MIT 6.035
Introduction to Shift-Reduce Parsing

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Orientation

- Specify Syntax Using Context-Free Grammar
  - Nonterminals
  - Terminals
  - Productions
- Given a grammar, Parser Generator produces a parser
  - Starts with input string
  - Produces parse tree

Expr → *Expr Op Expr*

Expr → (*Expr*)

Expr → - *Expr*

Expr → num

Op → +

Op → -

Op → *

Today's Lecture

- How generated parser works
- How parser generator produces parser
- Central mechanism
  - Pushdown automaton, which implements
  - Shift-reduce parser

Pushdown Automata

- Consists of
  - Pushdown stack (can have terminals and nonterminals)
  - Finite state automaton control
- Can do one of three actions (based on state and input):
  - Shift:
    - Shift current input symbol from input onto stack
  - Reduce:
    - If symbols on top of stack match right hand side of some grammar production NT $\rightarrow \beta$
    - Pop symbols (β) off of the stack
    - Push left hand side nonterminal (NT) onto stack
  - Accept the input string

Shift-Reduce Parser Example

Stack

Expr → *Expr Op Expr*

Expr → (*Expr*)

Expr → - *Expr*

Expr → num

Op → +

Op → -

Op → *

Input String

num	*	(	num	+	num	)

Shift-Reduce Parser Example

Expr → *Expr Op Expr*

Expr → (*Expr*)

Expr → - *Expr*

Expr → num

Op → +

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Op → *

Shift-Reduce Parser Example

Expr → *Expr Op Expr*

Expr → (*Expr*)

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SHIFT

num	*	(	num	+	num	)

Shift-Reduce Parser Example

Expr → *Expr Op Expr*

Expr → (*Expr*)

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SHIFT

*	(	num	+	num	)

Shift-Reduce Parser Example

Expr → *Expr Op Expr*

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Op → *

REDUCE

*	(	num	+	num	)

Shift-Reduce Parser Example

Expr → *Expr Op Expr*

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Expr → num

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Op → *

REDUCE

num

*	(	num	+	num	)

Shift-Reduce Parser Example

Expr → *Expr Op Expr*

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Expr → num

Op → +

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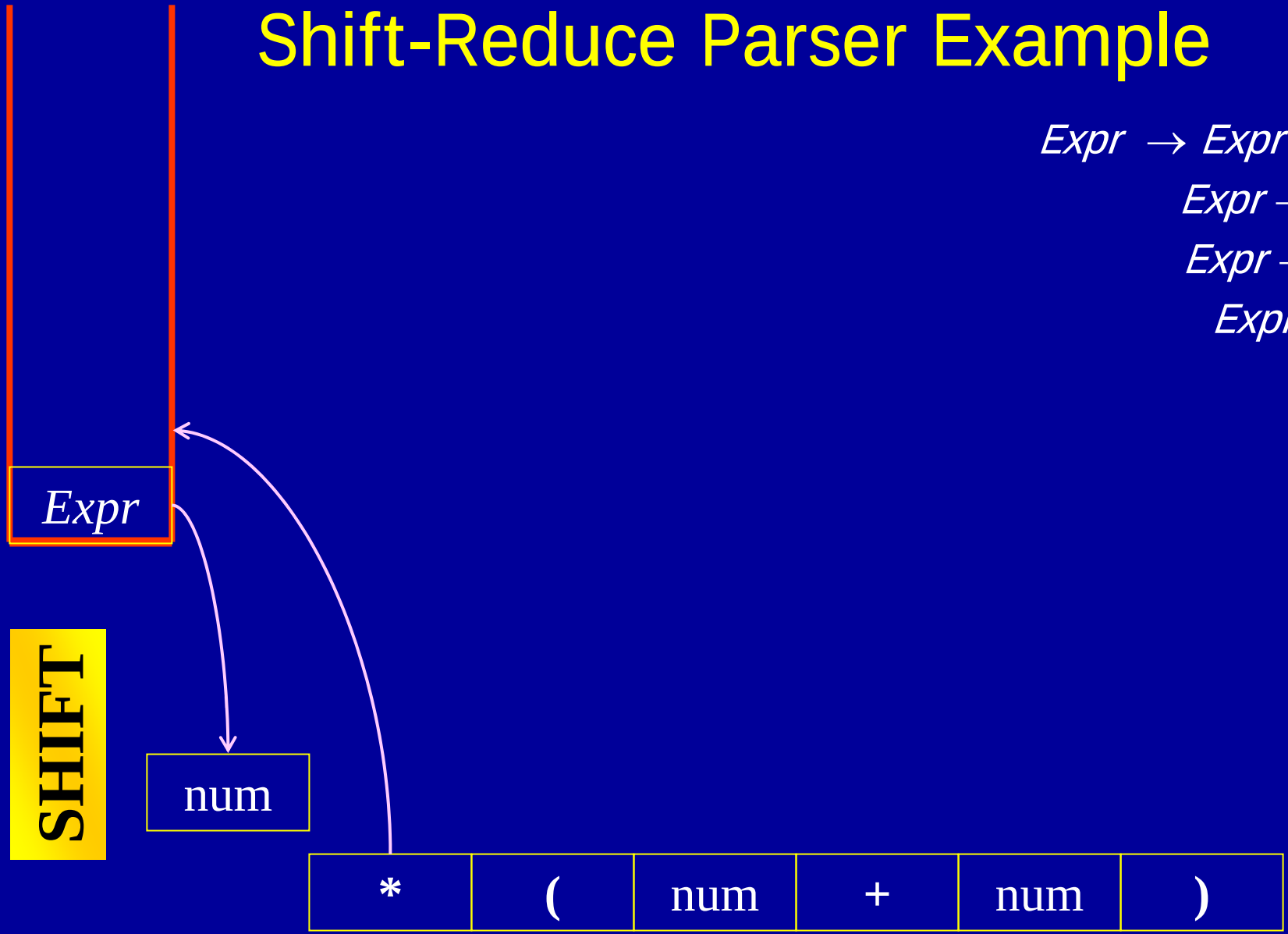

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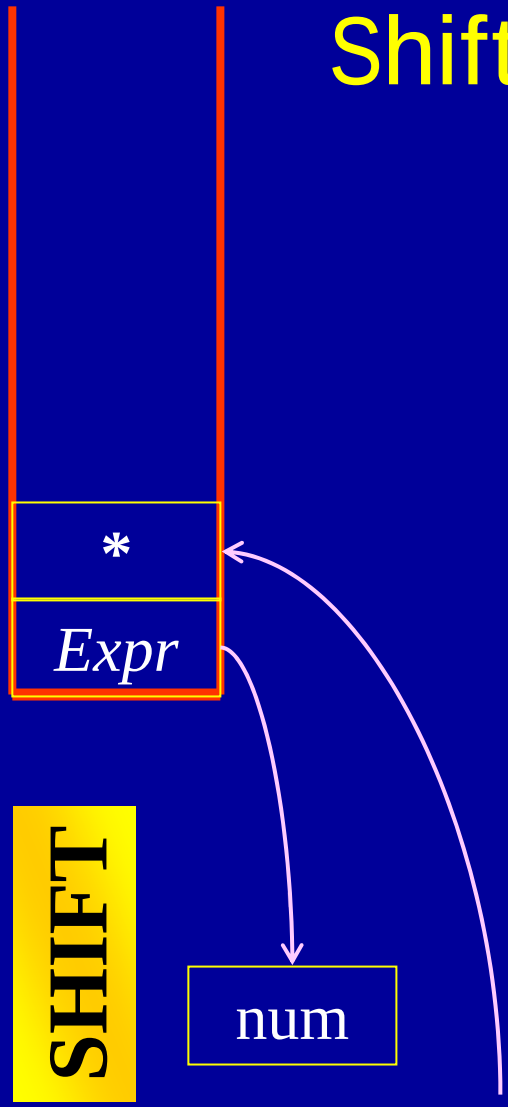

(	num	+	num	)

Shift-Reduce Parser Example

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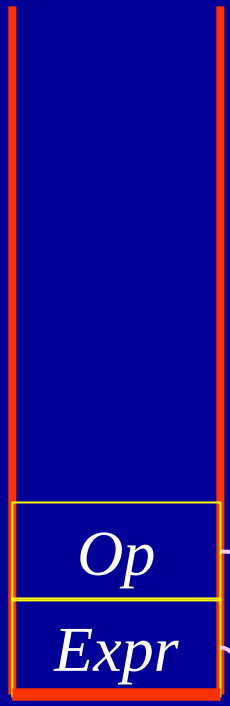
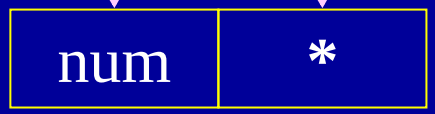
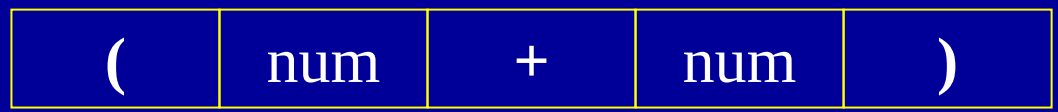


REDUCE





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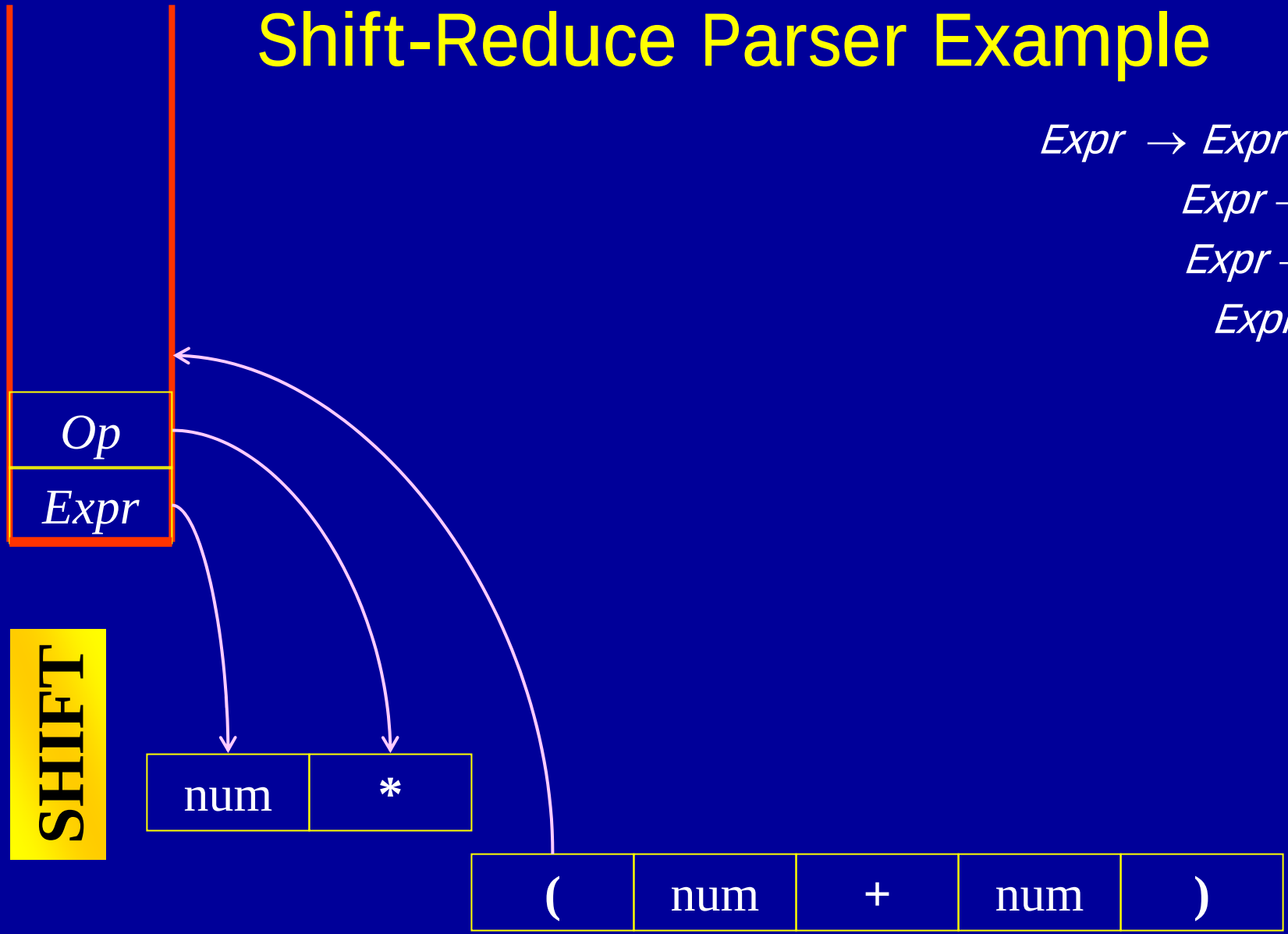

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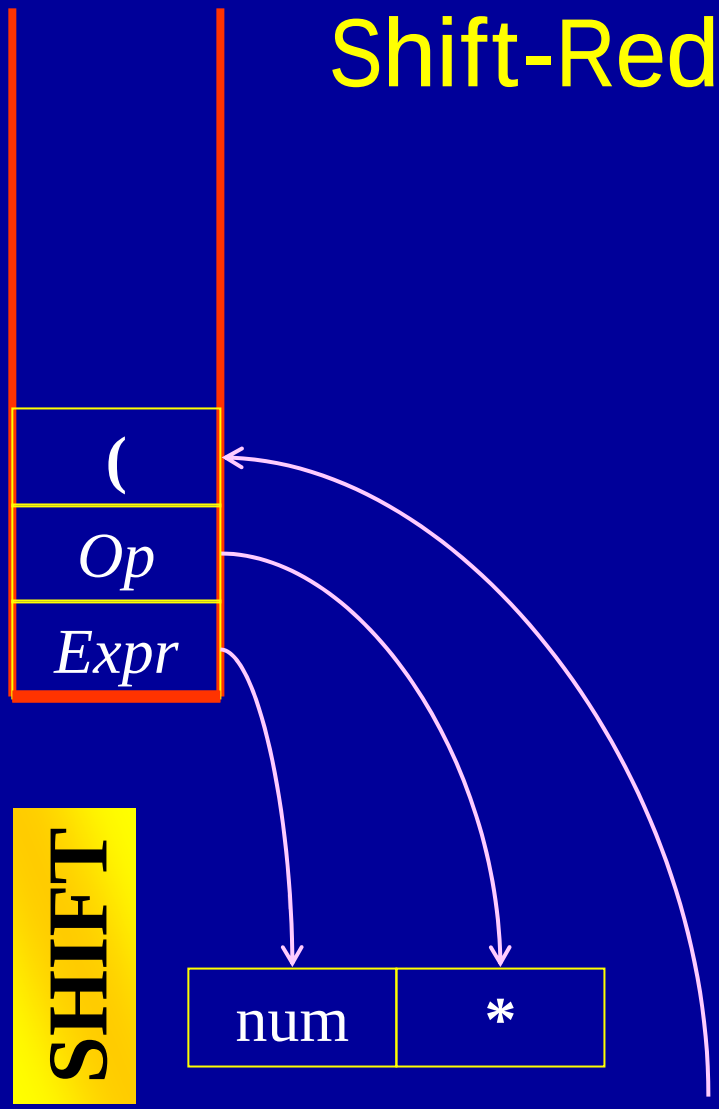

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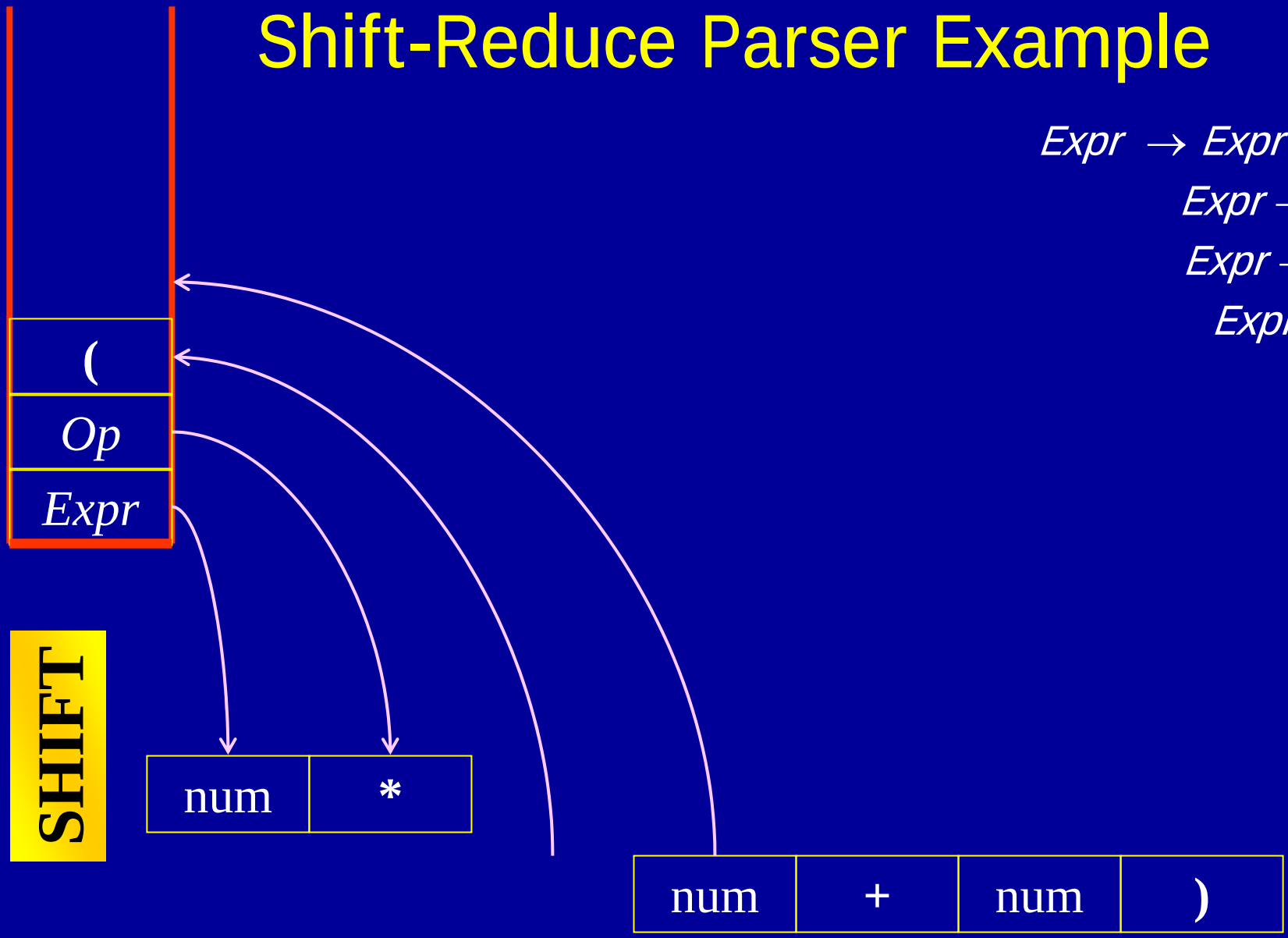

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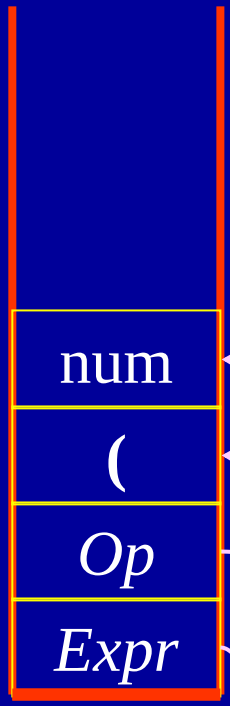

