software studio

object models: math structures

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basic structures

set

› an unordered, duplicate-free collection

tuple

› an ordered sequence

pair

› a tuple of length two

relation

› a set of pairs

graph

› a set (nodes) + a relation (edges)

examples

which of these is a set? a tuple? a relation?

- {1}
- {“hello”}
- (1, 2)
- (1)
- {(1)}
- {(1, 1), (2, 4)}
- {}
- ()
- {()}
- {({}, {})}

first-order structures

a structure is first-order if

- sets and relations aren't elements

which of these is first-order?

$\{1\}$

$\{(1, 2)\}$

$\{\{\}\}$

$\{()\}$

$\{(1, \{1\})\}$

reduction to first-order

a higher-order structure

› teams = {{“alice”, “bob”}, {“carol”, “dave”}}

a first-order structure

› teams = {t1, t2}

› members = {(t1, “alice”), (t1, “bob”), (t2, “carol”), (t2, “dave”)}

this is our approach

› first order modeling (with OMs)

› first order implementation (with RDBs)

operators & relation properties

cardinality of a set

{"hello", "there"} = 2

{} = 0

union, intersection, difference

{1,2} + {2,3} = {1,2,3}

{1,2} & {2,3} = {2}

{1,2} - {2,3} = {1}

domain and range

dom {("a", 1), ("b", 2)} = {"a", "b"}

ran {("a", 1), ("b", 2)} = {1,2}

image

{"a"} . {("a", 1), ("a", 2)} = {1,2}

{"a", "b"} . {("a", 1), ("b", 2)} = {1,2}

transpose

~ {(1,2), (3,4)} = {(2,1), (4,3)}

join

{("a", 1)} . {(1,2), (1,3), (2,4)} = {("a",2), ("a",3)}

a relation R is symmetric if

(a,b) in R implies (b,a) in R

a relation R is reflexive if

for all a, (a,a) in R

a relation R is transitive if

(a,b) and (b,c) in R implies (a,c) in R

a relation R is an equivalence if

it is symmetric, reflexive and transitive

a relation R is a function if

(a,b) and (a,c) in R implies b=c

a relation R is injective if

(a,c) and (b,c) in R implies a=b

and R is also a function