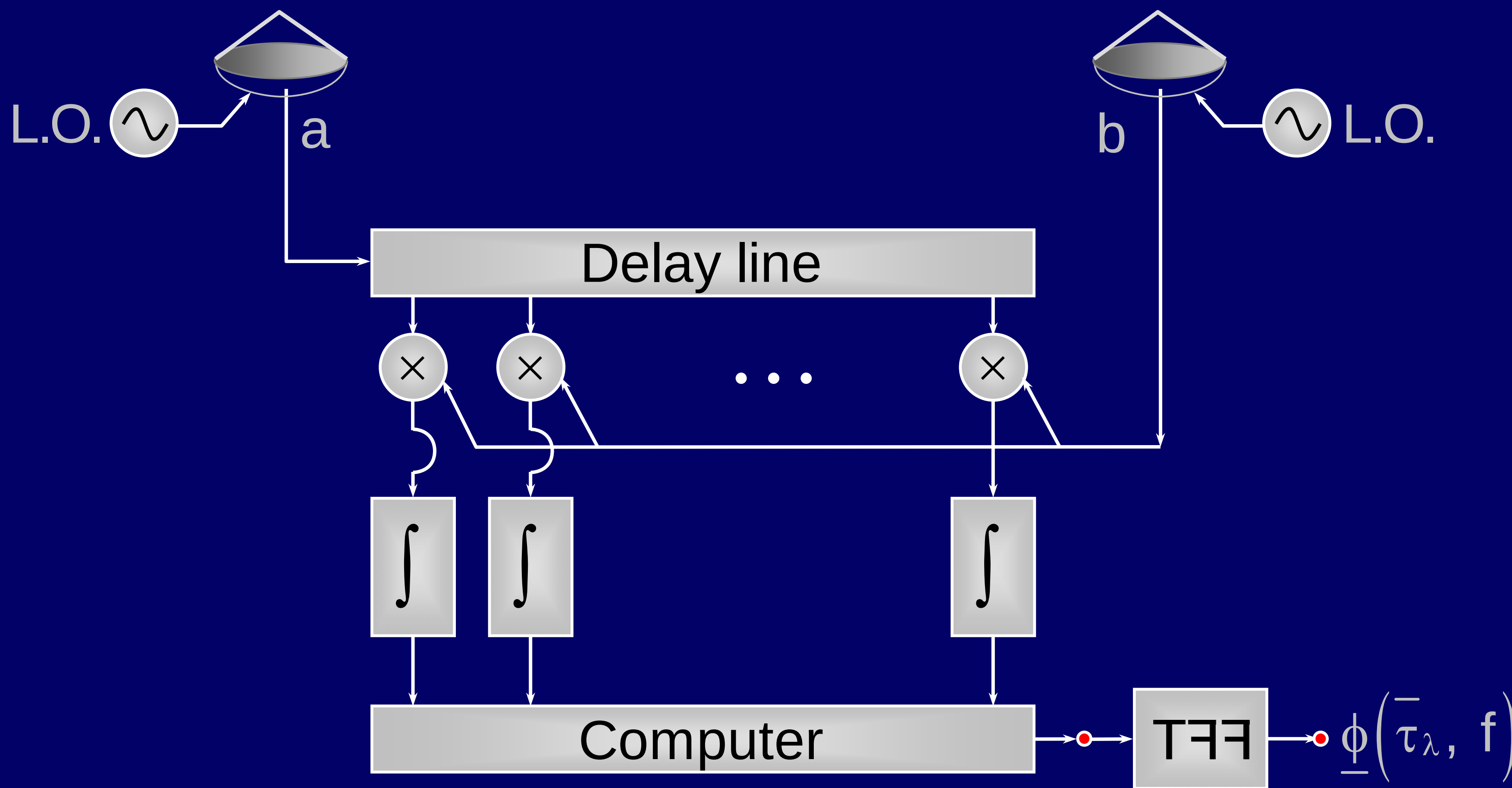
$c_+ = a_1a_2 + b_1b_2 + a_1b_2 + a_2b_1$

$c_- = -a_1a_2 - b_1b_2 + a_1b_2 + a_2b_1$

Note: if $a = b$, $\underline{\phi}(\bar{\tau}_\lambda, f) \to \underline{\phi}(0, f) = S(f)$