SHIFT

Shift-Reduce Parser Example

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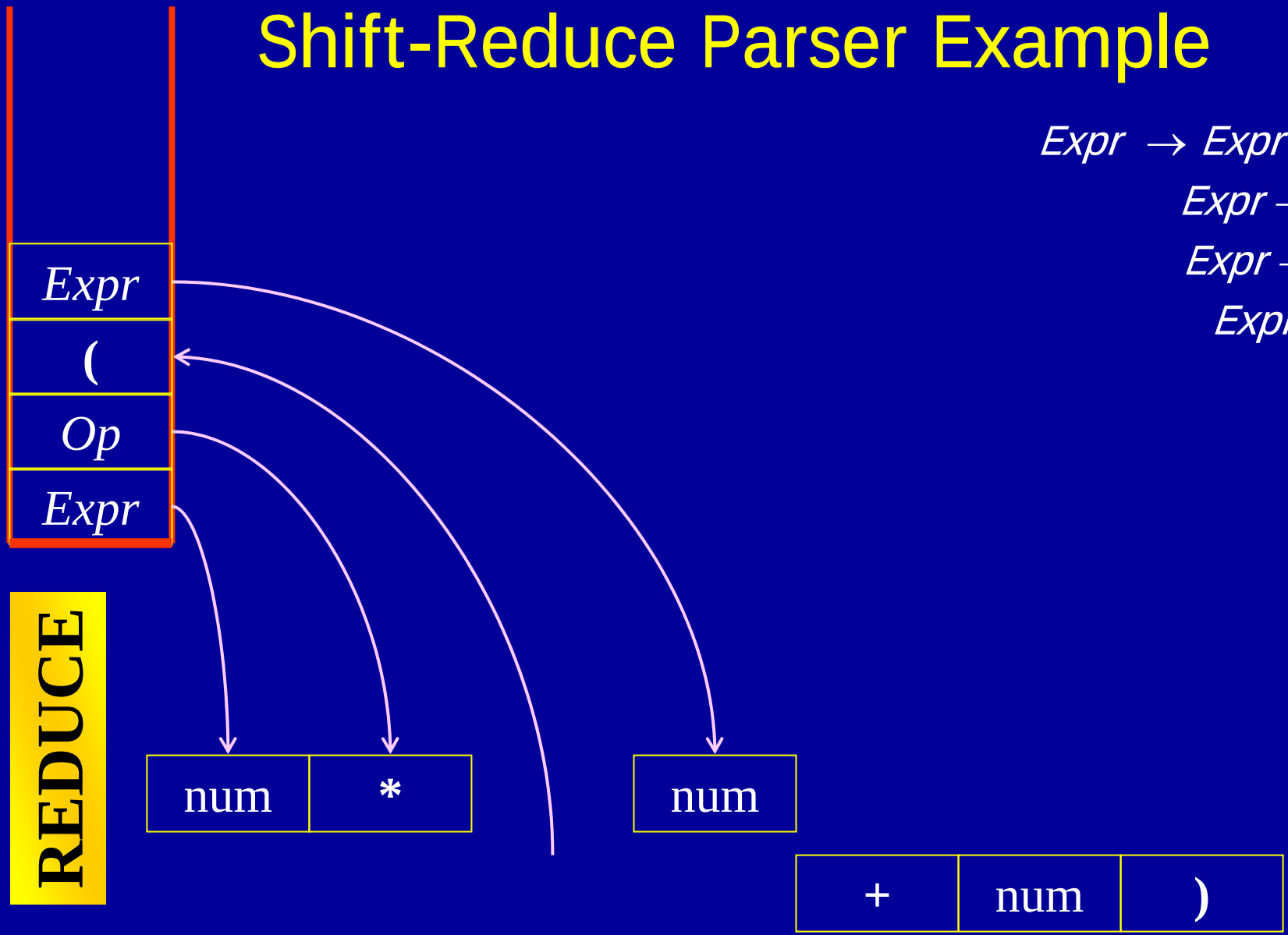

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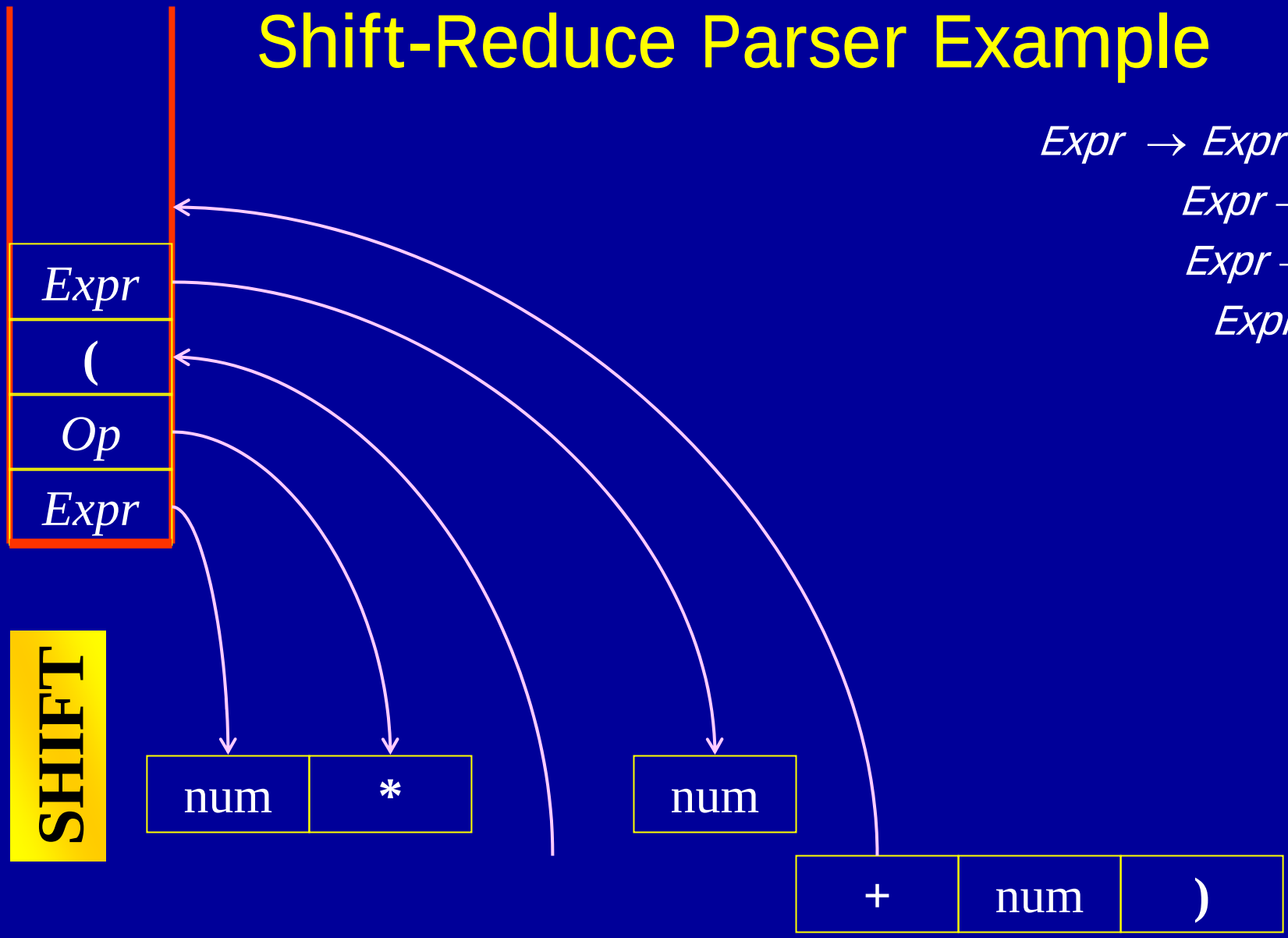

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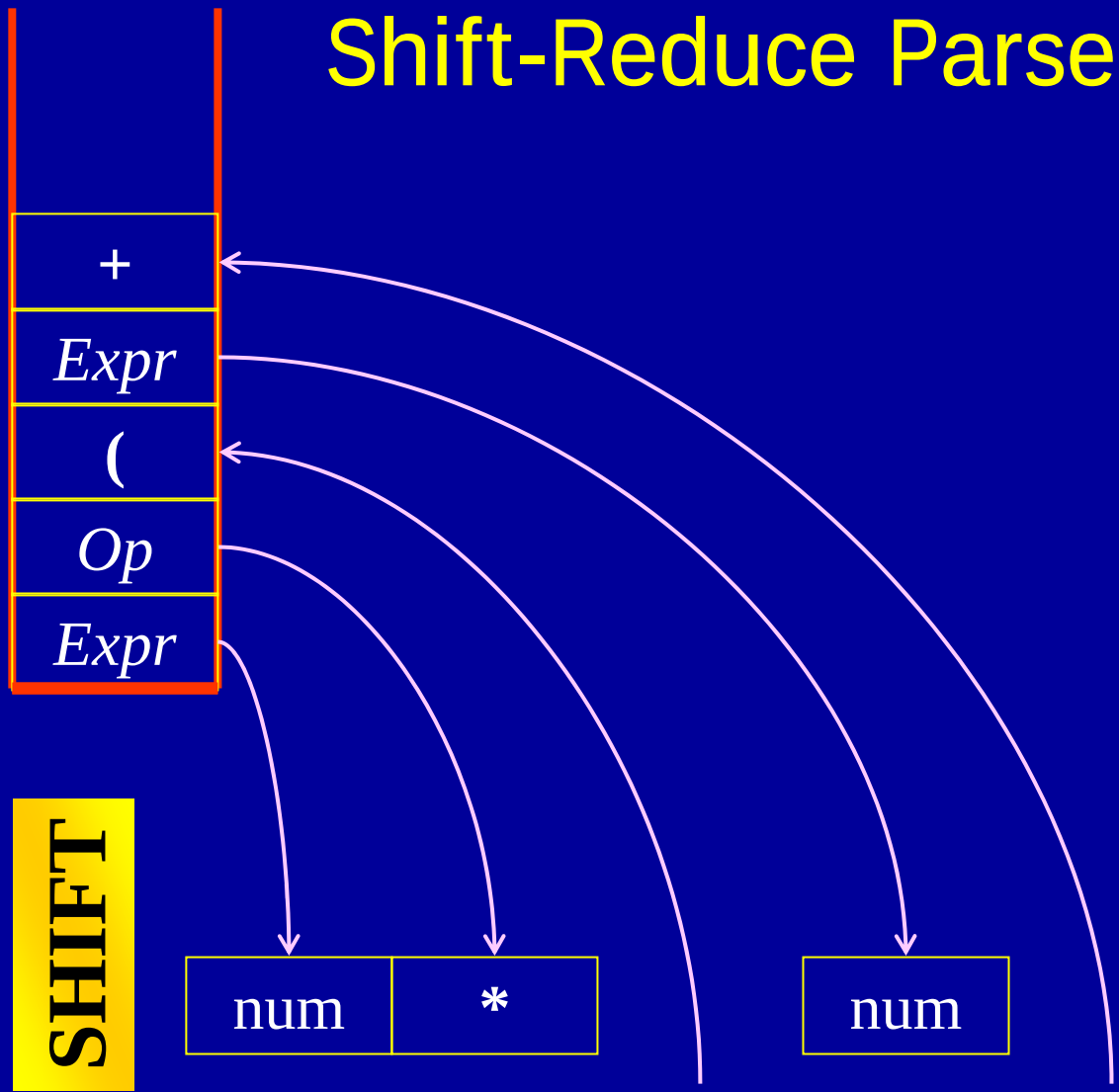

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num |)

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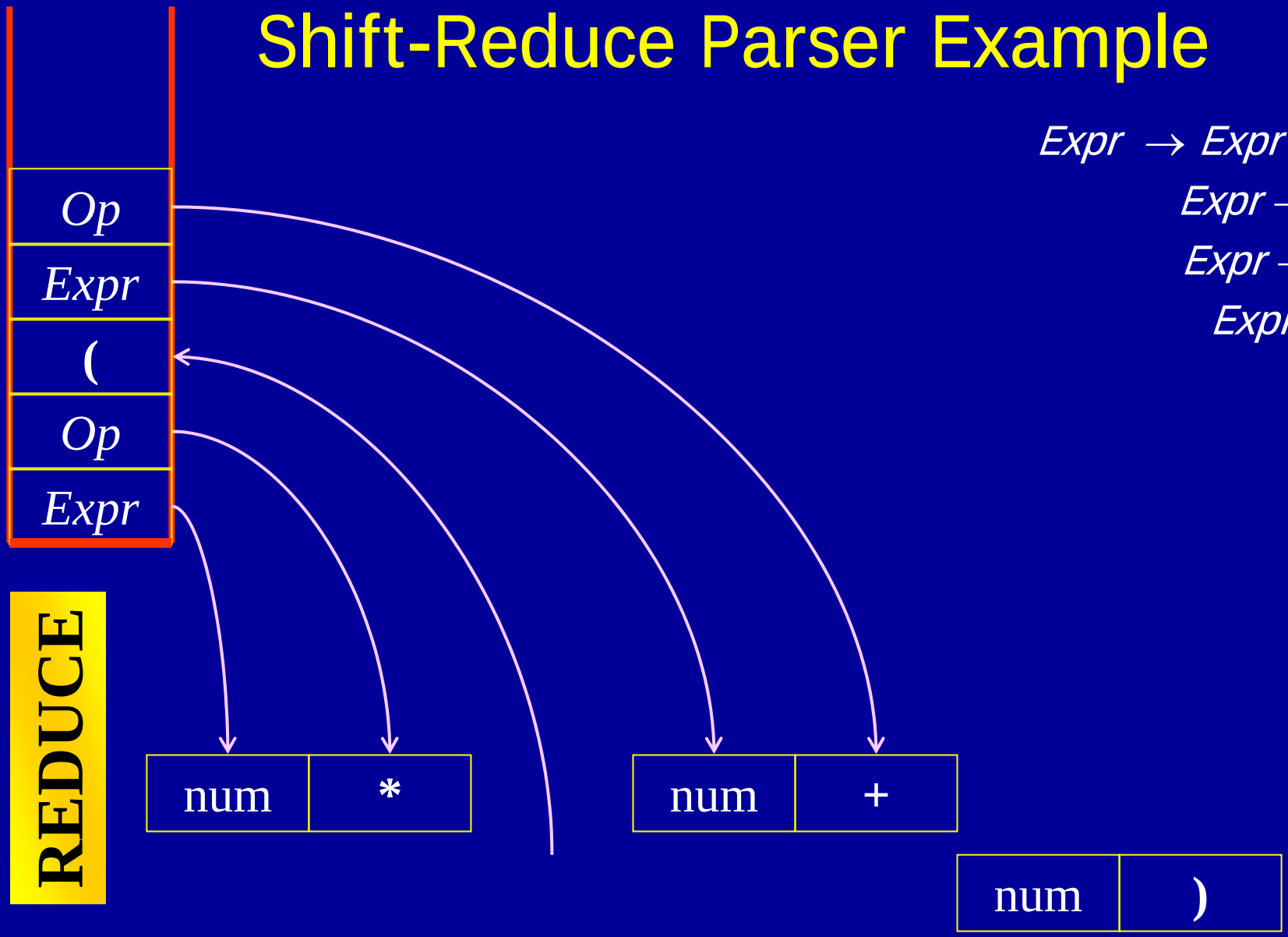

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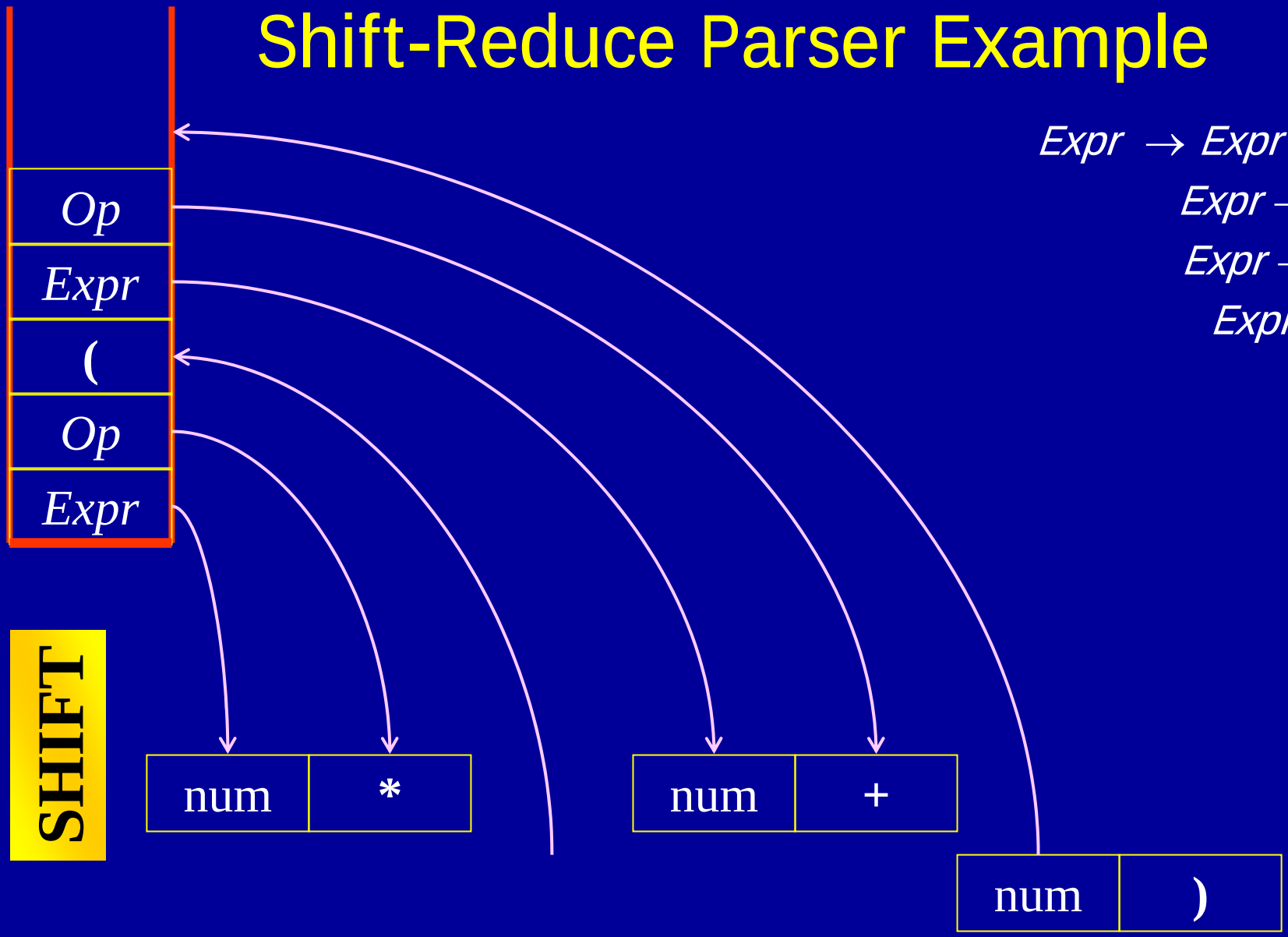

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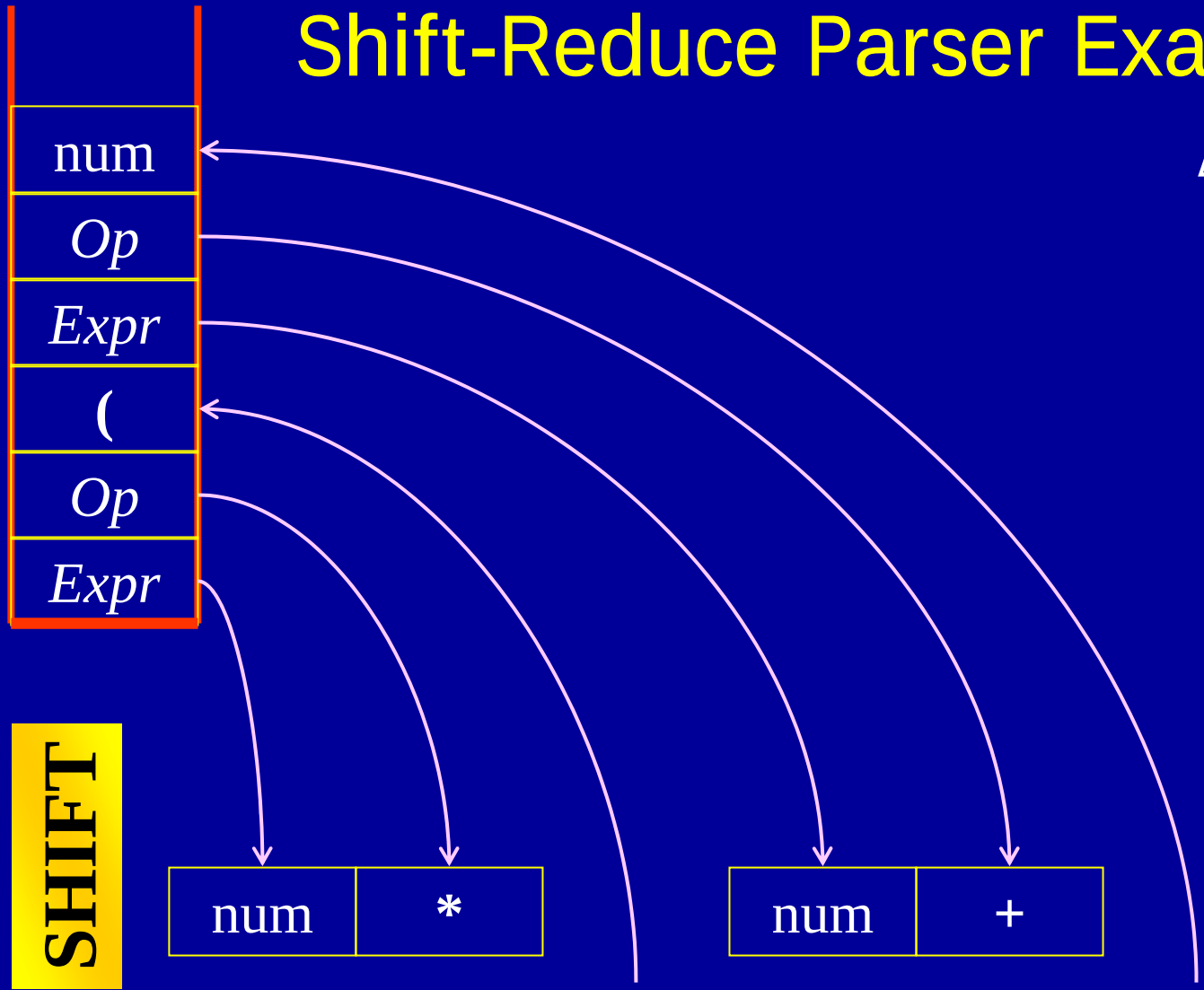

