

6.011: Signals, Systems & Inference

Lec 3

Energy spectral density

Inner (dot) product of signals

$$\langle \mathbf{x}, \mathbf{v} \rangle = \sum_k x[k]v[k] \quad \text{for real signals } x[\cdot], v[\cdot]$$

More generally, $p[n] = \sum_k x[k]v[k - n] = x * \overleftarrow{v}[n]$

where $\overleftarrow{v}[\ell] = v[-\ell]$

Transform of reversed signal

$$\text{DTFT of } v[n] = V(e^{j\Omega}) = \sum_n v[n]e^{-j\Omega n}$$

$$\begin{aligned} \text{DTFT of } \overleftarrow{v}[n] &= \sum_n v[-n]e^{-j\Omega n} = \sum_k v[k]e^{j\Omega k} \\ &= V^*(e^{j\Omega}) = V(e^{-j\Omega}) \end{aligned}$$

Transform of inner product

DTFT of $\overleftarrow{v}[n] = V(e^{-j\Omega})$ so

$$p[n] = \sum_k x[k]v[k-n] = x * \overleftarrow{v}[n]$$



$$P(e^{j\Omega}) = X(e^{j\Omega})V(e^{-j\Omega})$$

Zero lag ($n=0$):

Parseval, Rayleigh, Plancherel

$$p[0] = \sum_k x[k]v[k] = \frac{1}{2\pi} \int_{-\pi}^{\pi} P(e^{j\Omega})d\Omega$$

SO

$$\sum_k x[k]v[k] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\Omega})V(e^{-j\Omega})d\Omega$$

Energy

Setting $v[k] = x[k]$,

$$\begin{aligned}\mathcal{E}_x &= \sum_k |x[k]|^2 = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\Omega}) X(e^{-j\Omega}) d\Omega \\ &= \frac{1}{2\pi} \int_{-\pi}^{\pi} |X(e^{j\Omega})|^2 d\Omega\end{aligned}$$

Energy spectral density (ESD)

$$\sum_k x[k]x[k+n] = \underbrace{\bar{R}_{xx}[n]}_{\text{deterministic autocorrelation}}$$

↕ transform pair

$$|X(e^{j\Omega})|^2 = \underbrace{\bar{S}_{xx}(e^{j\Omega})}_{\text{energy spectral density}}$$

Cross (energy) spectral density

$$\bar{R}_{yx}[n] = \sum_k y[k]x[k-n] = y * \overleftarrow{x}[n]$$



$$\bar{S}_{yx}(e^{j\Omega}) = Y(e^{j\Omega})X(e^{-j\Omega})$$

So if $y[n] = h * x[n]$ then

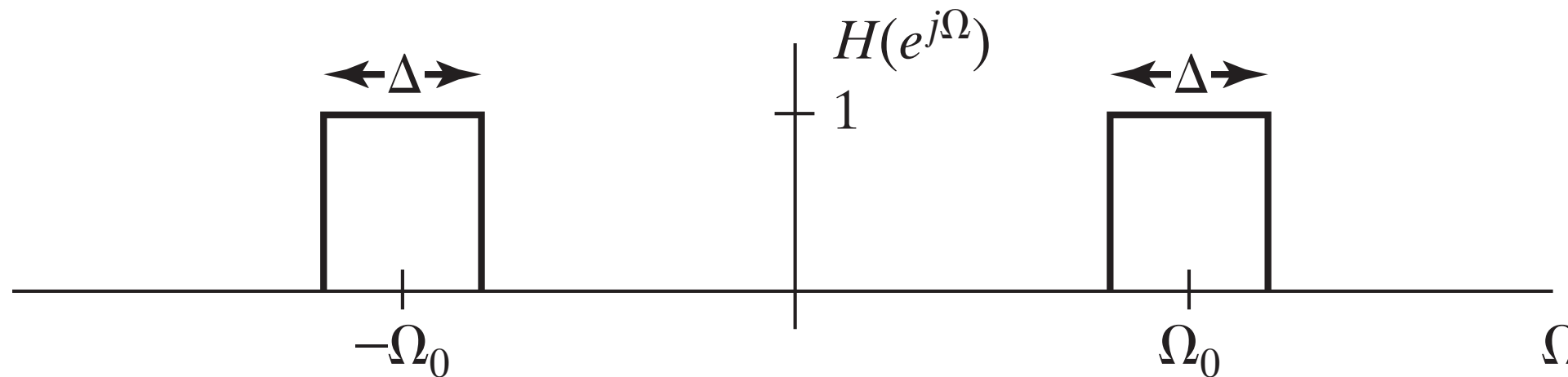
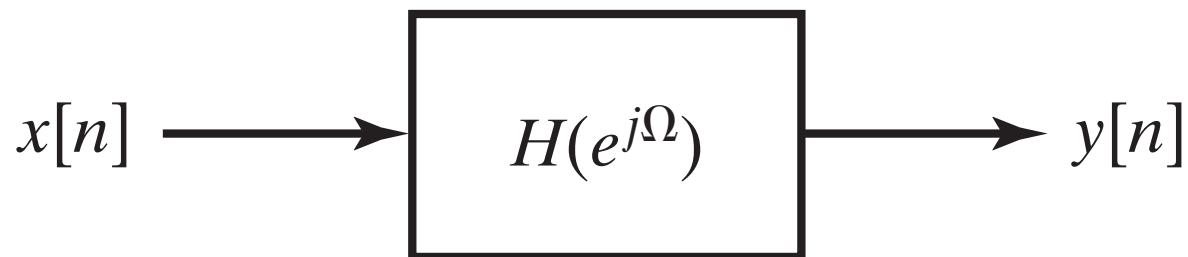
$$\bar{S}_{yx}(e^{j\Omega}) = H(e^{j\Omega})X(e^{j\Omega})X(e^{-j\Omega}) = H(e^{j\Omega})\bar{S}_{xx}(e^{j\Omega})$$