if $a \perp b$, $\underline{\phi}(\bar{\tau}_\lambda, f) \to 0$

Alternate Cross-Correlation Interferometer Spectrometer

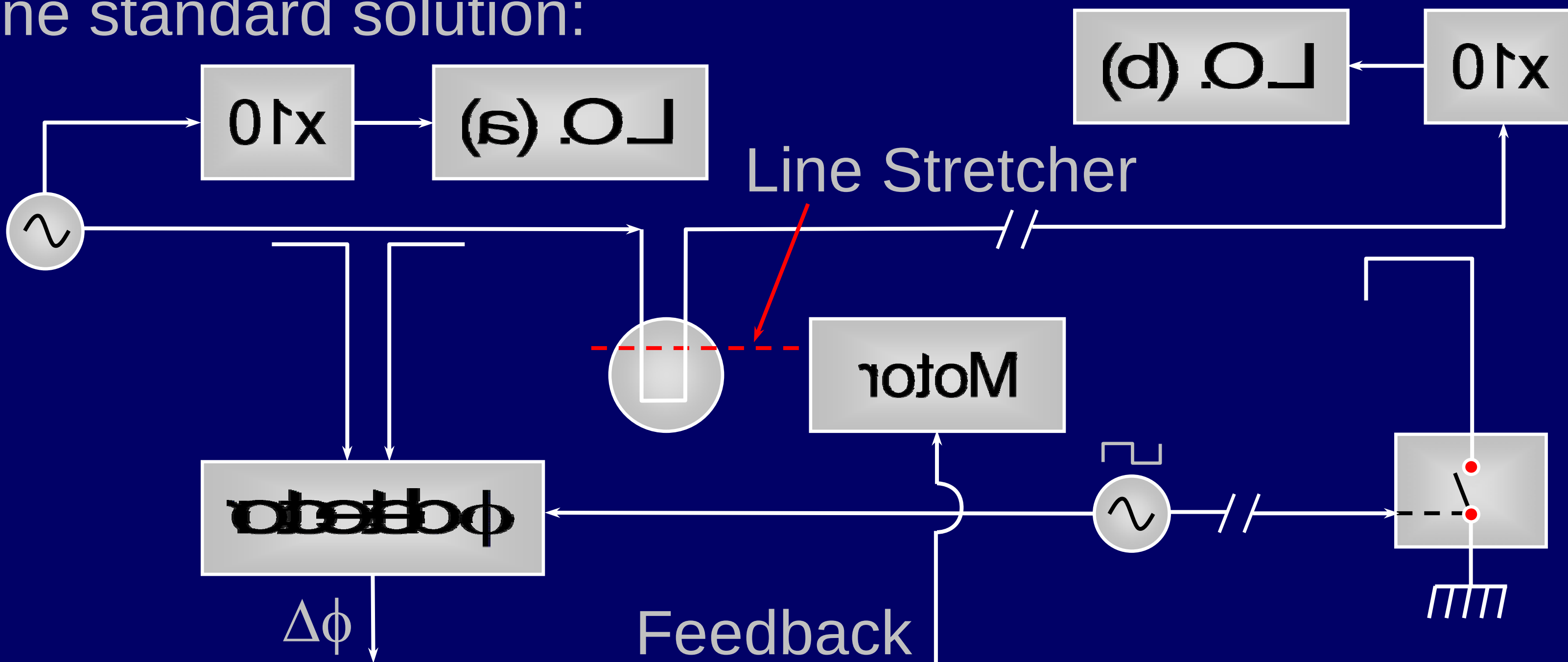


Note: Figures omit down-converters and bandlimiting filters

Mechanical Long Distance Phase Synchronization

For L $>>$ λ, L.O. synchronization can be degraded by random phase variations in path length L between two (or more) sites (due principally to thermal and acoustic variations)

One standard solution:

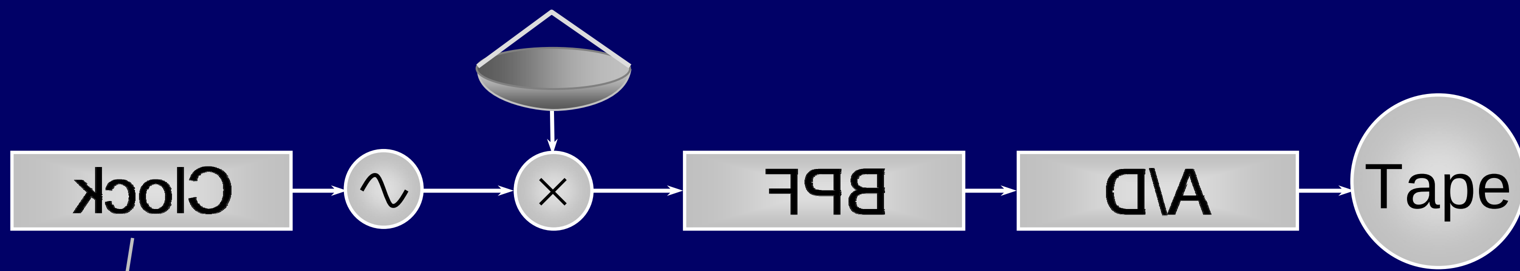


"Line stretcher" varies path so that $\Delta\phi \cong 0$

Communications and telemetry systems can similarly be phase synchronized

Synchronizing with Remote Atomic Clocks

If distance L is too great to synchronize L.O.'s, then we can use a remote clock, e.g. "very-long baseline" interferometry, "VLBI"