Expr → *Expr Op Expr*

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Op → +

Op → -

Op → *

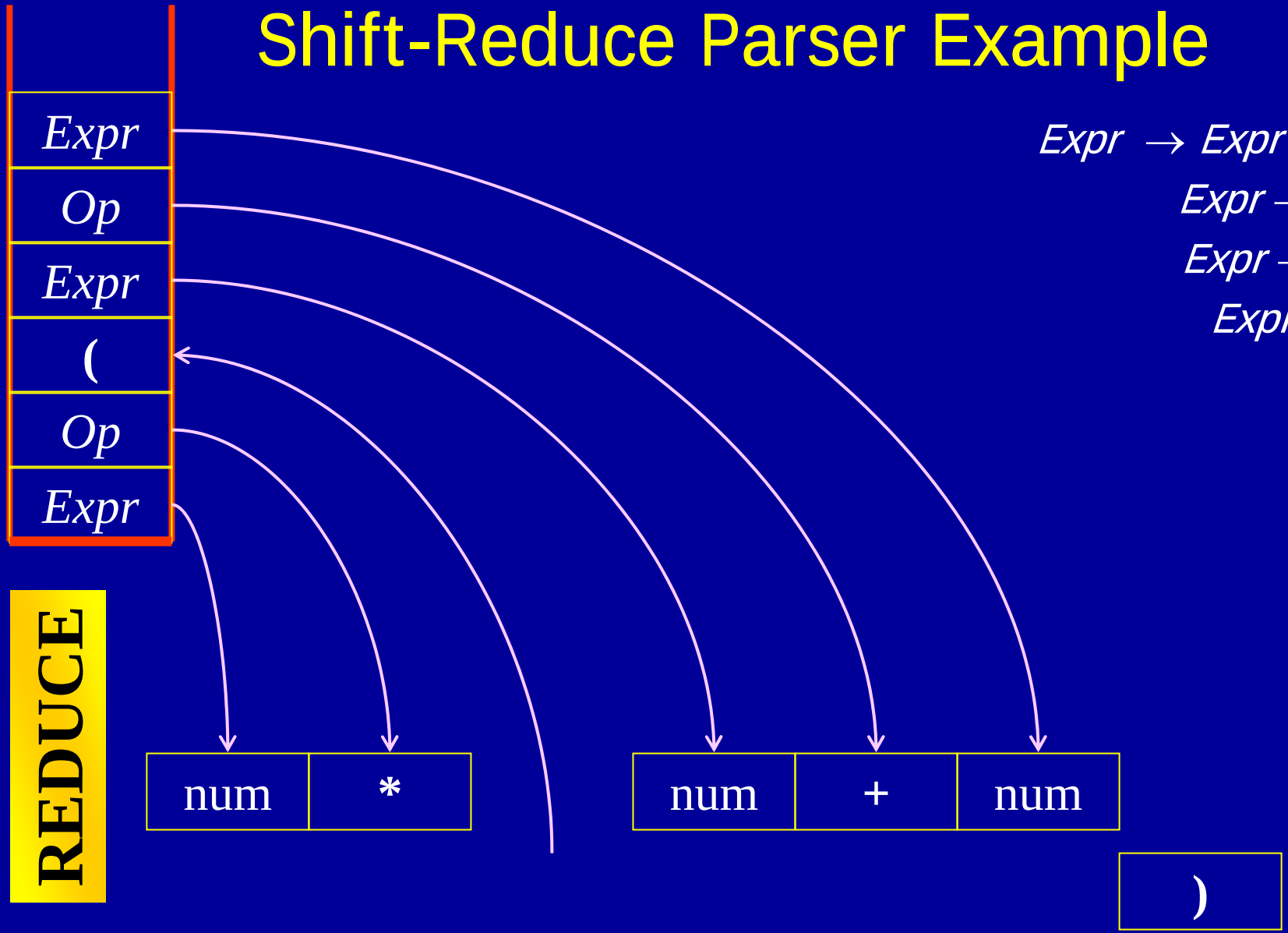
SHIFT

num | *

num | +

)

Shift-Reduce Parser Example



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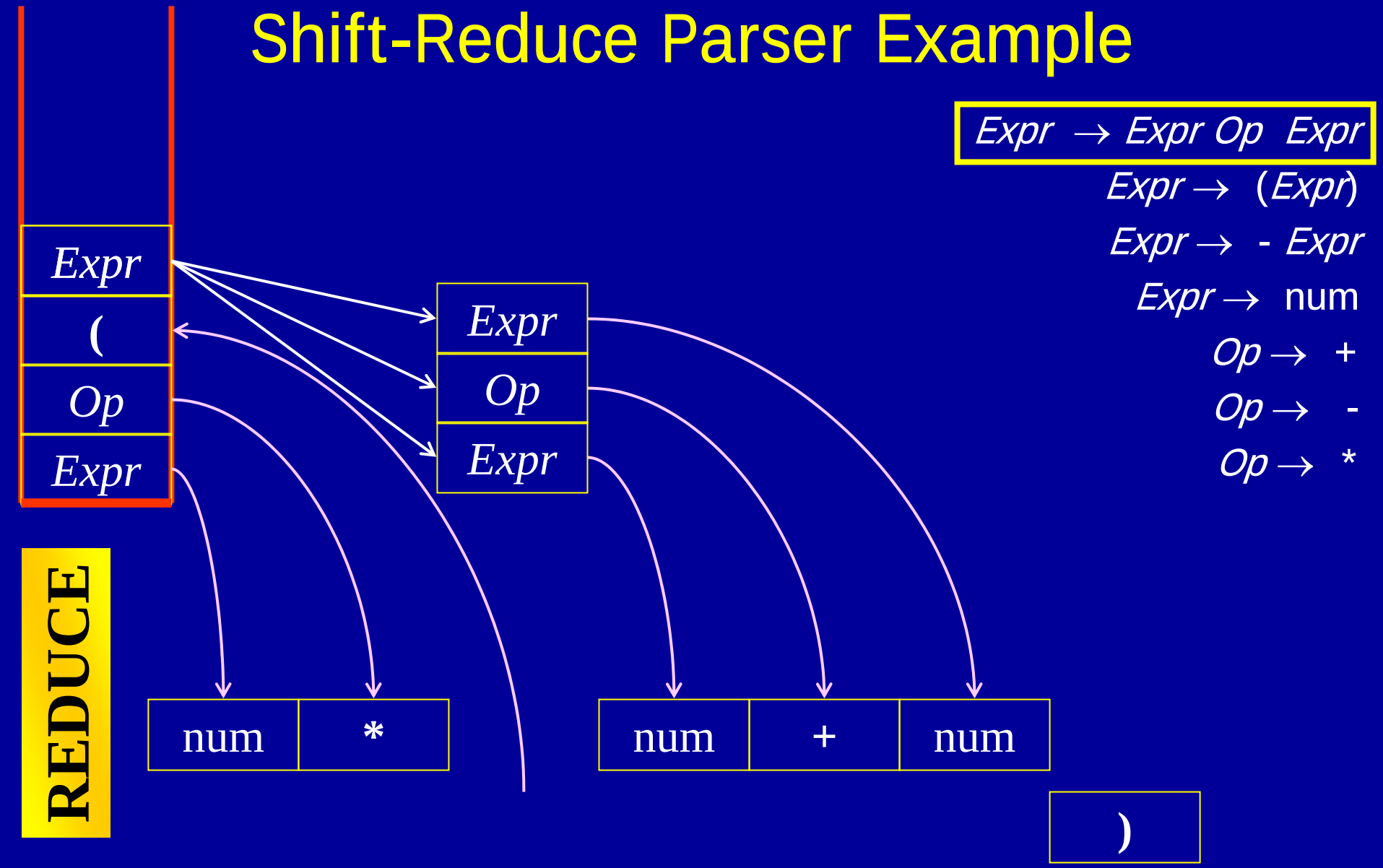

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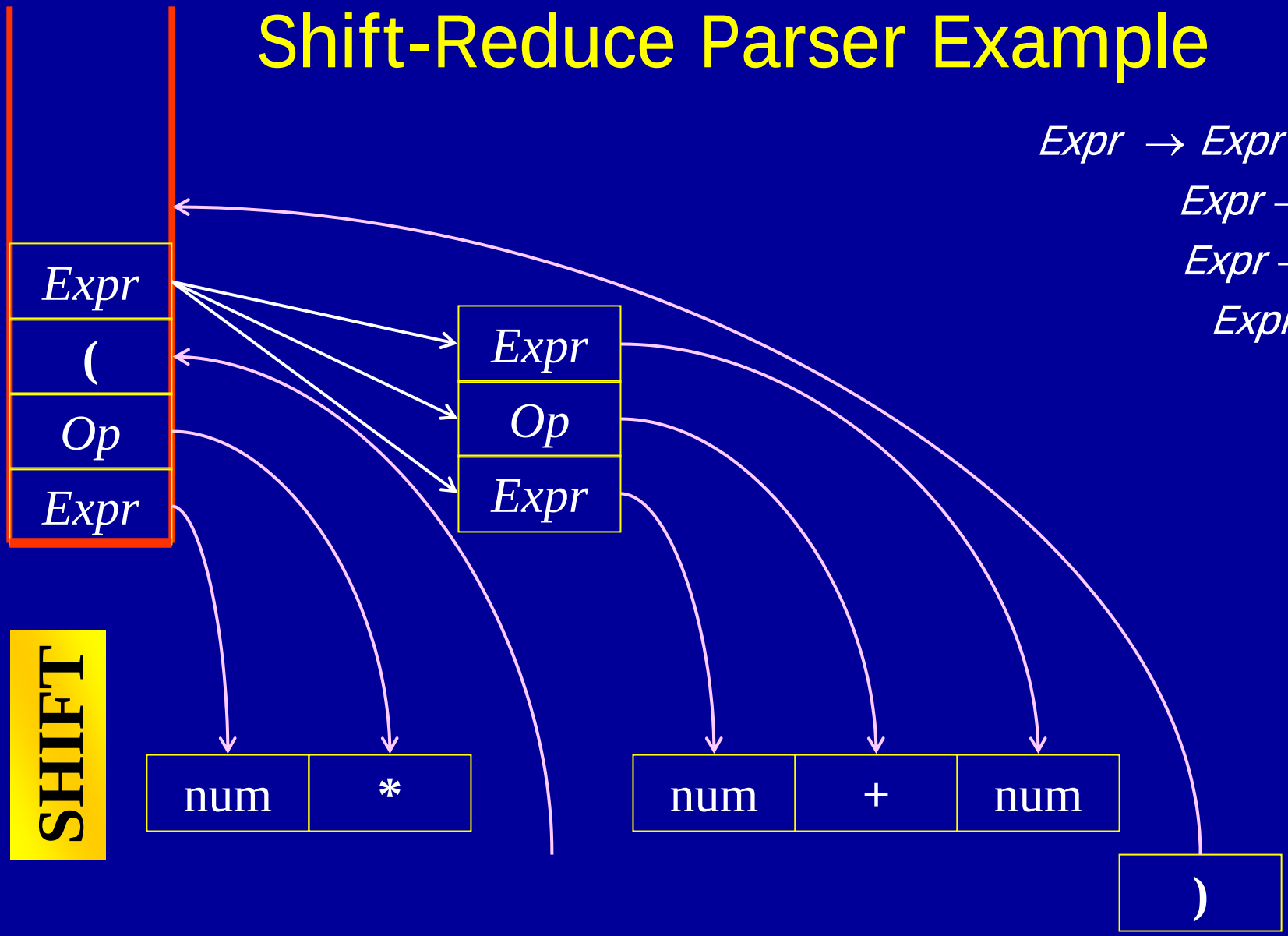

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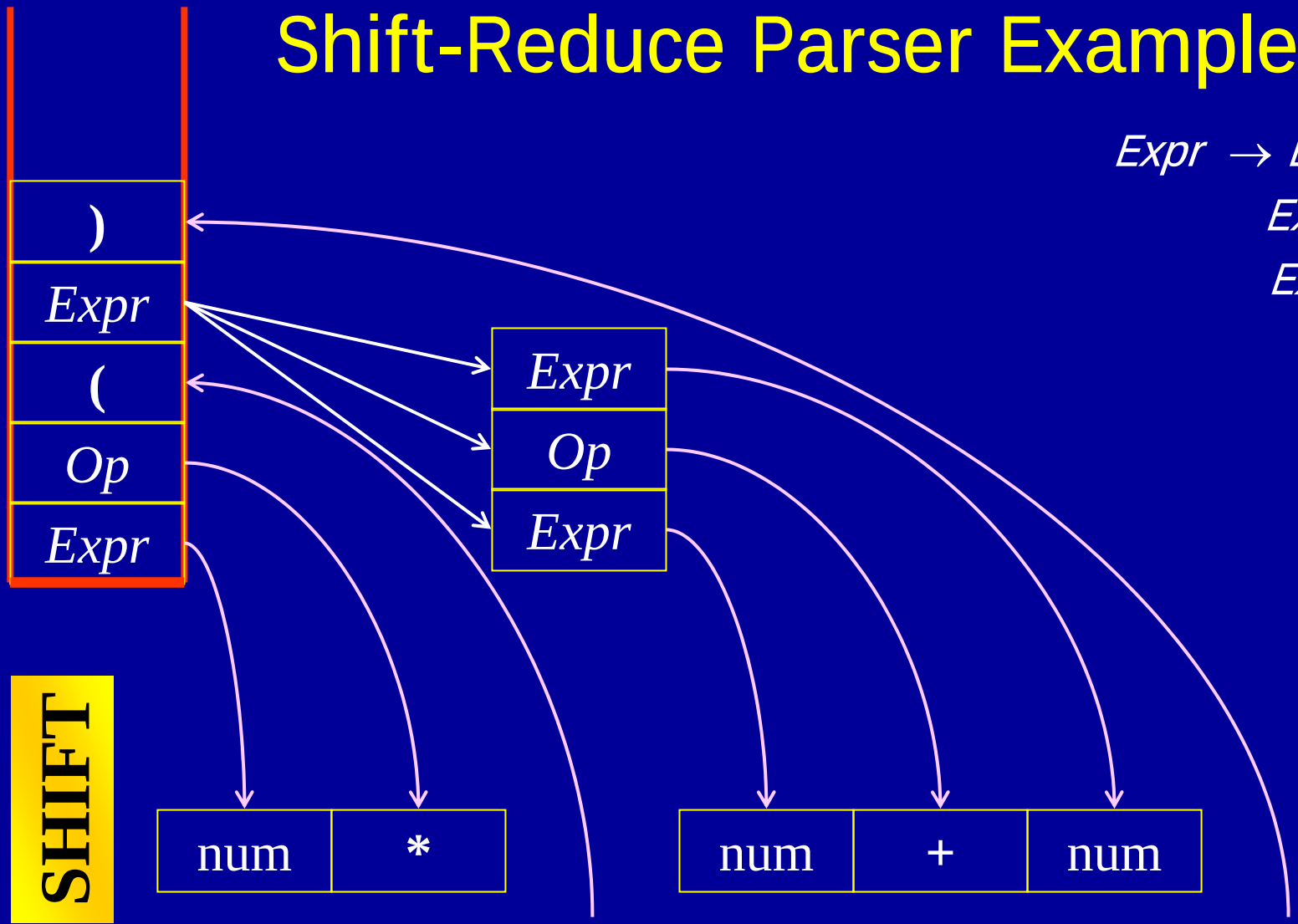

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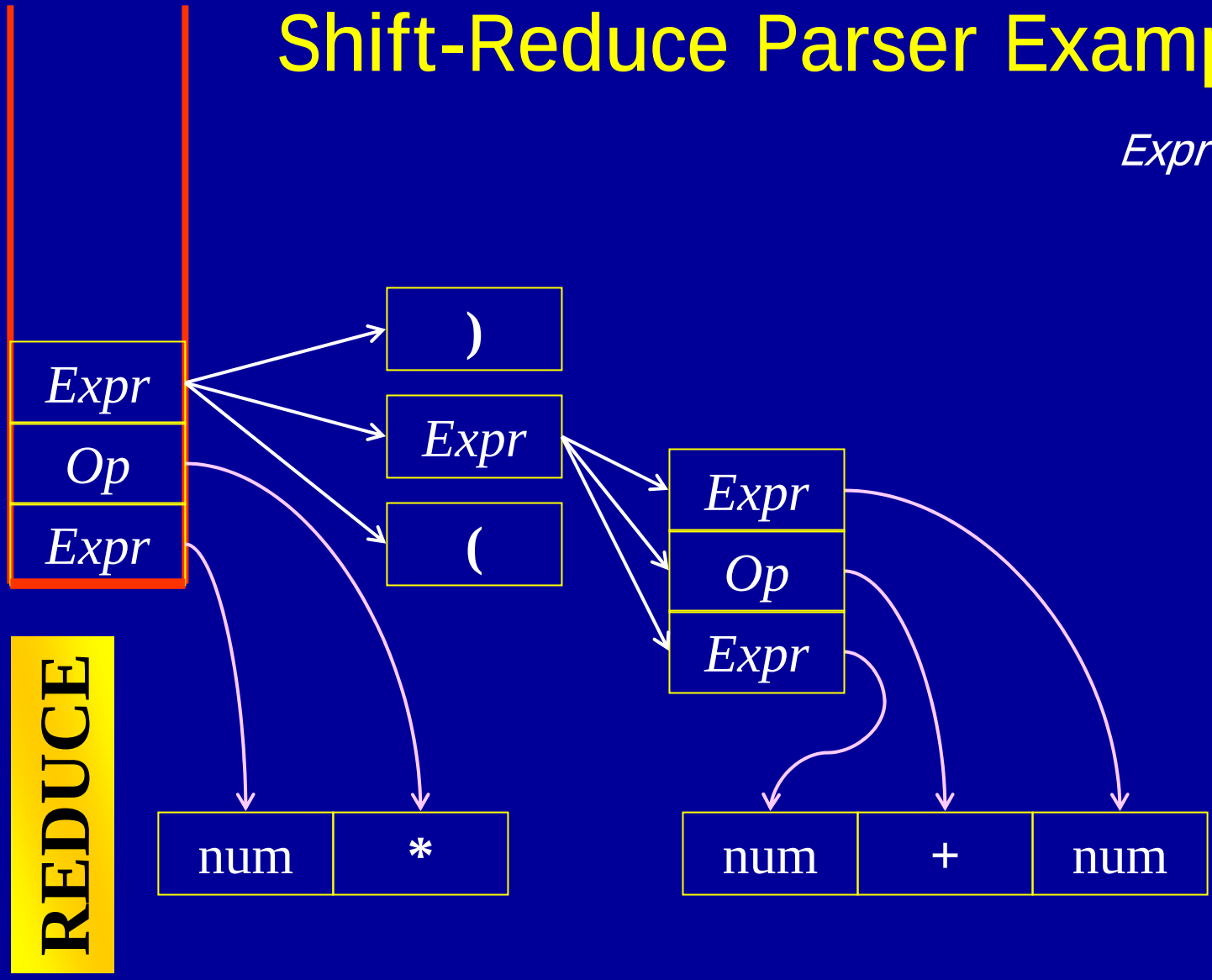