Similarly ...

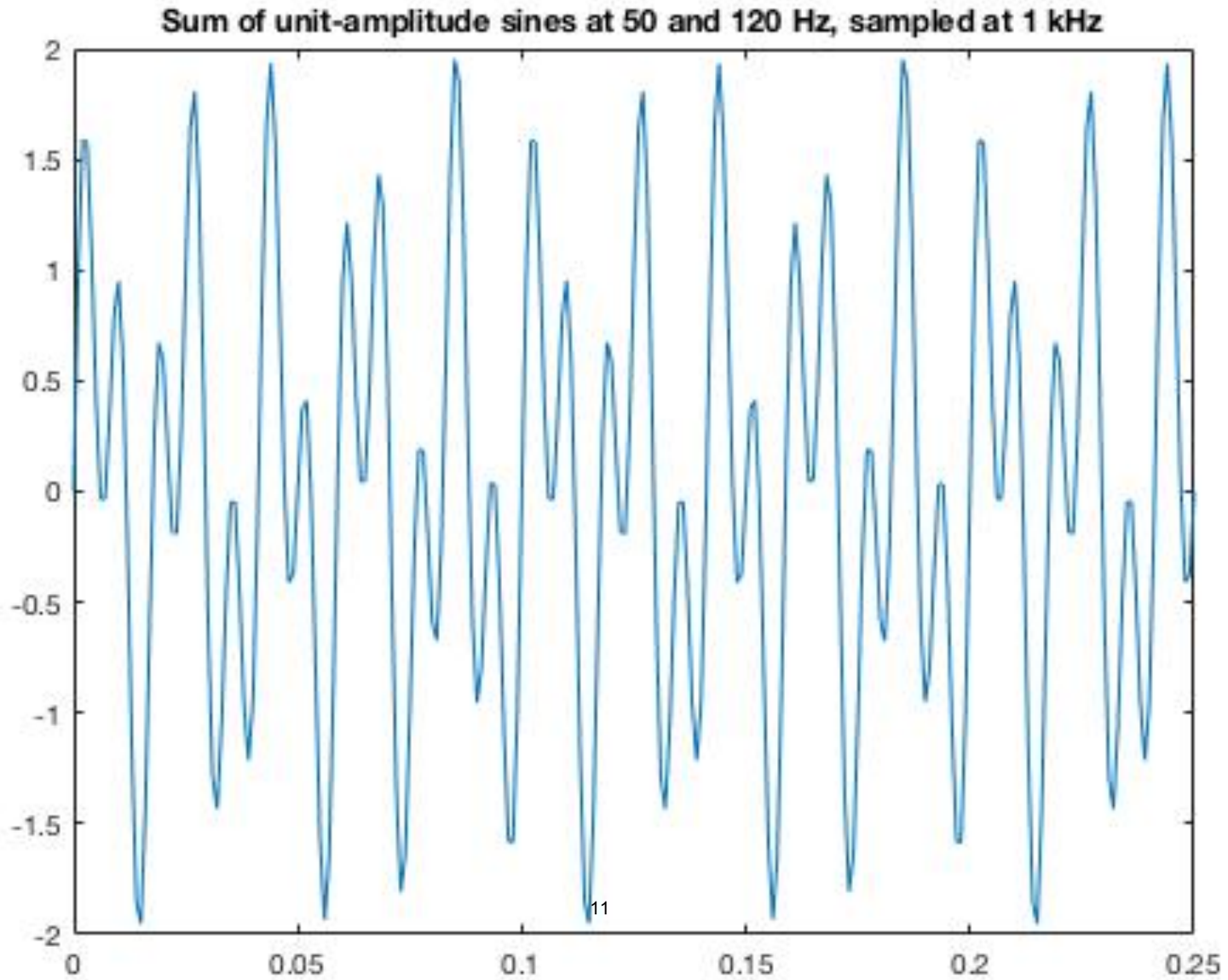
... if $y[n] = h * x[n]$ then

$$\begin{aligned}\bar{S}_{yy}(e^{j\Omega}) &= Y(e^{j\Omega})Y(e^{-j\Omega}) \\ &= H(e^{j\Omega})X(e^{j\Omega})X(e^{-j\Omega})H(e^{-j\Omega}) \\ &= \left| H(e^{j\Omega}) \right|^2 \bar{S}_{xx}(e^{j\Omega})\end{aligned}$$