Hydrogen maser	$\Rightarrow$	$\sim 10^{-14}, 10^{-15}$
Cesium beam	$\Rightarrow$	$\sim 10^{-12}$
Global Positioning System (GPS)	$\Rightarrow$	$\sim 10^{-8}$ sec
Loran – C	$\Rightarrow$	$\sim 10^{-6}$ sec
Temperature-stabilized crystal oscillators	$\Rightarrow$	$\sim 10^{-5} - 10^{-8}$
Crystal oscillators	$\Rightarrow$	$\sim 10^{-4} - 10^{-5}$

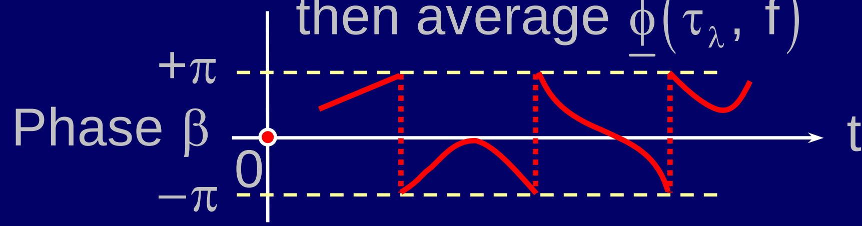
If clocks perfect: cross-correlate to find time offset, correct for it, then correlate the signals, albeit with an unknown fixed phase offset ϕ_0 [unless reference source (in the sky) or phase is available].

If clocks imperfect and delays each way are identical: at site A measure delay between clock B and A; do the same at B, and subtract results to yield twice the clock offsets. Use this offset to align A and B data streams.

Synchronizing with Remote Atomic Clocks

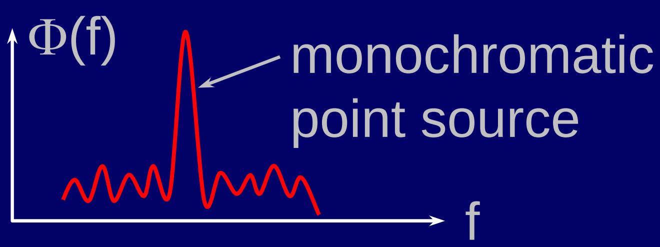
Alternative approaches if clocks and transmissions are imperfect:

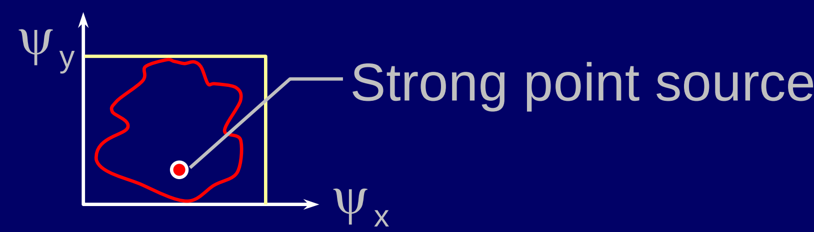
a) Track and correct phase shifts:



Example: A 10^{-12} cesium clock drifts 2π in ~100 sec, so $\underline{\hat{\phi}}(\tau_\lambda, f)$ might be computed for 5 – 10 sec blocks before averaging; then only $|\underline{\hat{\phi}}|$ is known.

b) Same, but set phase using strong resonant line point source,

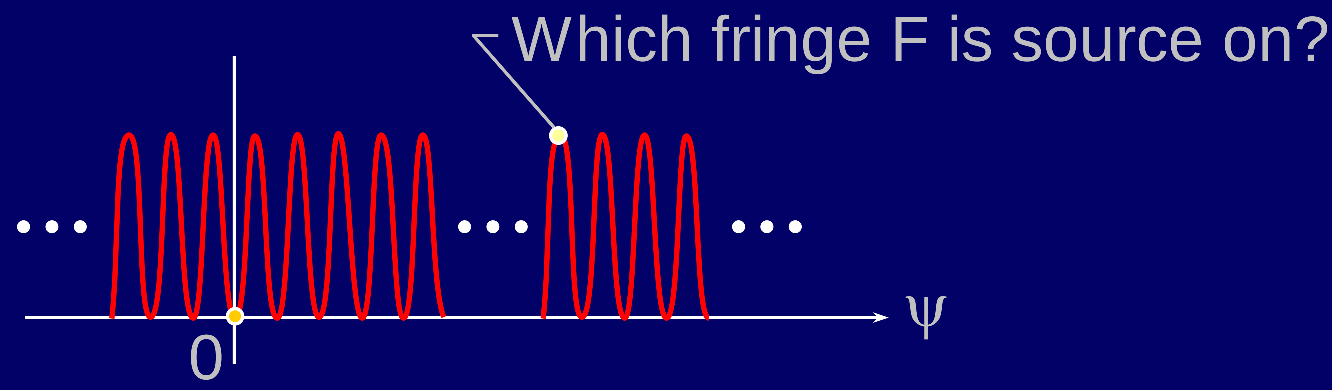




c) or separable point source in space, modulation, etc.

Multiband Synchronization of Clocks

Use multiple frequencies for wideband sources:



Switch across all f's within coherence time of clock.