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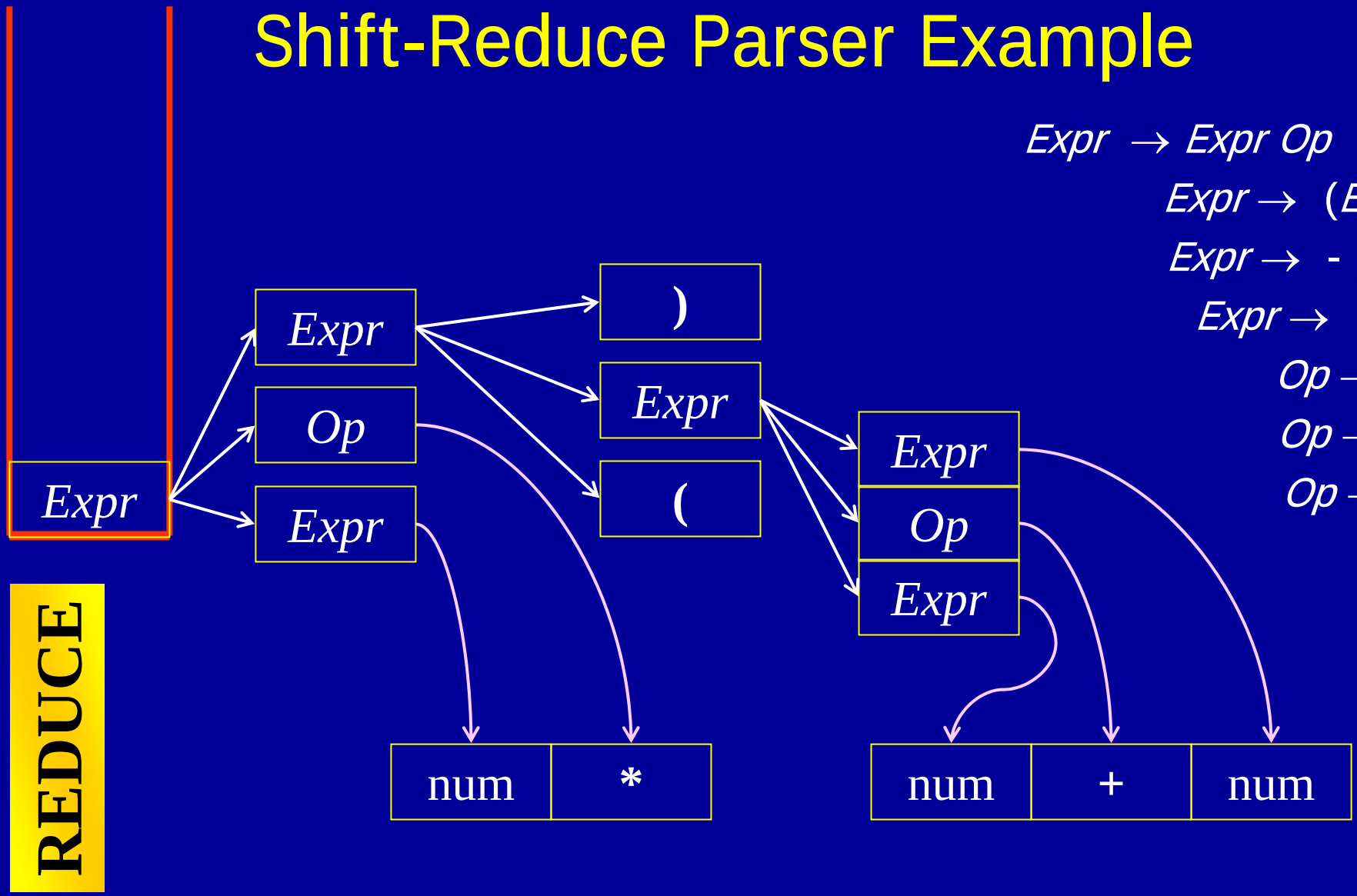

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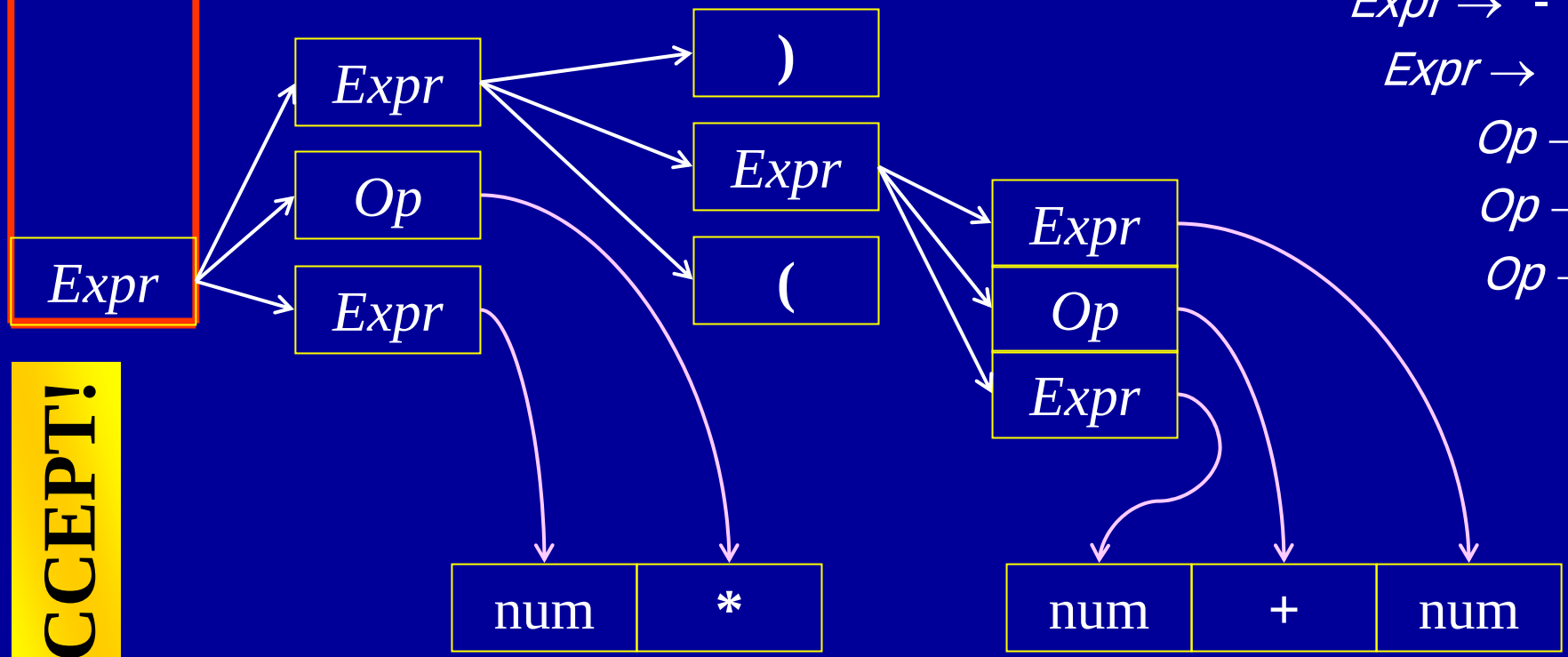


ACCEPT!

Basic Idea

- Goal: reconstruct parse tree for input string
- Read input from left to right
- Build tree in a bottom-up fashion
- Use stack to hold pending sequences of terminals and nonterminals

Potential Conflicts

- Reduce/Reduce Conflict
  - Top of the stack may match RHS of multiple productions
  - Which production to use in the reduction?
- Shift/Reduce Conflict
  - Stack may match RHS of production
  - But that may not be the right match
  - May need to shift an input and later find a different reduction

Conflicts

- Original Grammar

  Expr → *Expr Op Expr*

  Expr → (*Expr*)

  Expr → - *Expr*

  Expr → num

  Op → +

  Op → -

  Op → *

- New Grammar

  Expr → *Expr Op Expr*

  Expr → *Expr - Expr*

  Expr → (*Expr*)

  Expr → *Expr* -

  Expr → num

  Op → +

  Op → -

  Op → *

Conflicts

Expr → *Expr Op Expr*

Expr → *Expr* - *Expr*

Expr → (*Expr*)

Expr → *Expr* -

Expr → num

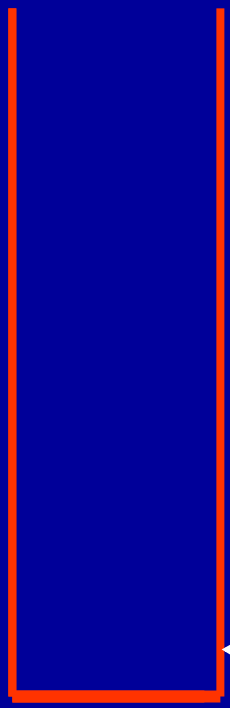
Op → +

Op → -

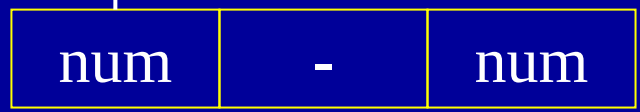
Op → *

num	-	num

Conflicts



SHIFT



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Expr → *Expr* - *Expr*

Expr → (*Expr*)

Expr → *Expr* -

Expr → num

Op → +

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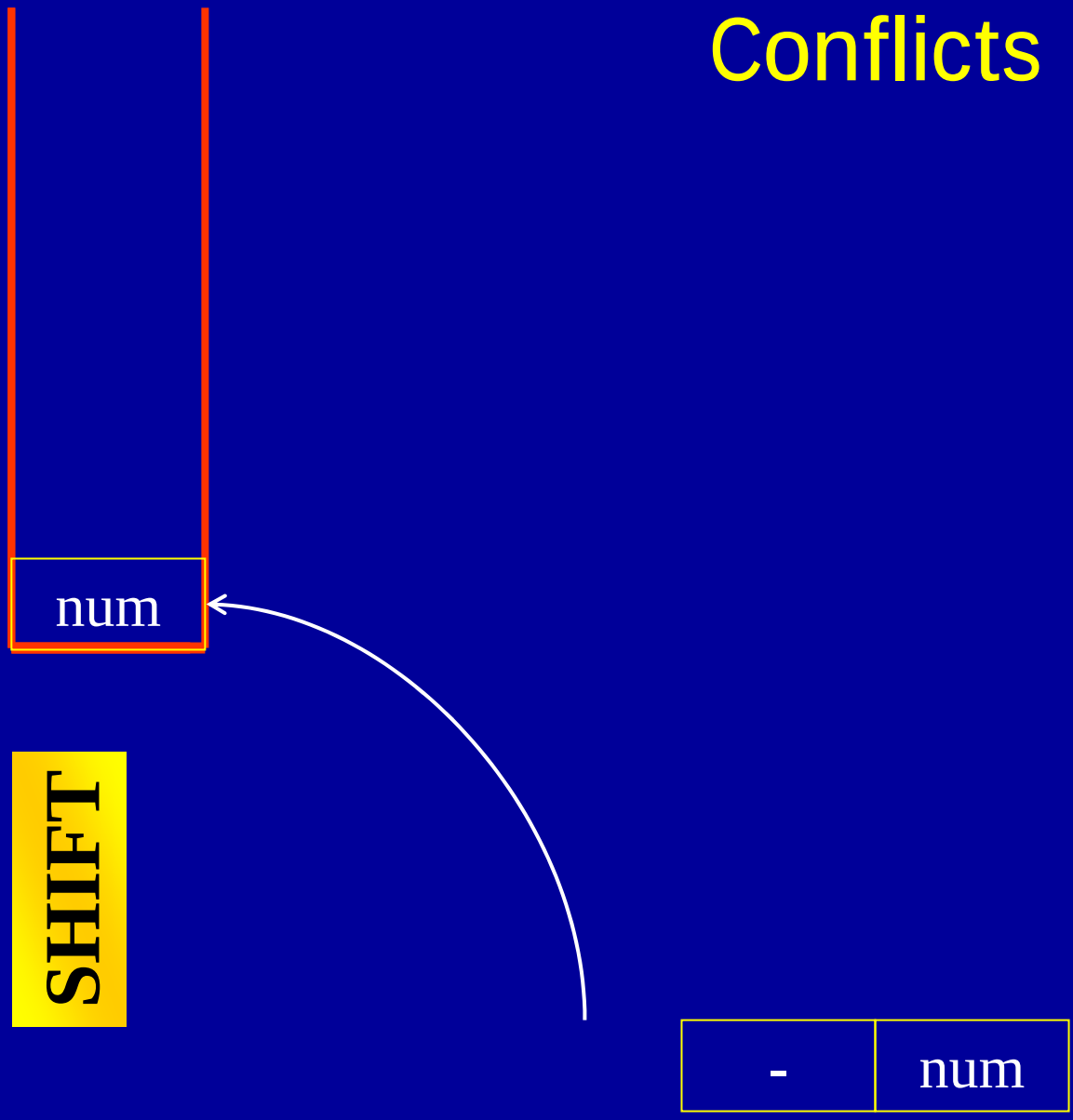

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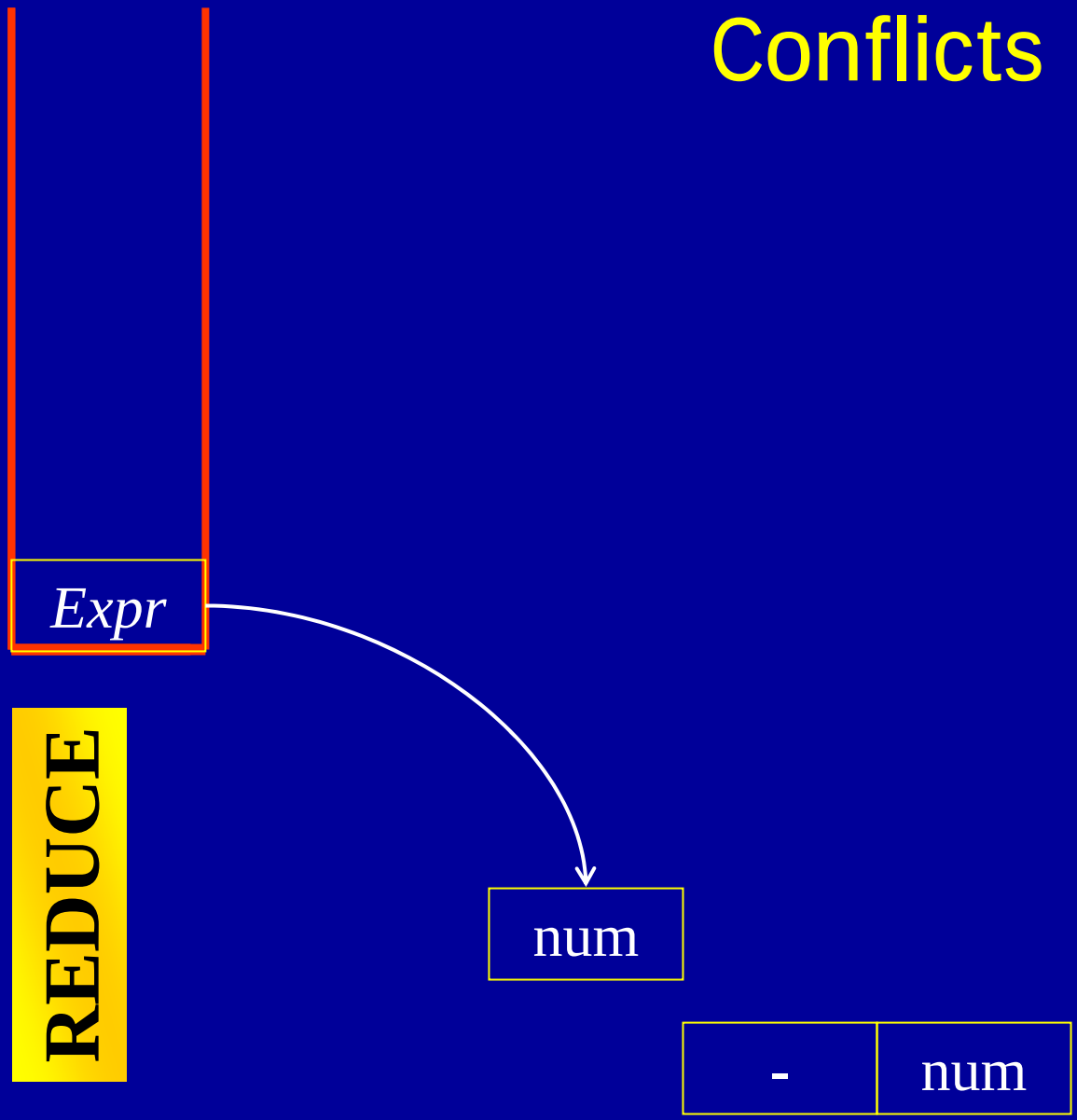

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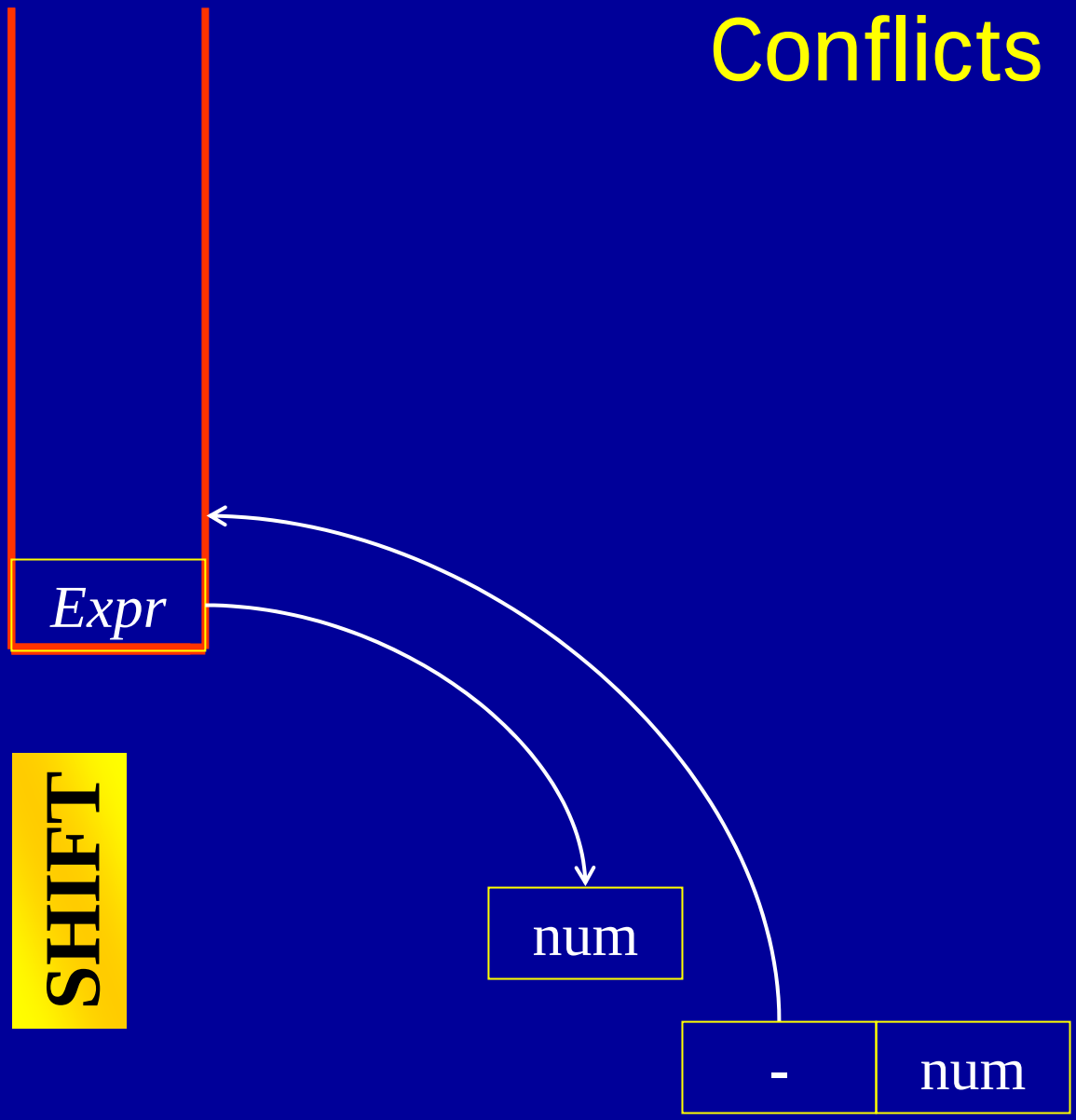
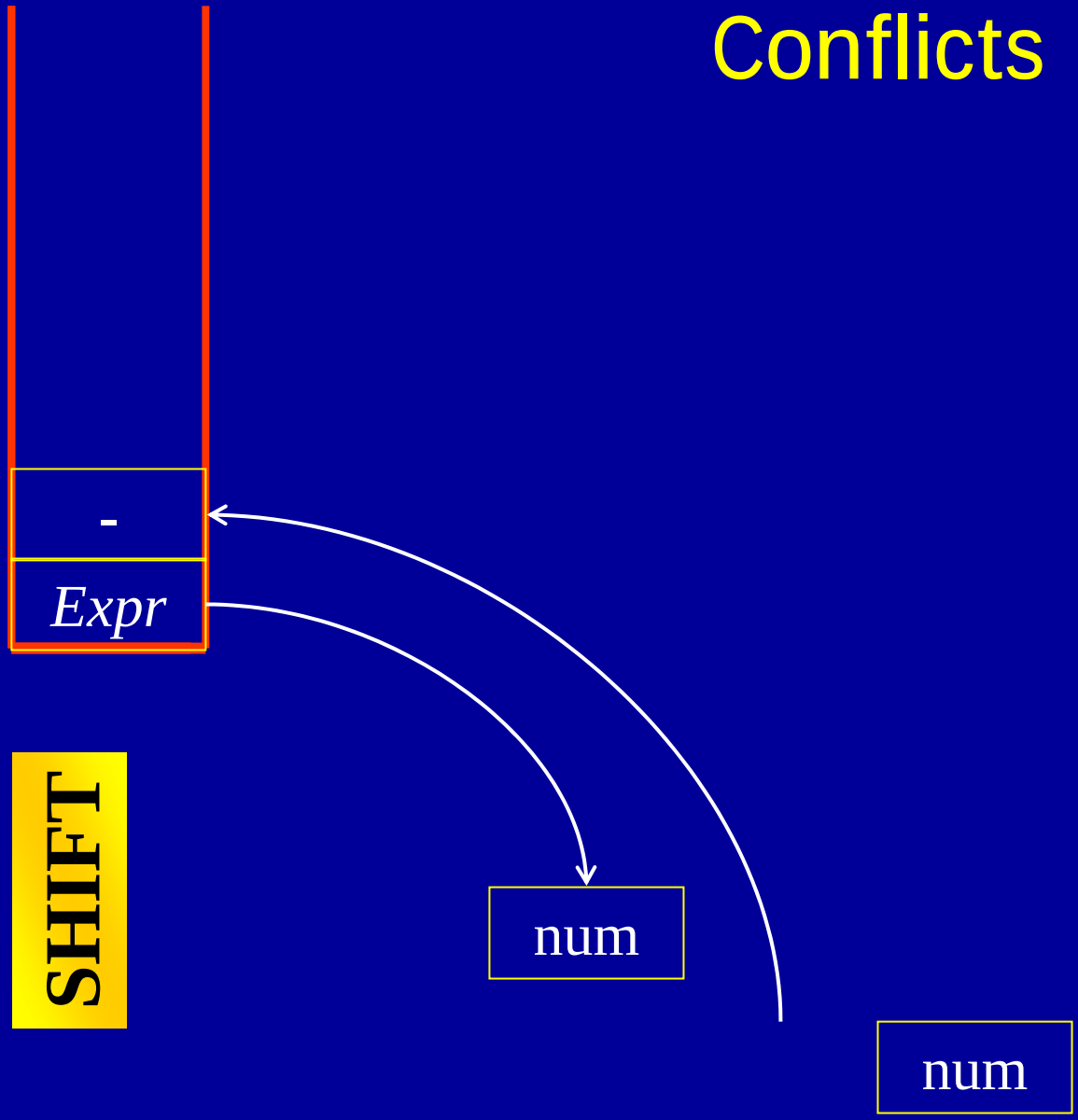