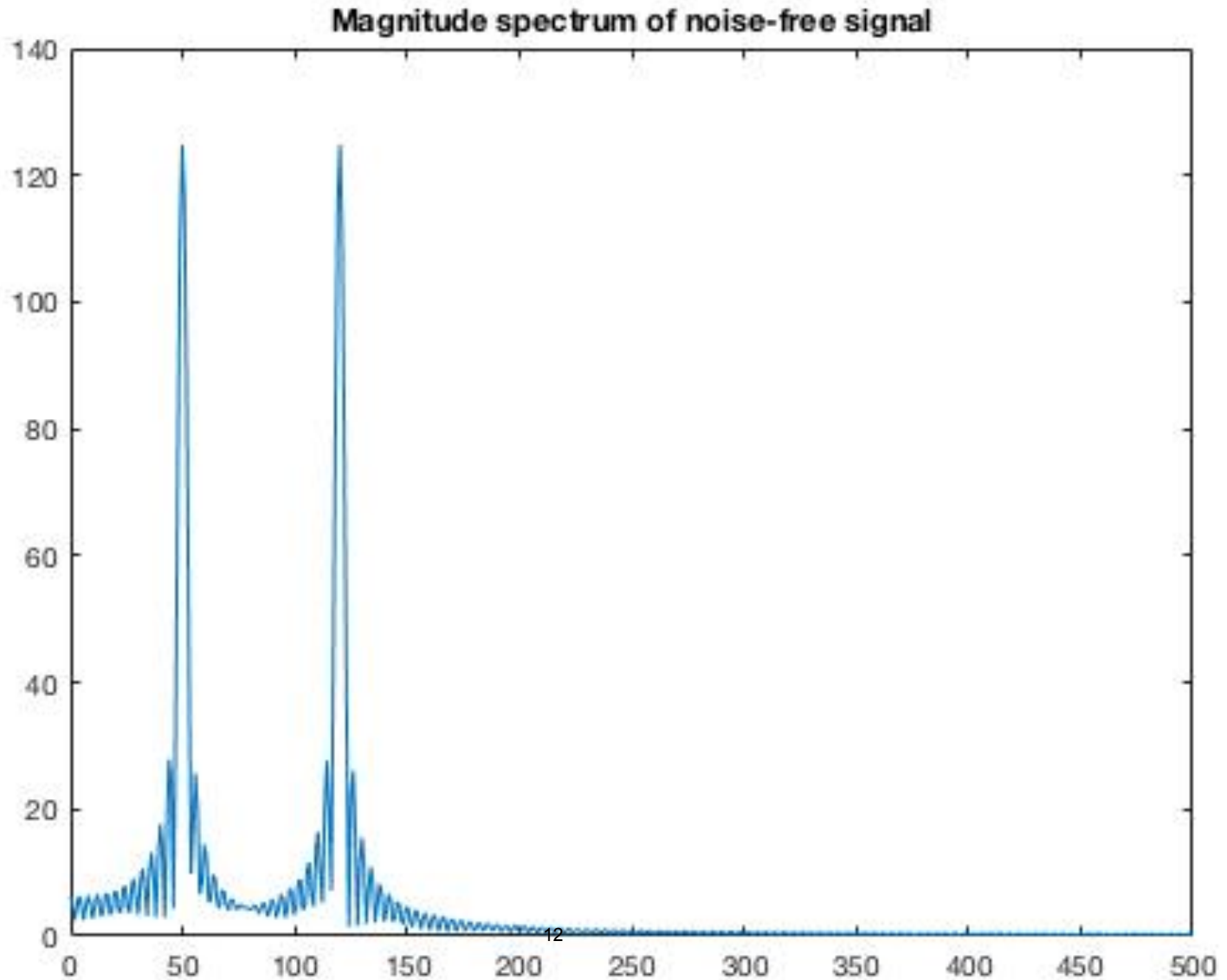
Energy of $x[.]$ in a specified band