Conflicts



Expr → *Expr Op Expr*

Expr → *Expr* - *Expr*

Expr → (*Expr*)

Expr → *Expr* -

Expr → num

Op → +

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Shift/Reduce/Reduce Conflict

Options:

- Reduce
- Reduce
- Shift

Expr → *Expr Op Expr*

Expr → *Expr* - *Expr*

Expr → (*Expr*)

Expr → *Expr* -

Expr → num

Op → +

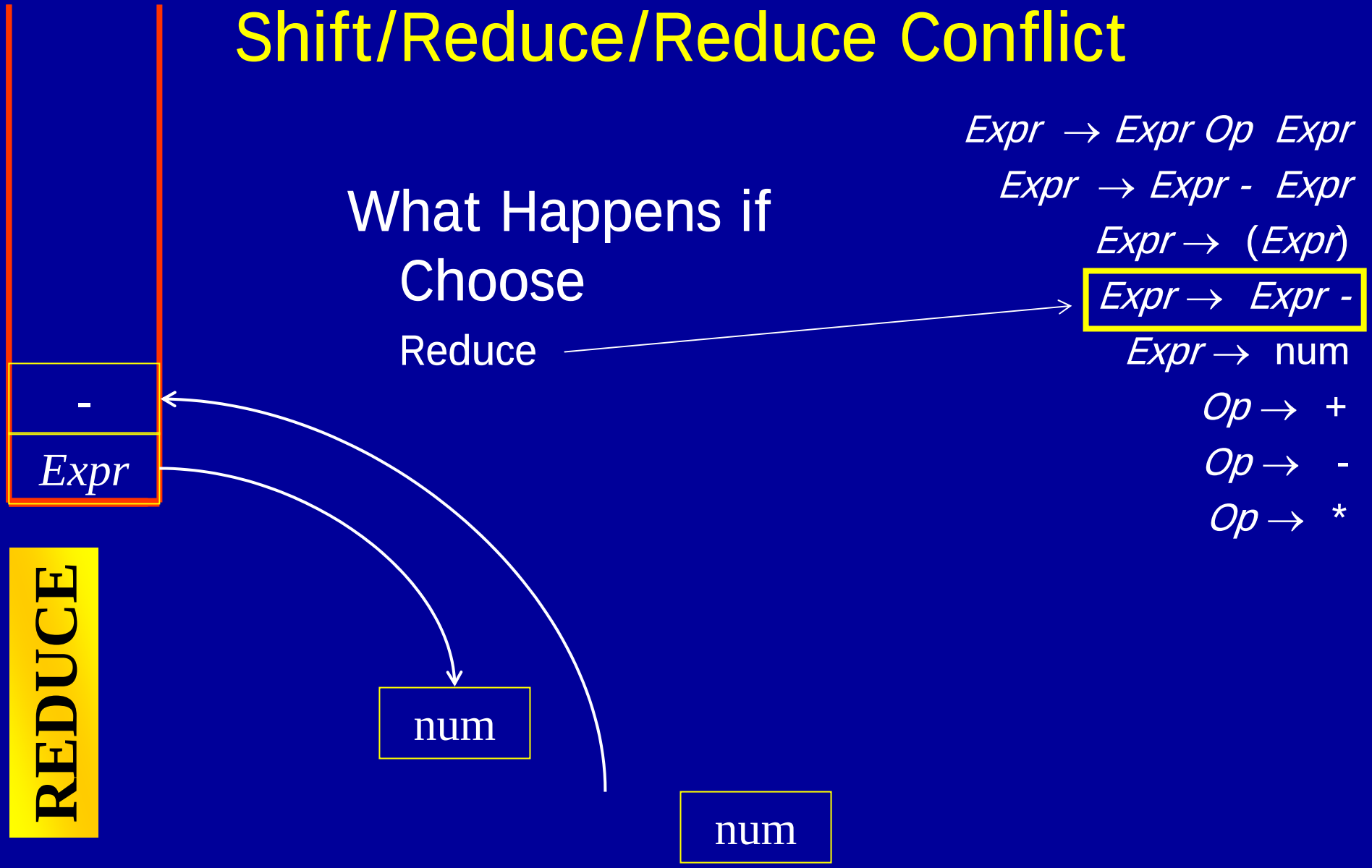
Op → -

Op → *

num

num

Shift/Reduce/Reduce Conflict



Shift/Reduce/Reduce Conflict

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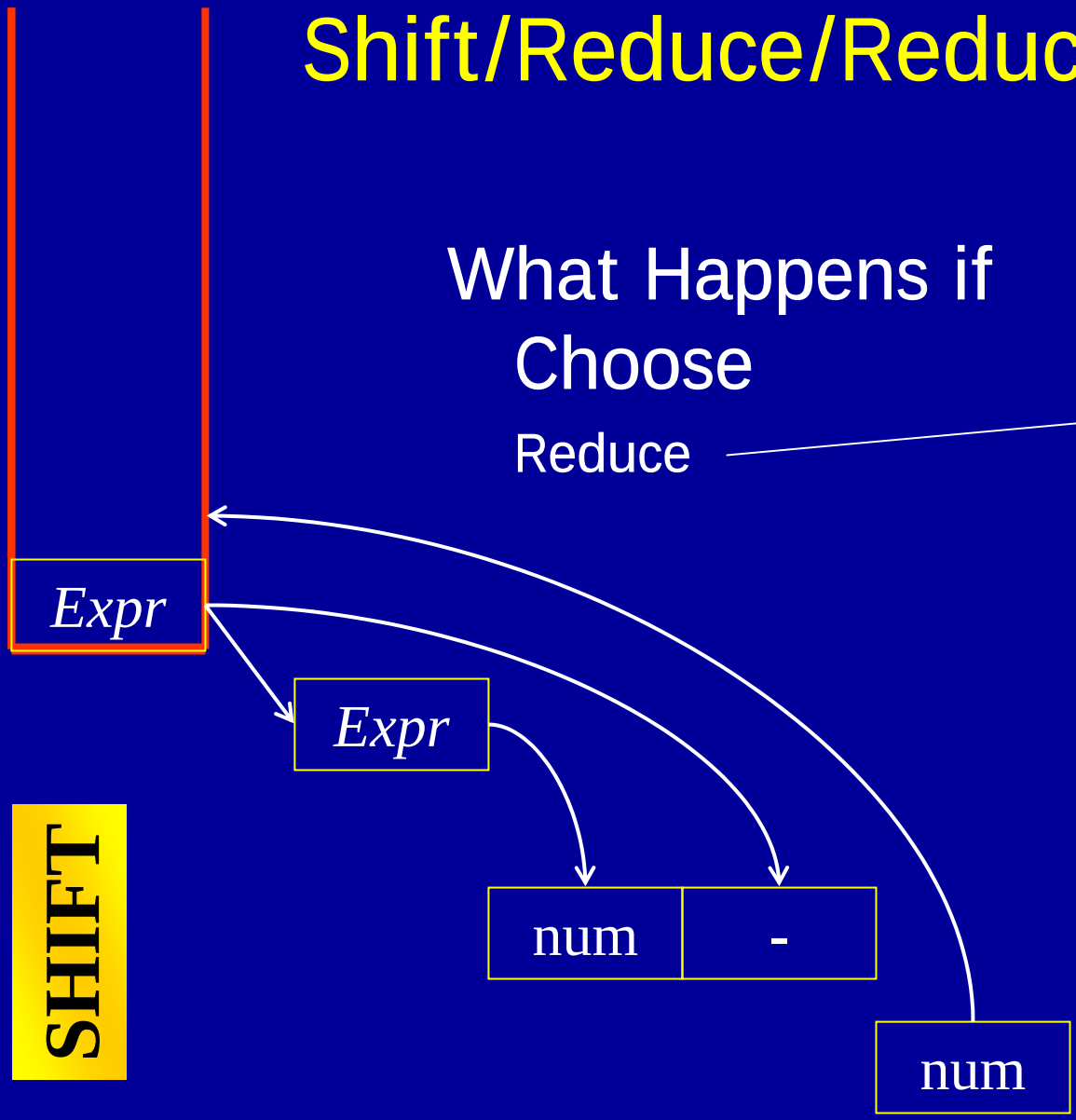
What Happens if Choose

Reduce



Shift/Reduce/Reduce Conflict

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What Happens if Choose

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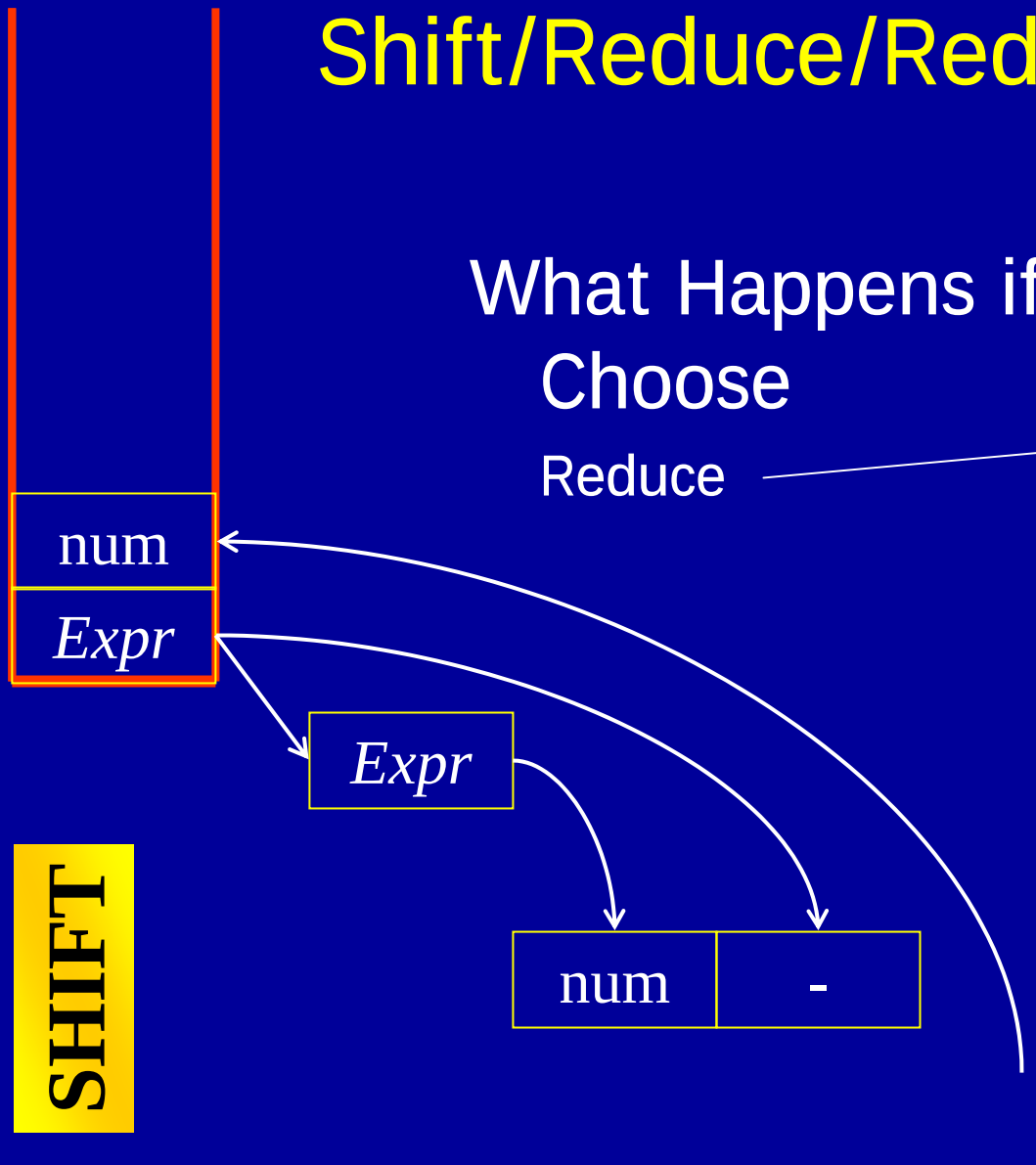
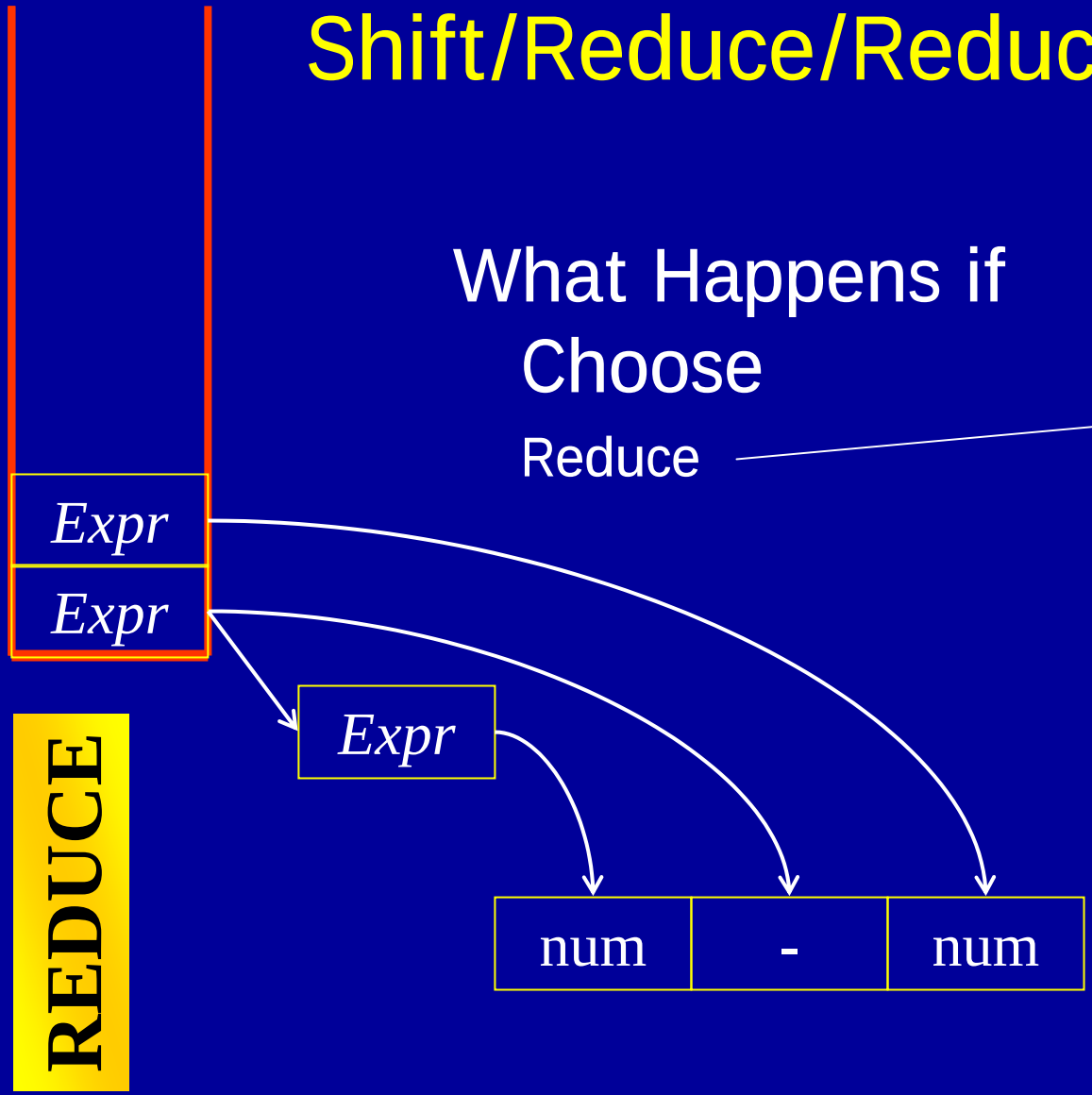


Shift/Reduce/Reduce Conflict



What Happens if Choose

Reduce

$Expr \rightarrow Expr\ Op\ Expr$

$Expr \rightarrow Expr$ - $Expr$

$Expr \rightarrow$ ($Expr$)

$Expr \rightarrow Expr$ -

$Expr \rightarrow$ num

$Op \rightarrow$ +

$Op \rightarrow$ -

$Op \rightarrow$ *

Shift/Reduce/Reduce Conflict

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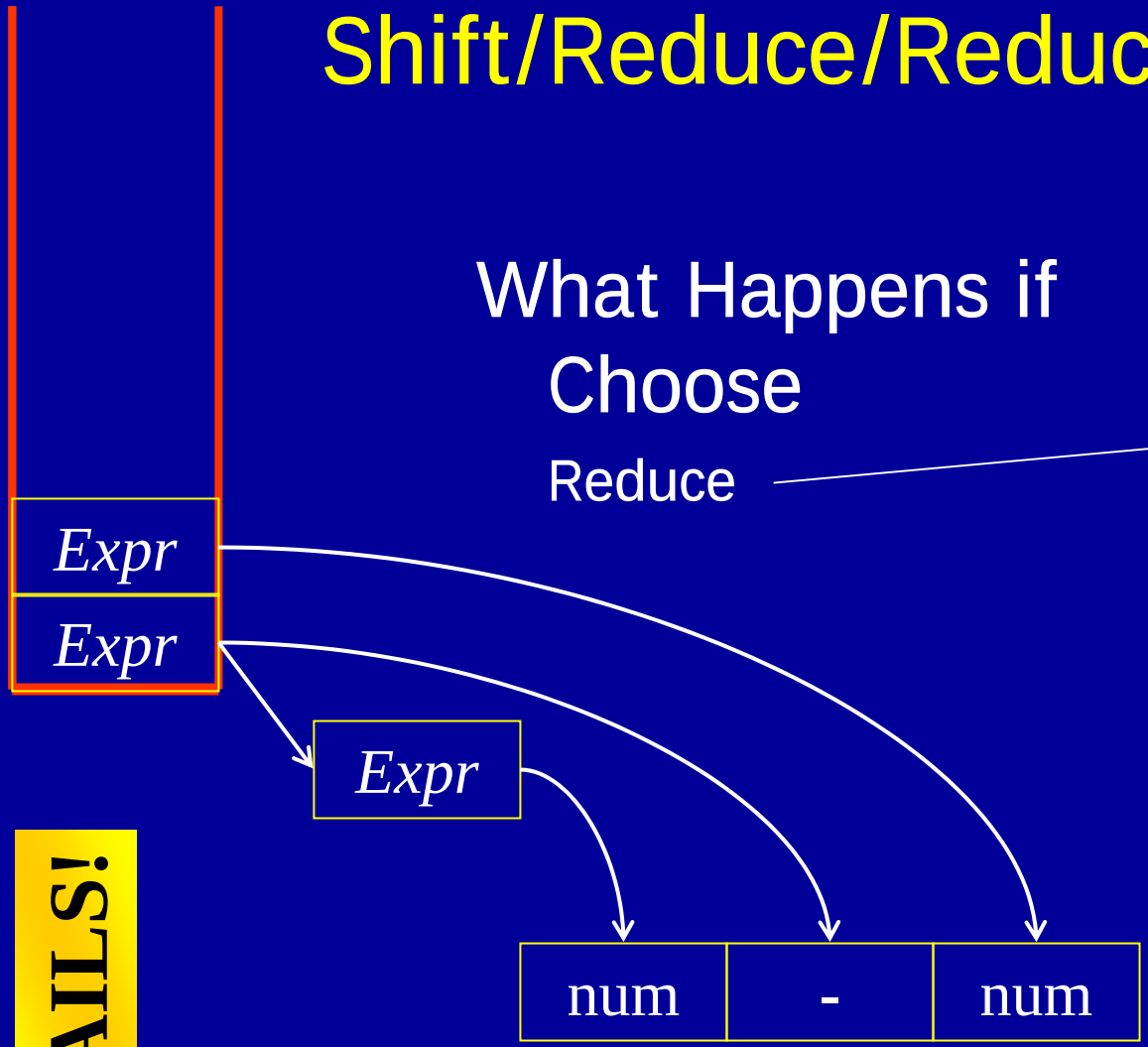
What Happens if Choose

Reduce



FAILS!