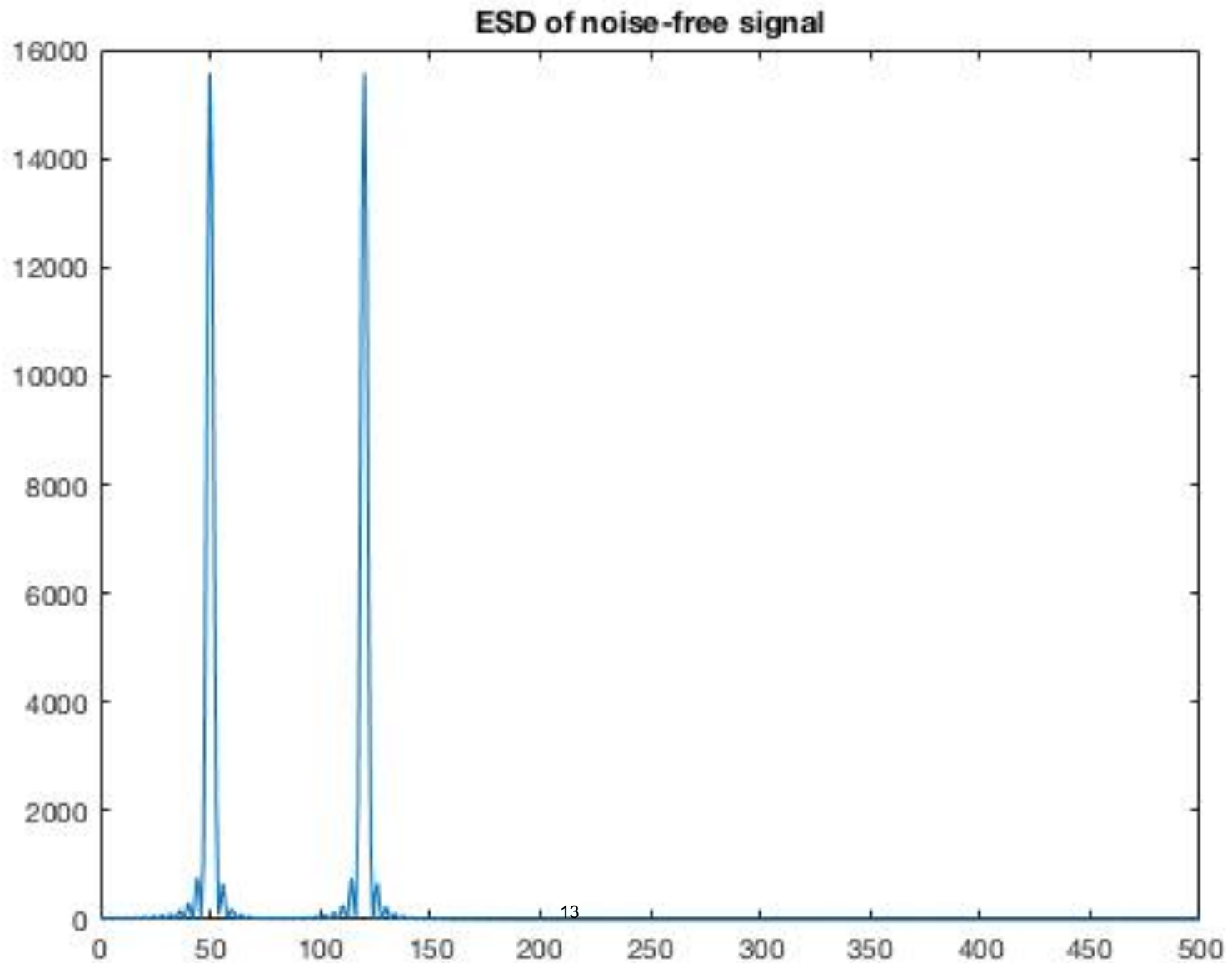
Noise-free signal