Shift/Reduce/Reduce Conflict

Both of These Actions Work

Reduce

Shift

$Expr \rightarrow Expr\ Op\ Expr$

$Expr \rightarrow Expr\ -\ Expr$

$Expr \rightarrow (Expr)$

$Expr \rightarrow Expr\ -$

$Expr \rightarrow$ num

$Op \rightarrow +$

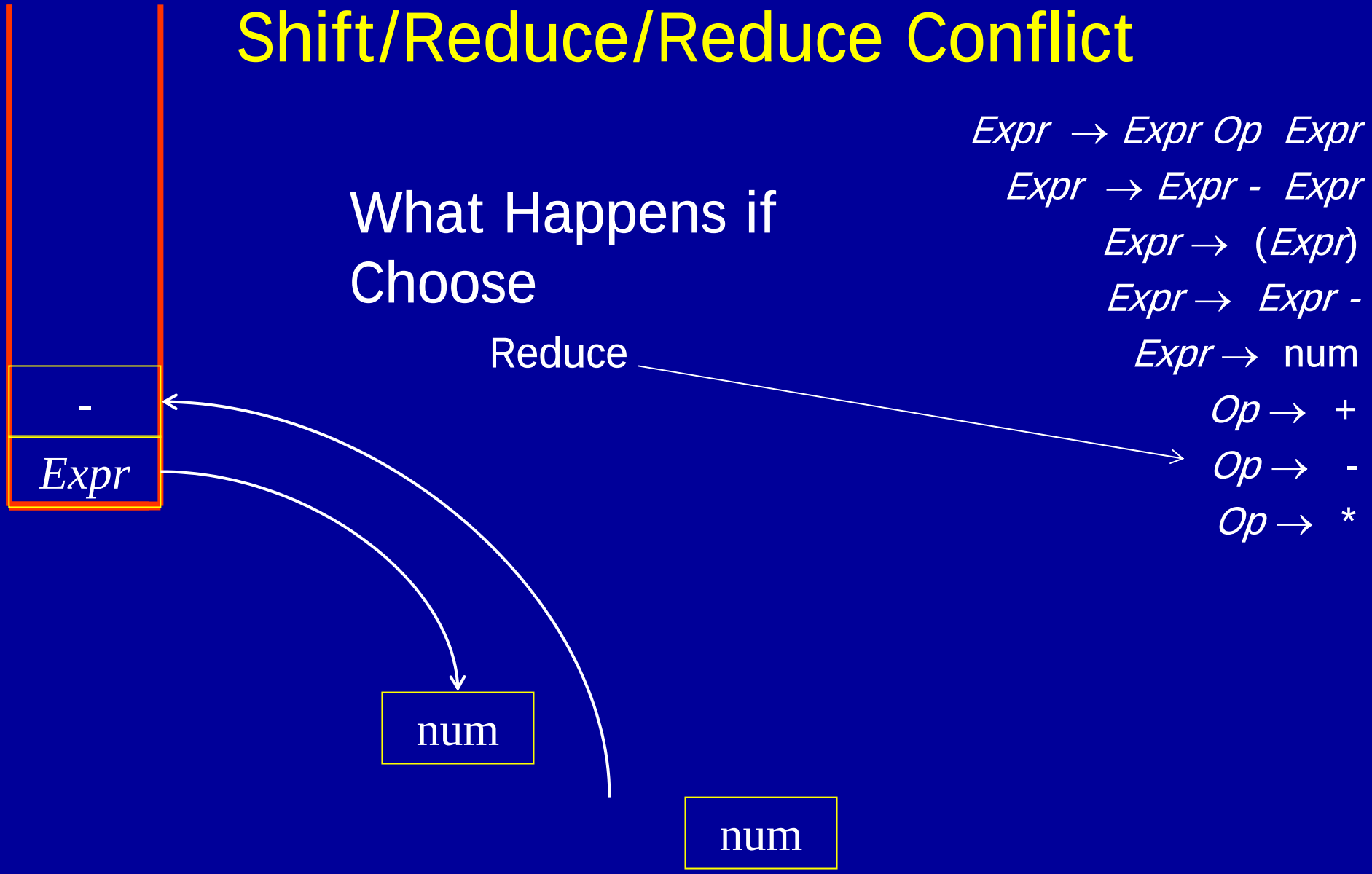
$Op \rightarrow -$

$Op \rightarrow *$

num

num

Shift/Reduce/Reduce Conflict



Shift/Reduce/Reduce Conflict

$Expr \rightarrow Expr\ Op\ Expr$

$Expr \rightarrow Expr$ - $Expr$

$Expr \rightarrow$ ($Expr$)

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$Expr \rightarrow$ num

$Op \rightarrow$ +

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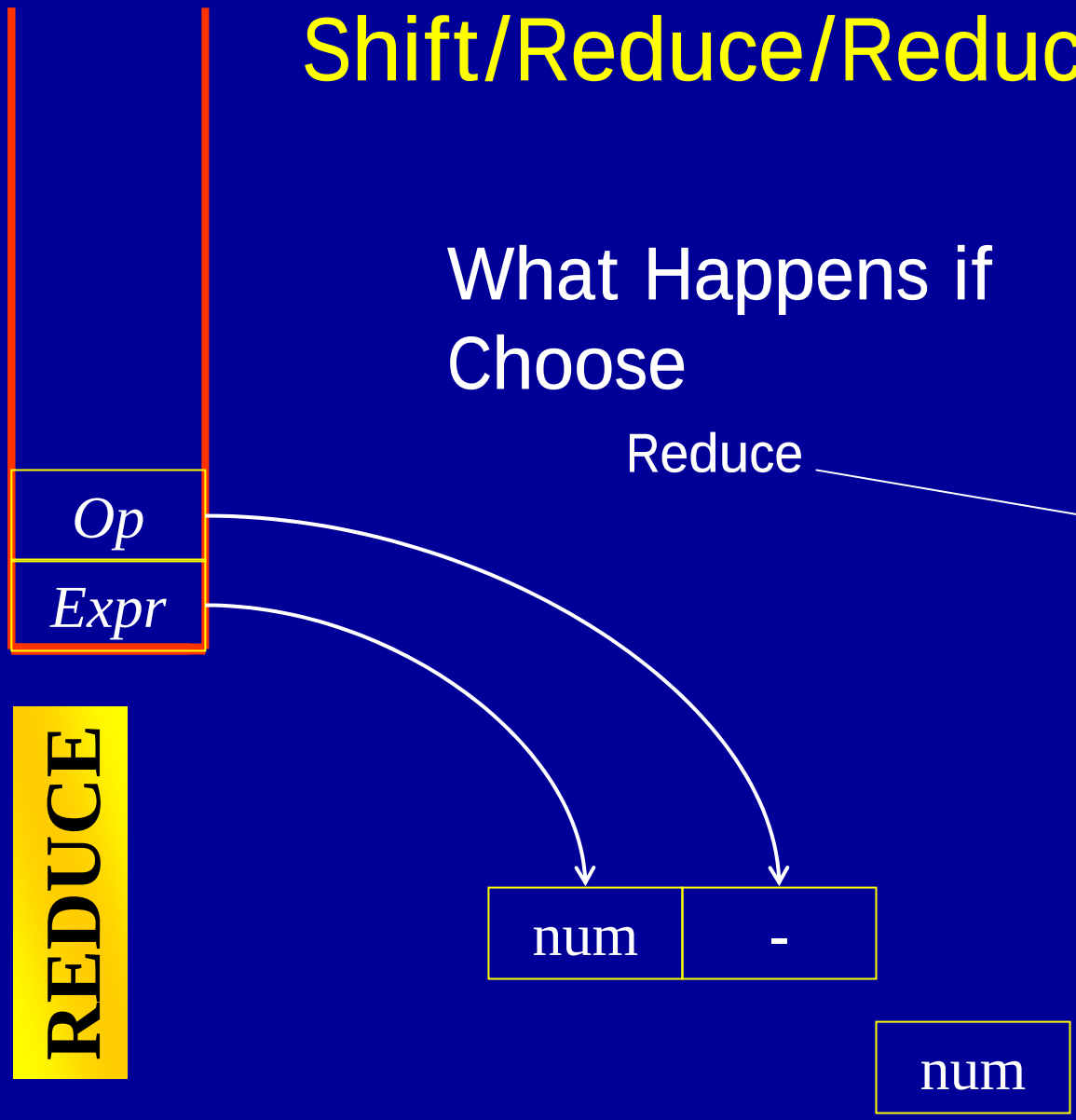
$Op \rightarrow$ *

What Happens if Choose

Reduce



Shift/Reduce/Reduce Conflict

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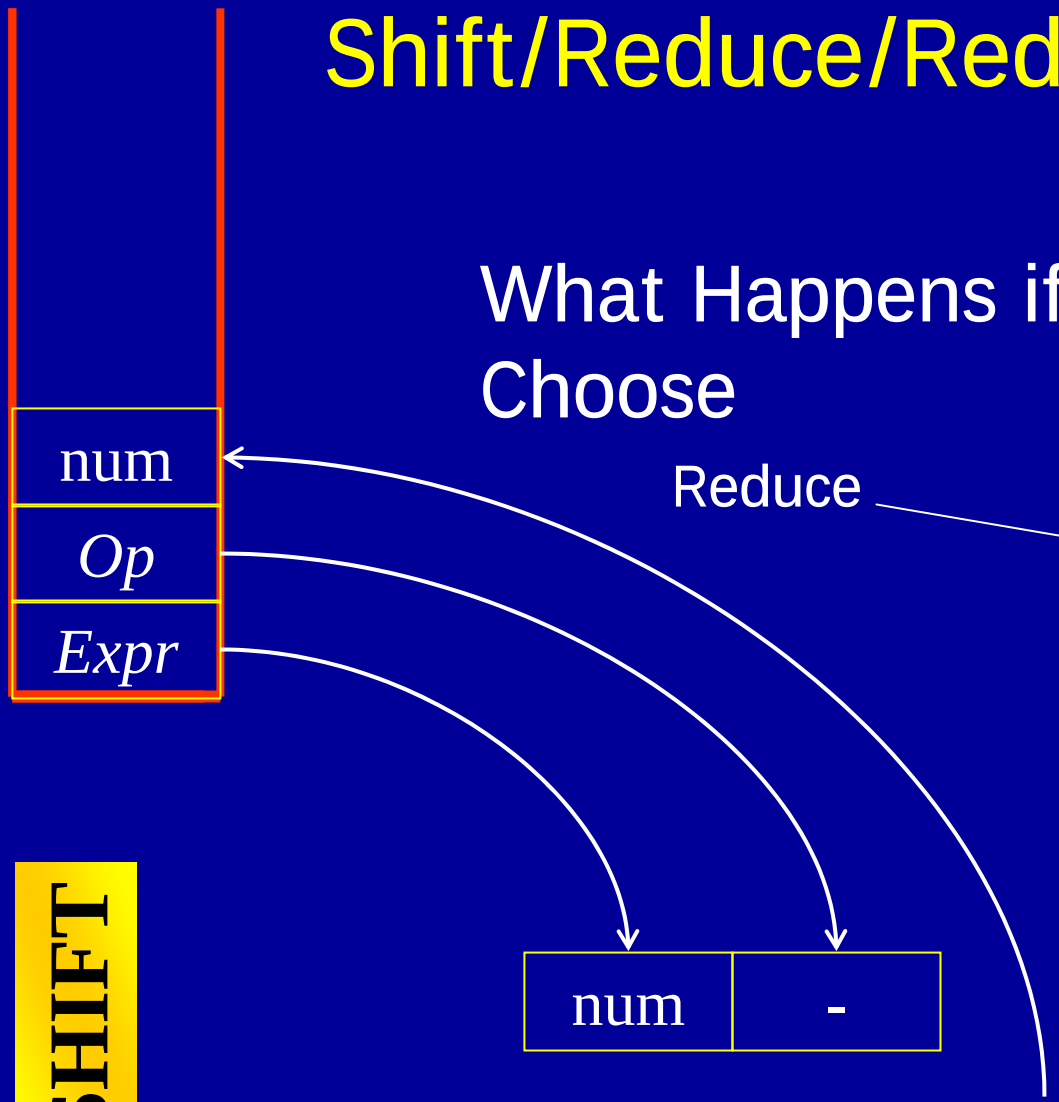
What Happens if Choose



Shift/Reduce/Reduce Conflict

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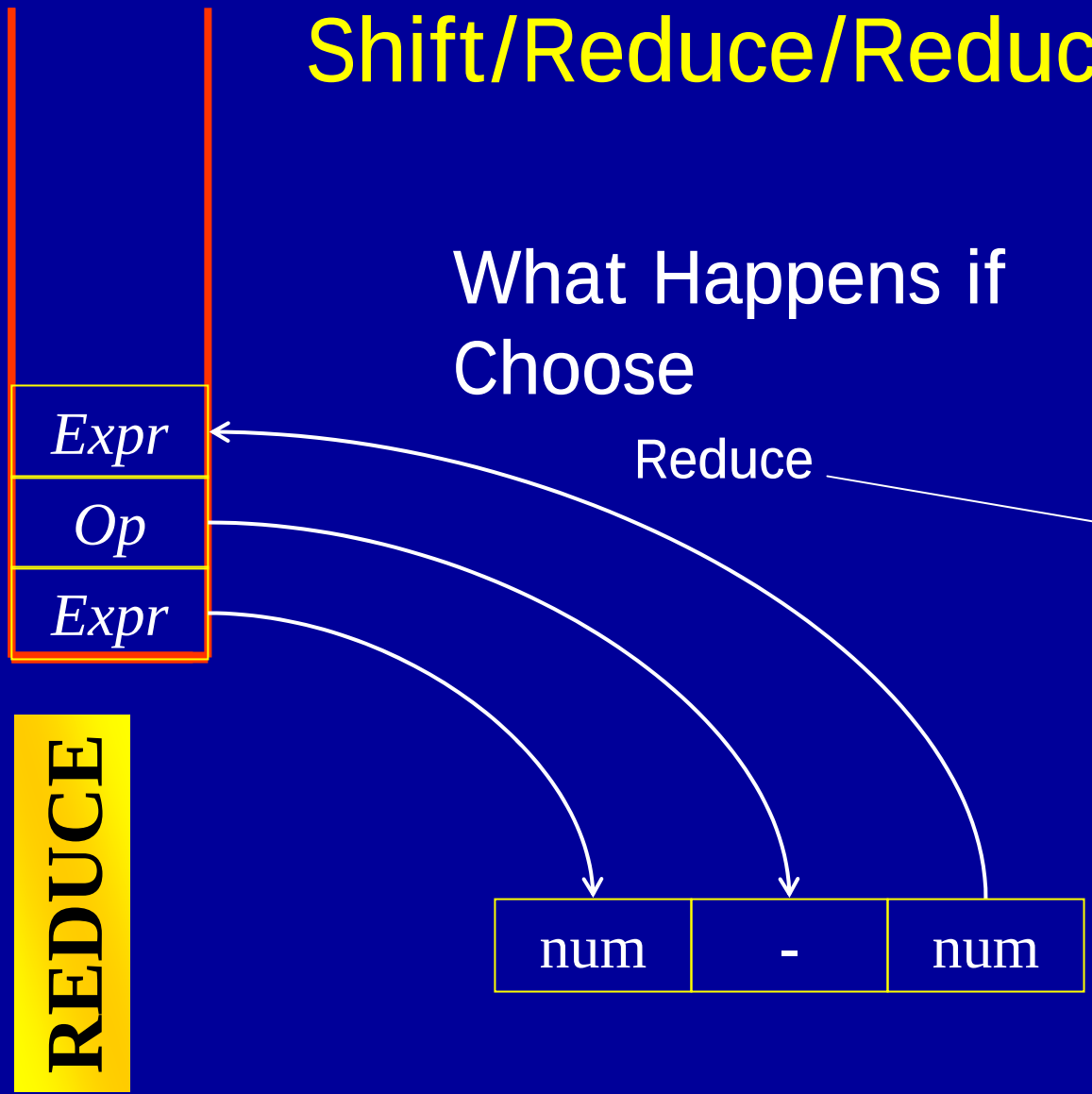
What Happens if Choose



Shift/Reduce/Reduce Conflict

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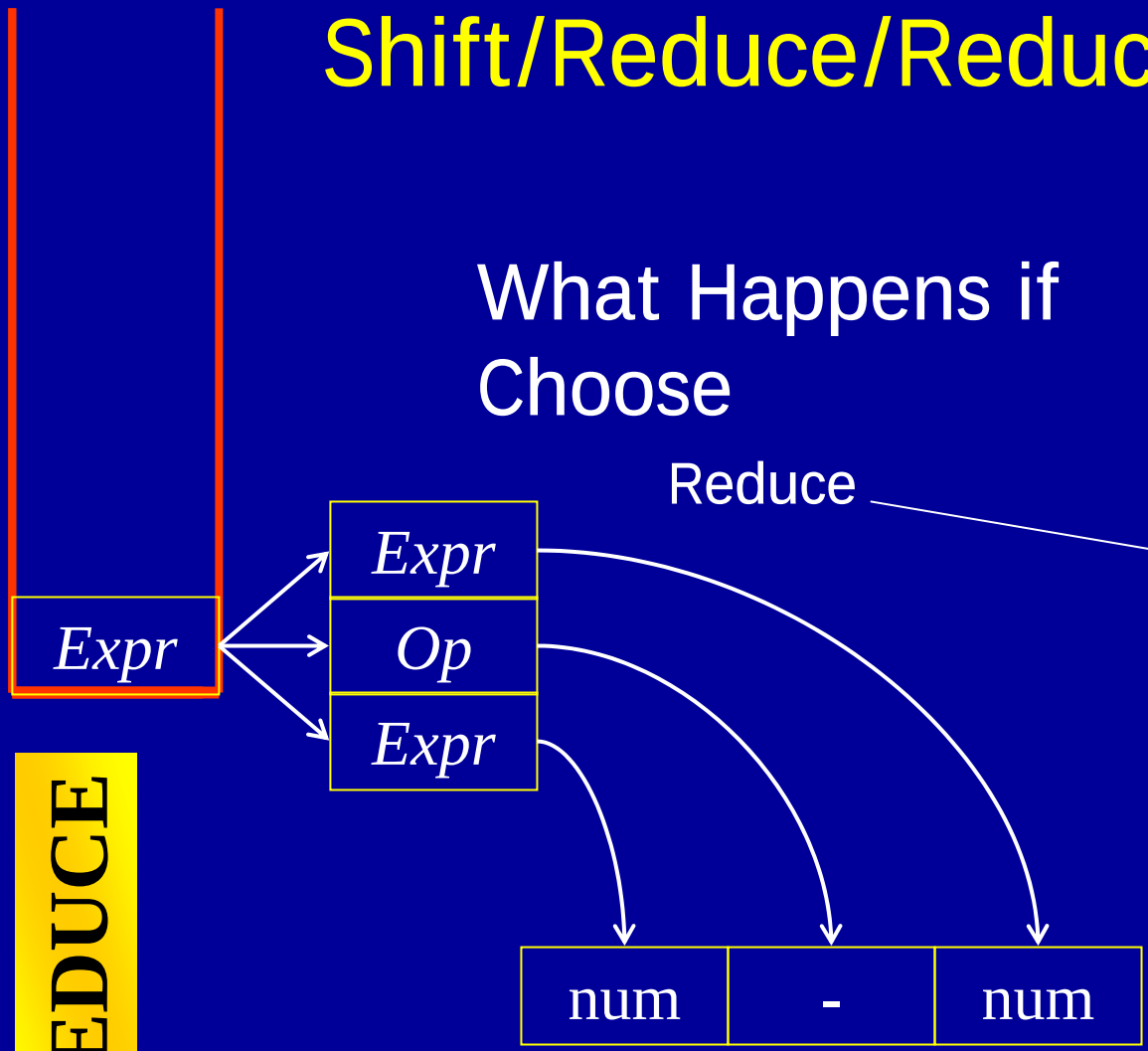
What Happens if Choose

Reduce

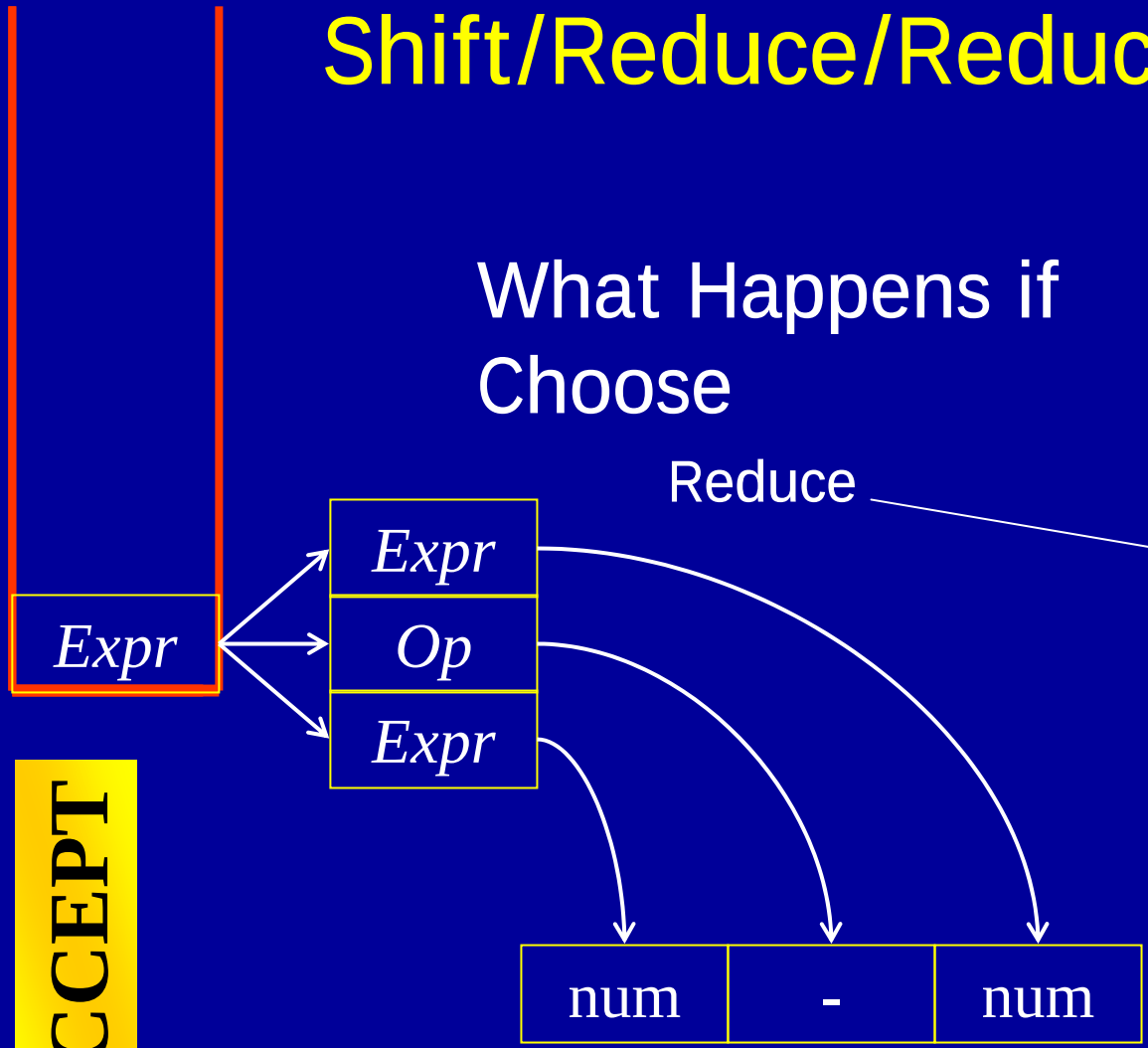


Shift/Reduce/Reduce Conflict

Expr → *Expr Op Expr*
Expr → *Expr* - *Expr*
Expr → (*Expr*)
Expr → *Expr* -
Expr → num
Op → +
Op → -
Op → *

What Happens if Choose

Reduce



ACCEPT

Conflicts

Expr → *Expr Op Expr*

Expr → *Expr* - *Expr*

Expr → (*Expr*)

Expr → *Expr* -

Expr → num

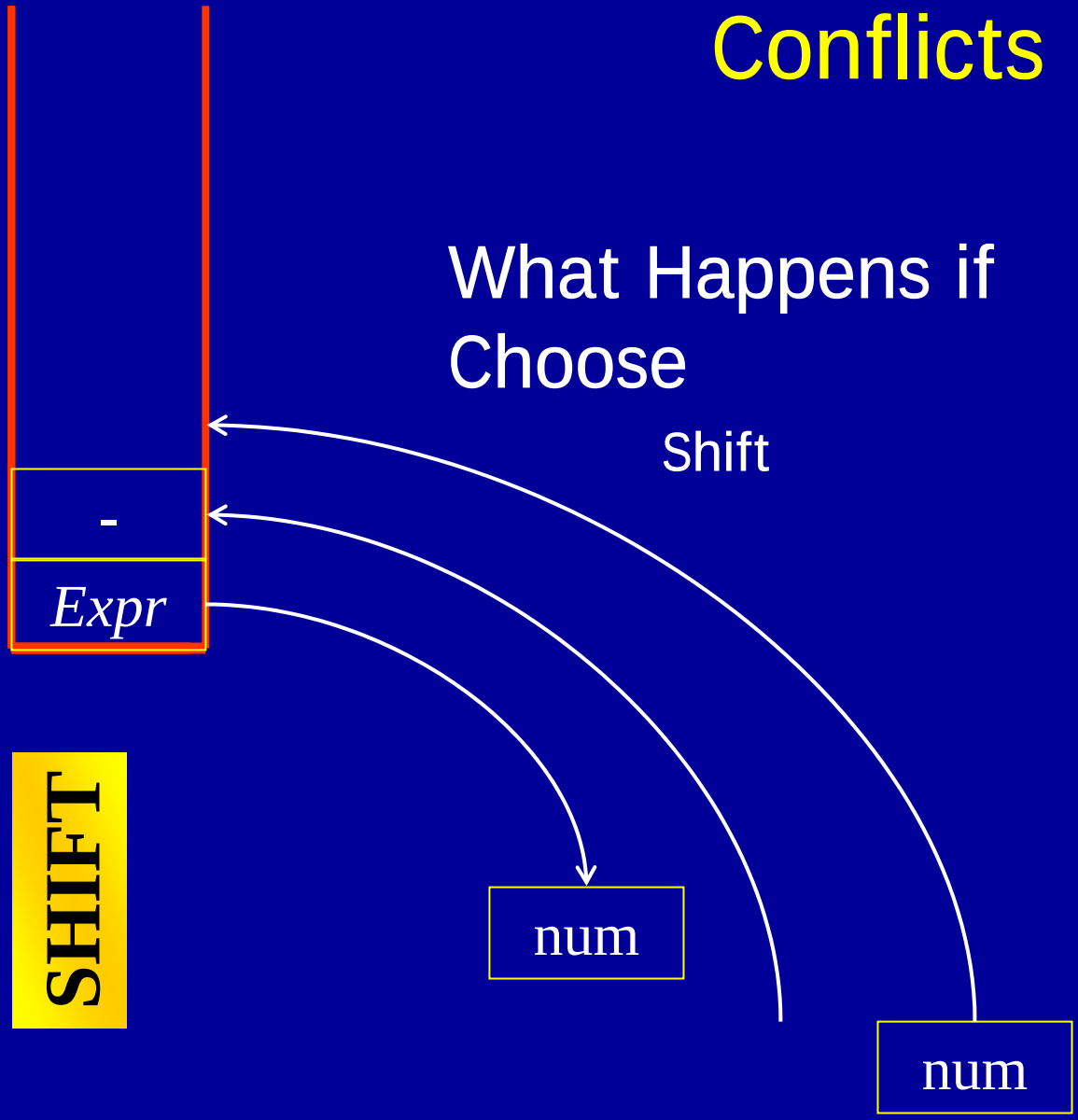
Op → +

Op → -

Op → *

What Happens if Choose

Shift



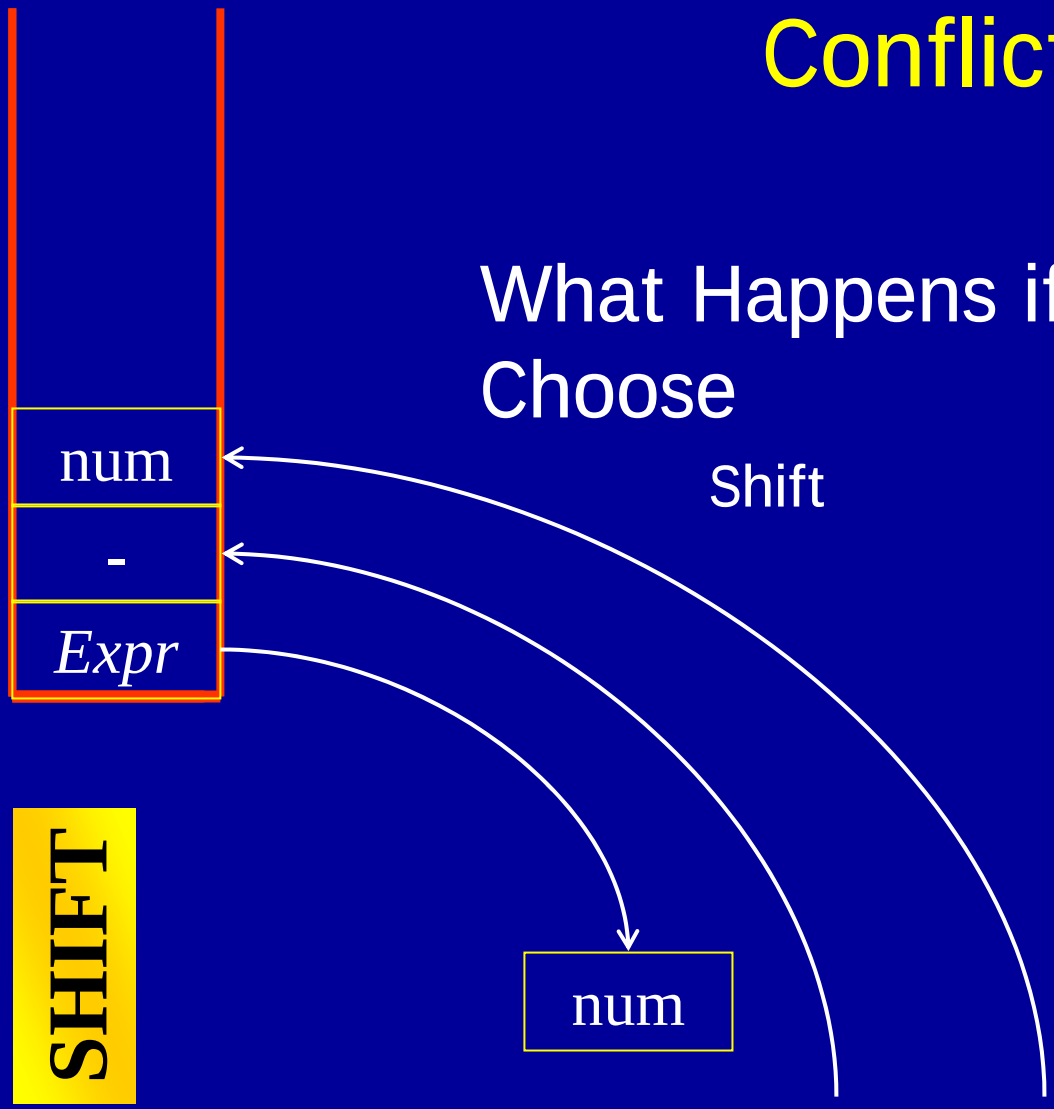
SHIFT

Conflicts

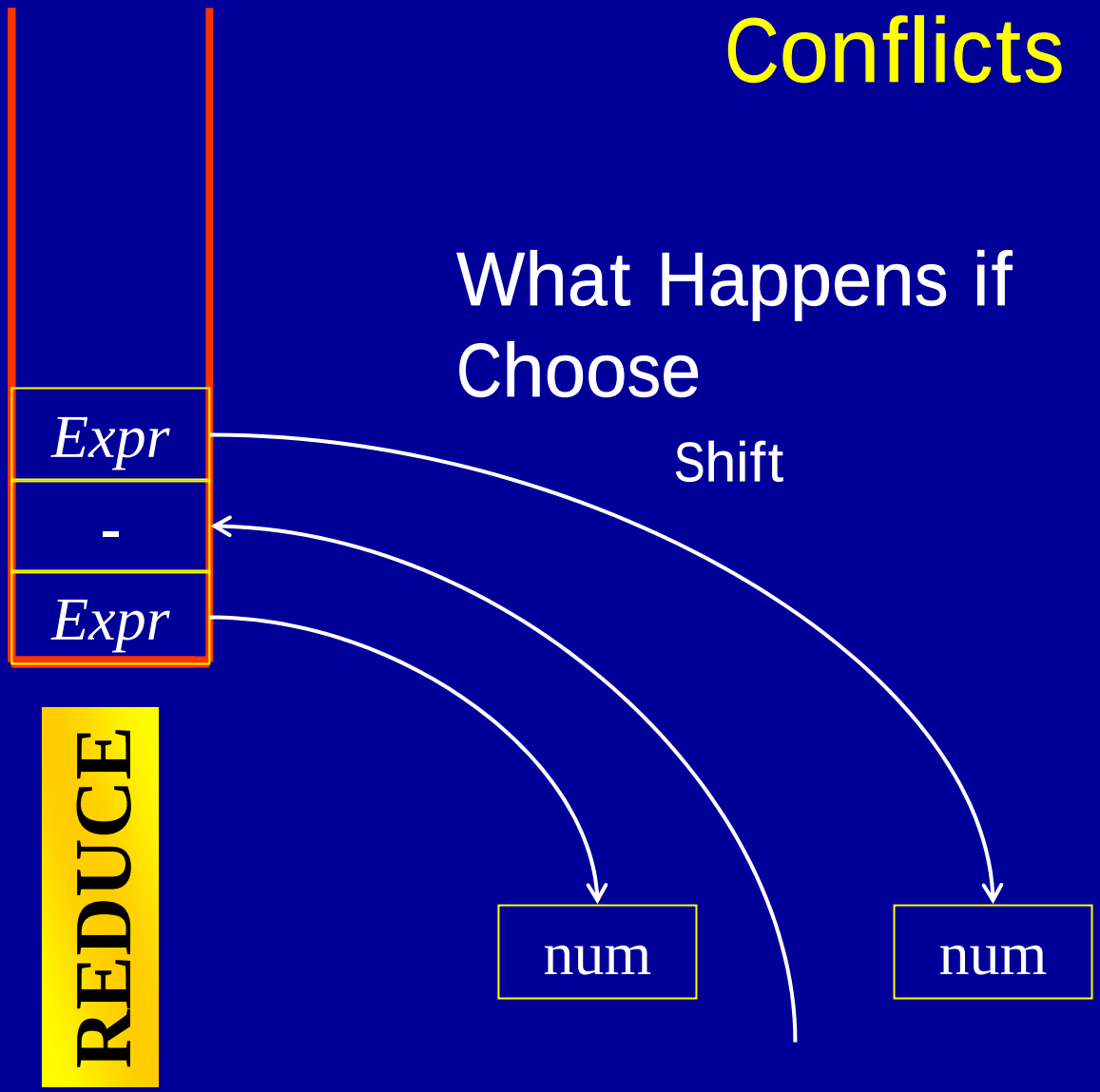
Expr → *Expr Op Expr*
Expr → *Expr* - *Expr*
Expr → (*Expr*)
Expr → *Expr* -
Expr → num
Op → +
Op → -
Op → *

What Happens if Choose

Shift



Conflicts



$Expr \rightarrow Expr\ Op\ Expr$

$Expr \rightarrow Expr$ - $Expr$

$Expr \rightarrow$ ($Expr$)

$Expr \rightarrow Expr$ -

$Expr \rightarrow$ num

$Op \rightarrow$ +

$Op \rightarrow$ -

$Op \rightarrow$ *

Conflicts

Expr → *Expr Op Expr*

Expr → *Expr* - *Expr*

Expr → (*Expr*)

Expr → *Expr* -

Expr → num

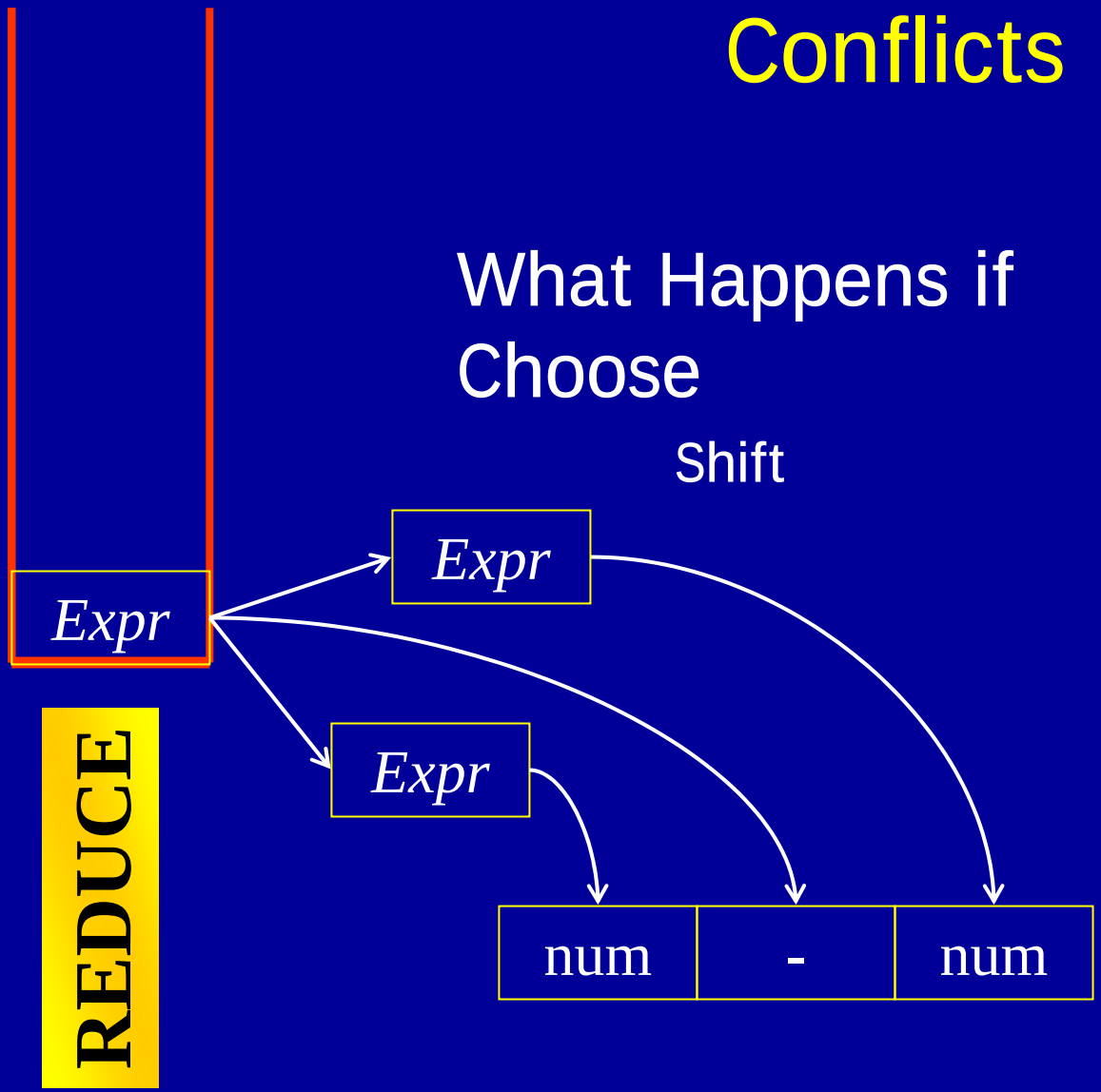
Op → +

Op → -

Op → *

What Happens if Choose

Shift

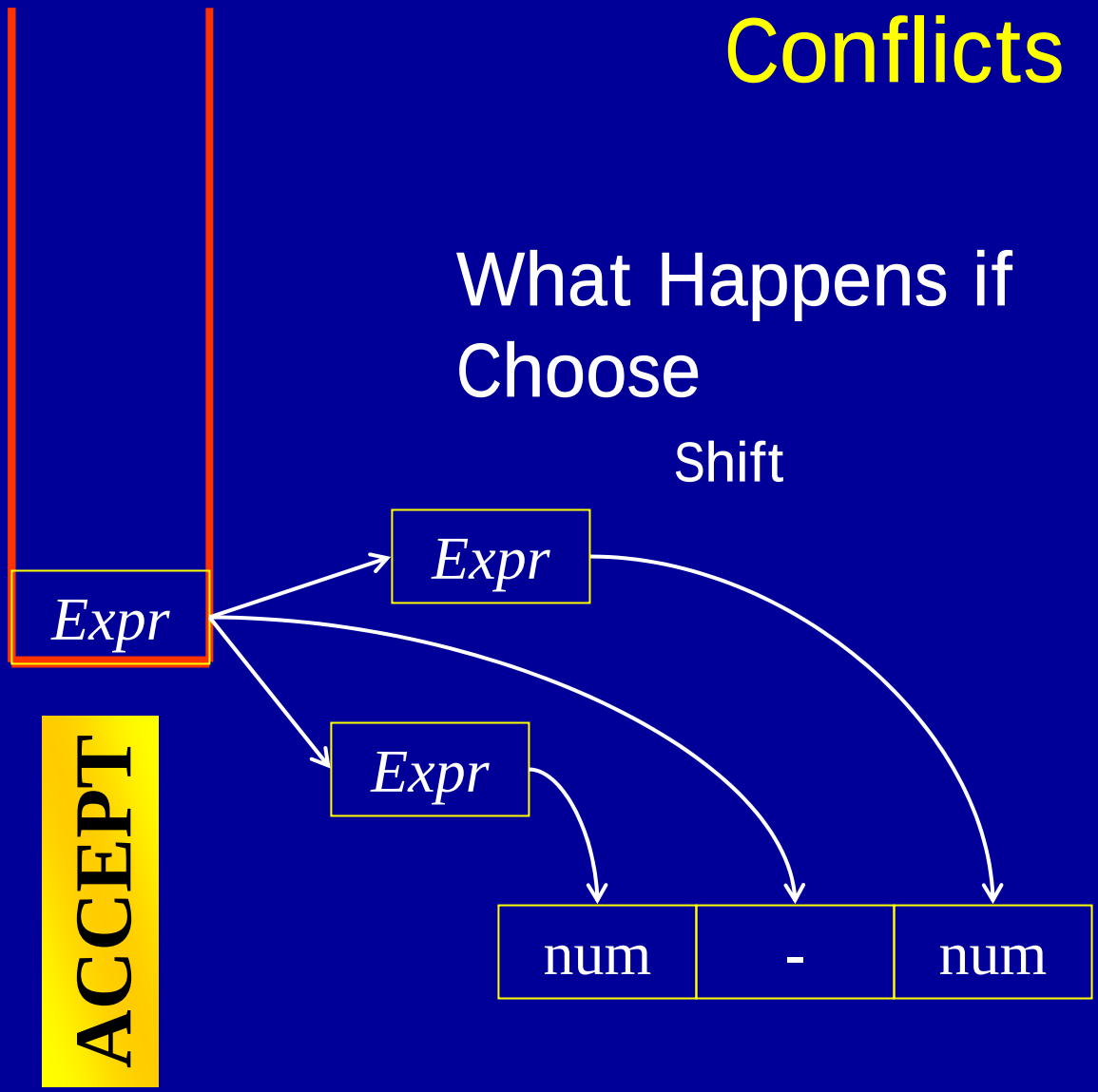


Conflicts

Expr → *Expr Op Expr*
Expr → *Expr* - *Expr*
Expr → (*Expr*)
Expr → *Expr* -
Expr → num
Op → +
Op → -
Op → *

What Happens if Choose

Shift



Shift/Reduce/Reduce Conflict

This Shift/Reduce Conflict Reflects Ambiguity in Grammar

Expr → *Expr Op Expr*

Expr → *Expr* - *Expr*

Expr → (*Expr*)

Expr → *Expr* -

Expr → num

Op → +

Op → -

Op → *

num

num

Shift/Reduce/Reduce Conflict

This Shift/Reduce Conflict Reflects Ambiguity in Grammar

Expr → *Expr Op Expr*

~~*Expr* → *Expr* - *Expr*~~

Expr → (*Expr*)

Expr → *Expr* -

Expr → num

Op → +

Op → -

Op → *

Eliminate by Hacking Grammar

num

num

Shift/Reduce/Reduce Conflict

This Shift/Reduce Conflict Can Be Eliminated By Lookahead of One Symbol

$Expr \rightarrow Expr\ Op\ Expr$

$Expr \rightarrow Expr\ -\ Expr$

$Expr \rightarrow (Expr)$

$Expr \rightarrow Expr\ -$

$Expr \rightarrow num$

$Op \rightarrow +$

$Op \rightarrow -$

$Op \rightarrow *$

Parser Generator Should Handle It

num

num

Constructing a Parser

- We will construct version with no lookahead
- Key Decisions
  - Shift or Reduce
  - Which Production to Reduce
- Basic Idea
  - Build a DFA to control shift and reduce actions
  - In effect, convert grammar to pushdown automaton
  - Encode finite state control in parse table

Parser State

- Input Token Sequence ($ for end of input)
- Current State from Finite State Automaton
- Two Stacks
  - State Stack (implements finite state automaton)
  - Symbol Stack (terminals from input and nonterminals from reductions)

Integrating Finite State Control

- Actions
  - Push Symbols and States Onto Stacks
  - Reduce According to a Given Production
  - Accept
- Selected action is a function of
  - Current input symbol
  - Current state of finite state control
- Each action specifies next state
- Implement control using parse table

Parse Tables

	ACTION			Goto
State	**(**	**)**	**$**	**X**
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (3)	reduce (3)	reduce (3)	

- Implements finite state control
- At each step, look up
  - Table[top of state stack] [input symbol]
- Then carry out the action

Parse Table Example

	ACTION			Goto
State	**(**	**)**	**$**	**X**
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (3)	reduce (3)	reduce (3)	

State Stack

Symbol Stack

Input

(())

Grammar

$S \rightarrow X\$$ (1)

$X \rightarrow (X)$ (2)

$X \rightarrow (\ )$ (3)

s0

X

Parser Tables

	ACTION			Goto
State	**(**	**)**	**$**	**X**
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (3)	reduce (3)	reduce (3)	

- Shift to s*n*
  - Push input token into the symbol stack
  - Push s*n* into state stack
  - Advance to next input symbol

Parser Tables

	ACTION			Goto
State	**(**	**)**	**$**	**X**
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (3)	reduce (3)	reduce (3)	

- Reduce (*n*)
  - Pop both stacks as many times as the number of symbols on the RHS of rule *n*
  - Push LHS of rule *n* into symbol stack

Parser Tables

	ACTION			Goto
State	**(**	**)**	**$**	**X**
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (3)	reduce (3)	reduce (3)	

- Reduce (*n*) (continued)
  - Look up
  - Table[top of the state stack][top of symbol stack]
  - Push that state (in goto part of table) onto state stack

Parser Tables

	ACTION			Goto
State	**(**	**)**	**$**	**X**
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (3)	reduce (3)	reduce (3)	

- Accept
  - Stop parsing and report success

Parse Table In Action

	ACTION			Goto
State	**(**	**)**	**$**	**X**
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (3)	reduce (3)	reduce (3)	

State Stack | Symbol Stack | Input | Grammar

(())$

$S \rightarrow X\$$ (1)

$X \rightarrow (X)$ (2)

$X \rightarrow (\ )$ (3)

s0

Parse Table In Action

	ACTION			Goto
State	**(**	**)**	**$**	**X**
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (3)	reduce (3)	reduce (3)	

State Stack | Symbol Stack | Input | Grammar

(())$

$S \rightarrow X\$$ (1)

$X \rightarrow (X)$ (2)

$X \rightarrow (\ )$ (3)

s0

Parse Table In Action

	ACTION			Goto
State	**(**	**)**	**$**	**X**
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (3)	reduce (3)	reduce (3)	

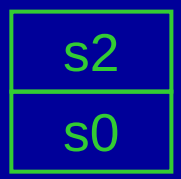
State Stack | Symbol Stack | Input | Grammar

())$

$S \rightarrow X\$$ (1)

$X \rightarrow (X)$ (2)

$X \rightarrow (\ )$ (3)



Parse Table In Action

	ACTION			Goto
State	**(**	**)**	**$**	**X**
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (3)	reduce (3)	reduce (3)	

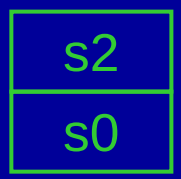
State Stack | Symbol Stack | Input | Grammar

())$

$S \rightarrow X\$$ (1)

$X \rightarrow (X)$ (2)

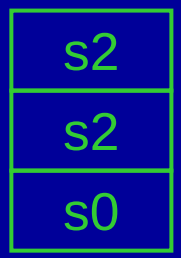
$X \rightarrow (\ )$ (3)



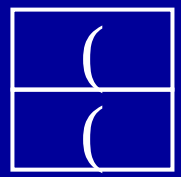
Parse Table In Action

	ACTION			Goto
State	**(**	**)**	**$**	**X**
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (3)	reduce (3)	reduce (3)	

State Stack



Symbol Stack



Input

))$

Grammar

$S \rightarrow X\$$ (1)

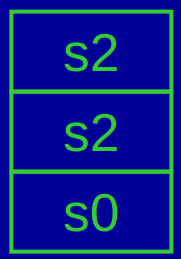
$X \rightarrow (X)$ (2)

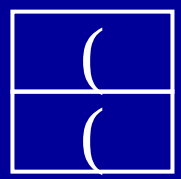
$X \rightarrow (\ )$ (3)

Parse Table In Action

	ACTION			Goto
State	**(**	**)**	**$**	**X**
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (3)	reduce (3)	reduce (3)	

State Stack Symbol Stack Input Grammar





))$

$S \rightarrow X\$$ (1)

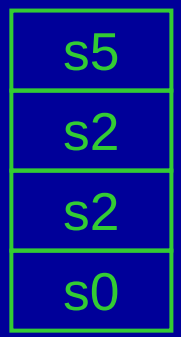
$X \rightarrow (X)$ (2)

$X \rightarrow (\ )$ (3)

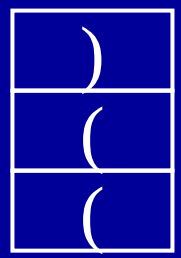
Parse Table In Action

	ACTION			Goto
State	**(**	**)**	**$**	**X**
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (3)	reduce (3)	reduce (3)	

State Stack



Symbol Stack



Input

)$

Grammar

$S \rightarrow X\$$ (1)

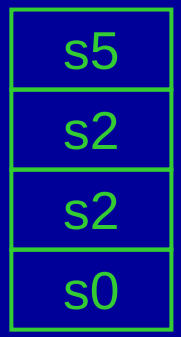
$X \rightarrow (X)$ (2)

$X \rightarrow (\ )$ (3)

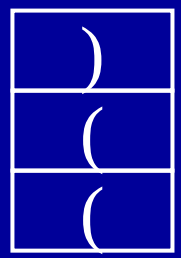
Parse Table In Action

	ACTION			Goto
State	**(**	**)**	**$**	**X**
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (3)	reduce (3)	reduce (3)	

State Stack



Symbol Stack



Input

)$

Grammar

$S \rightarrow X\$$ (1)

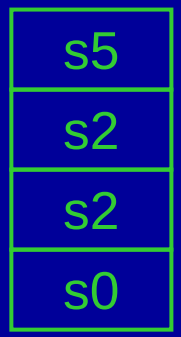
$X \rightarrow (X)$ (2)

$X \rightarrow (\ )$ (3)

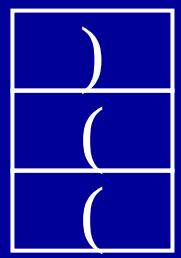
Step One: Pop Stacks

		ACTION		Goto
State	**(**	**)**	**$**	**X**
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (3)	reduce (3)	reduce (3)	

State Stack



Symbol Stack



Input

)$

Grammar

$S \rightarrow X\$$ (1)

$X \rightarrow (X)$ (2)

$X \rightarrow (\ )$ (3)

Step One: Pop Stacks

		ACTION		Goto
State	**(**	**)**	**$**	**X**
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (3)	reduce (3)	reduce (3)	

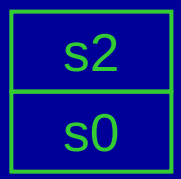
State Stack | Symbol Stack | Input | Grammar

)$

$S \rightarrow X\$$ (1)

$X \rightarrow (X)$ (2)

$X \rightarrow (\ )$ (3)



Step Two: Push Nonterminal

		ACTION		Goto
State	**(**	**)**	**$**	**X**
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (3)	reduce (3)	reduce (3)	

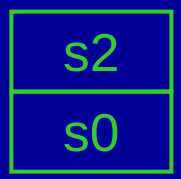
State Stack | Symbol Stack | Input | Grammar

)$

$S \rightarrow X\$$ (1)

$X \rightarrow (X)$ (2)

$X \rightarrow (\ )$ (3)



Step Two: Push Nonterminal

		ACTION		Goto
State	**(**	**)**	**$**	**X**
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (3)	reduce (3)	reduce (3)	

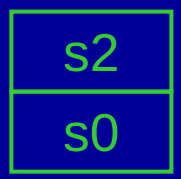
State Stack | Symbol Stack | Input | Grammar

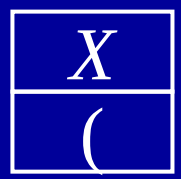
)$

$S \rightarrow X\$$ (1)

$X \rightarrow (X)$ (2)

$X \rightarrow (\ )$ (3)





Step Three: Use Goto, Push New State

	ACTION			Goto
State	**(**	**)**	**$**	**X**
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (3)	reduce (3)	reduce (3)	

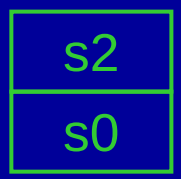
State Stack Symbol Stack Input Grammar

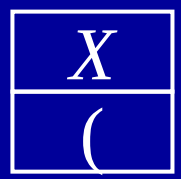
)$

$S \rightarrow X\$$ (1)

$X \rightarrow (X)$ (2)

$X \rightarrow (\ )$ (3)

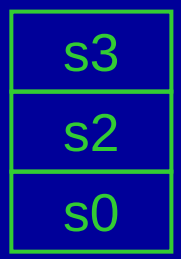


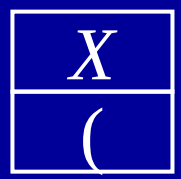


Step Three: Use Goto, Push New State

		ACTION		Goto
State	**(**	**)**	**$**	**X**
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (3)	reduce (3)	reduce (3)	

State Stack | Symbol Stack | Input | Grammar





)$

$S \rightarrow X\$$ (1)

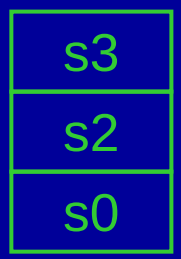
$X \rightarrow (X)$ (2)

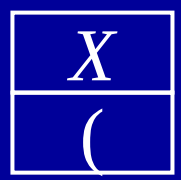
$X \rightarrow (\ )$ (3)

Parse Table In Action

	ACTION			Goto
State	**(**	**)**	**$**	**X**
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (3)	reduce (3)	reduce (3)	

State Stack | Symbol Stack | Input | Grammar





)$

$S \rightarrow X\$$ (1)

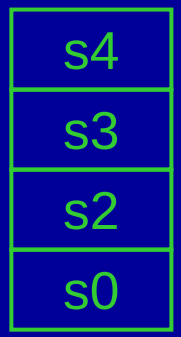
$X \rightarrow (X)$ (2)

$X \rightarrow (\ )$ (3)

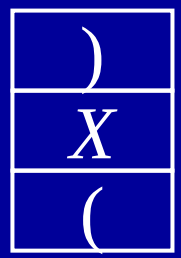
Parse Table In Action

	ACTION			Goto
State	**(**	**)**	**$**	**X**
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (3)	reduce (3)	reduce (3)	

State Stack



Symbol Stack



Input

$

Grammar

$S \rightarrow X\$$ (1)

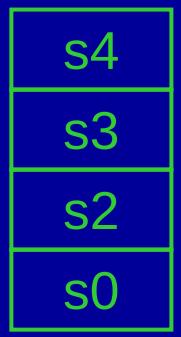
$X \rightarrow (X)$ (2)

$X \rightarrow (\ )$ (3)

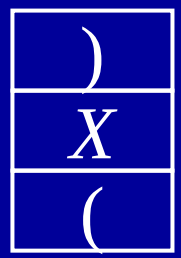
Parse Table In Action

	ACTION			Goto
State	**(**	**)**	**$**	**X**
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (3)	reduce (3)	reduce (3)	

State Stack



Symbol Stack



Input

$

Grammar

$S \rightarrow X\$$ (1)

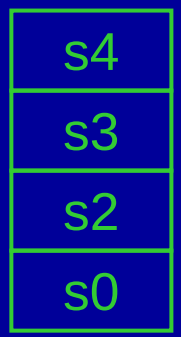
$X \rightarrow (X)$ (2)

$X \rightarrow (\ )$ (3)

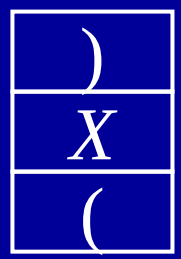
Step One: Pop Stacks

		ACTION		Goto
State	**(**	**)**	**$**	**X**
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (3)	reduce (3)	reduce (3)	

State Stack



Symbol Stack



Input

$

Grammar

$S \rightarrow X\$$ (1)

$X \rightarrow (X)$ (2)

$X \rightarrow (\ )$ (3)

Step One: Pop Stacks

		ACTION		Goto
State	**(**	**)**	**$**	**X**
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (3)	reduce (3)	reduce (3)	

State Stack | Symbol Stack | Input | Grammar

$

$S \rightarrow X\$$ (1)

$X \rightarrow (X)$ (2)

$X \rightarrow (\ )$ (3)

s0

Step Two: Push Nonterminal

	ACTION			Goto
State	**(**	**)**	**$**	**X**
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (3)	reduce (3)	reduce (3)	

State Stack | Symbol Stack | Input | Grammar

$

$S \rightarrow X\$$ (1)

$X \rightarrow (X)$ (2)

$X \rightarrow (\ )$ (3)

s0

Step Two: Push Nonterminal

		ACTION		Goto
State	**(**	**)**	**$**	**X**
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (3)	reduce (3)	reduce (3)	

State Stack | Symbol Stack | Input | Grammar

$

$S \rightarrow X\$$ (1)

$X \rightarrow (X)$ (2)

$X \rightarrow (\ )$ (3)

s0

X

Step Three: Use Goto, Push New State

		ACTION		Goto
State	(	)	$	X
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (3)	reduce (3)	reduce (3)	

State Stack | Symbol Stack | Input | Grammar

$

$S \rightarrow X\$$ (1)

$X \rightarrow (X)$ (2)

$X \rightarrow (\ )$ (3)

s0

X

Step Three: Use Goto, Push New State

		ACTION		Goto
State	(	)	$	X
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (3)	reduce (3)	reduce (3)	

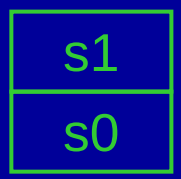
State Stack | Symbol Stack | Input | Grammar

$

$S \rightarrow X\$$ (1)

$X \rightarrow (X)$ (2)

$X \rightarrow (\ )$ (3)



Accept the String!

		ACTION		Goto
State	(	)	$	X
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (3)	reduce (3)	reduce (3)	

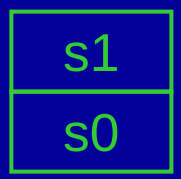
State Stack | Symbol Stack | Input | Grammar

$

$S \rightarrow X\$$ (1)

$X \rightarrow (X)$ (2)

$X \rightarrow (\ )$ (3)



Key Concepts

- Pushdown automaton for parsing
  - Stack, Finite state control
  - Parse actions: shift, reduce, accept
- Parse table for controlling parser actions
  - Indexed by parser state and input symbol
  - Entries specify action and next state
  - Use state stack to help control
- Parse tree construction
  - Reads input from left to right
  - Bottom-up construction of parse tree