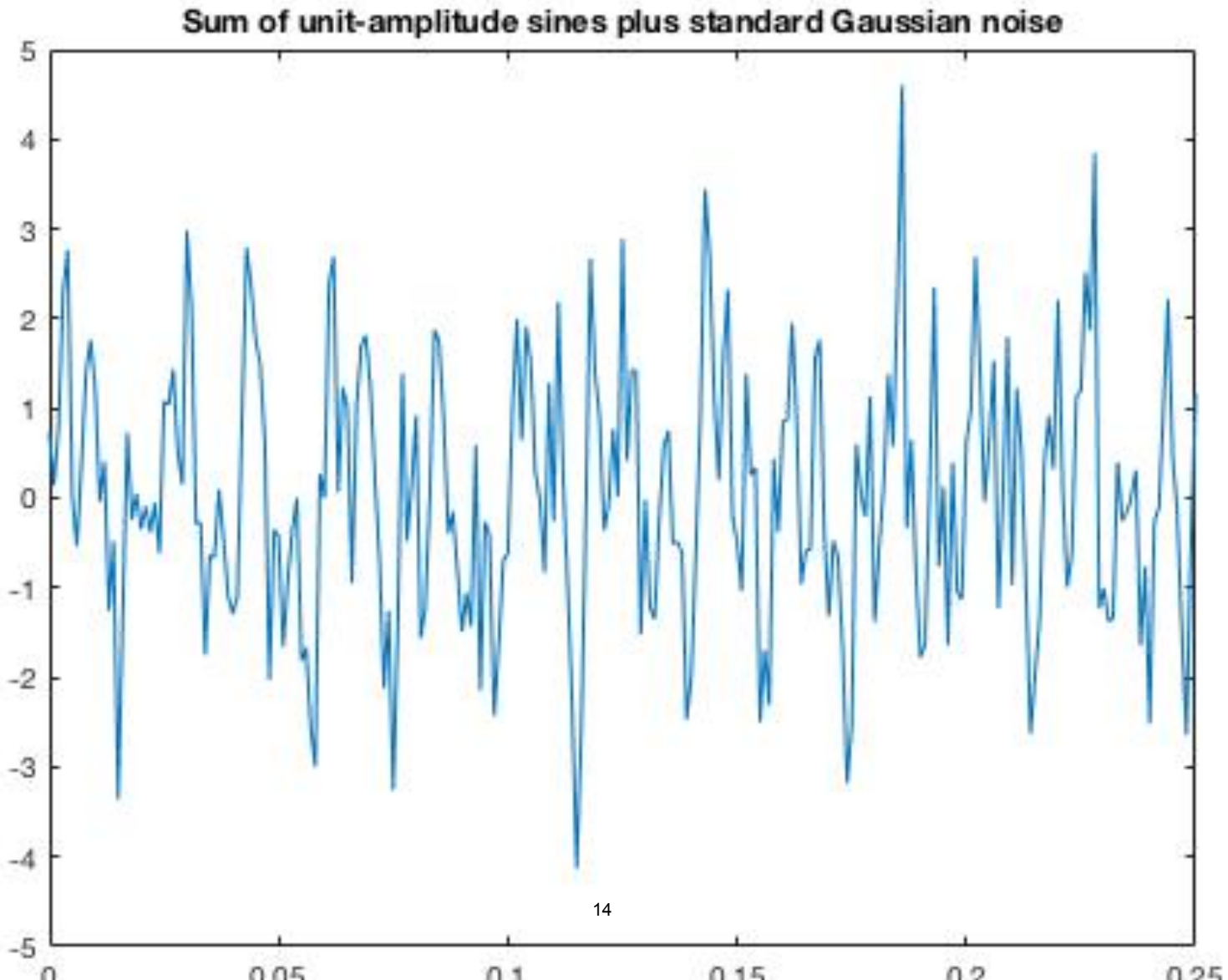
Magnitude spectrum of noise-free signal



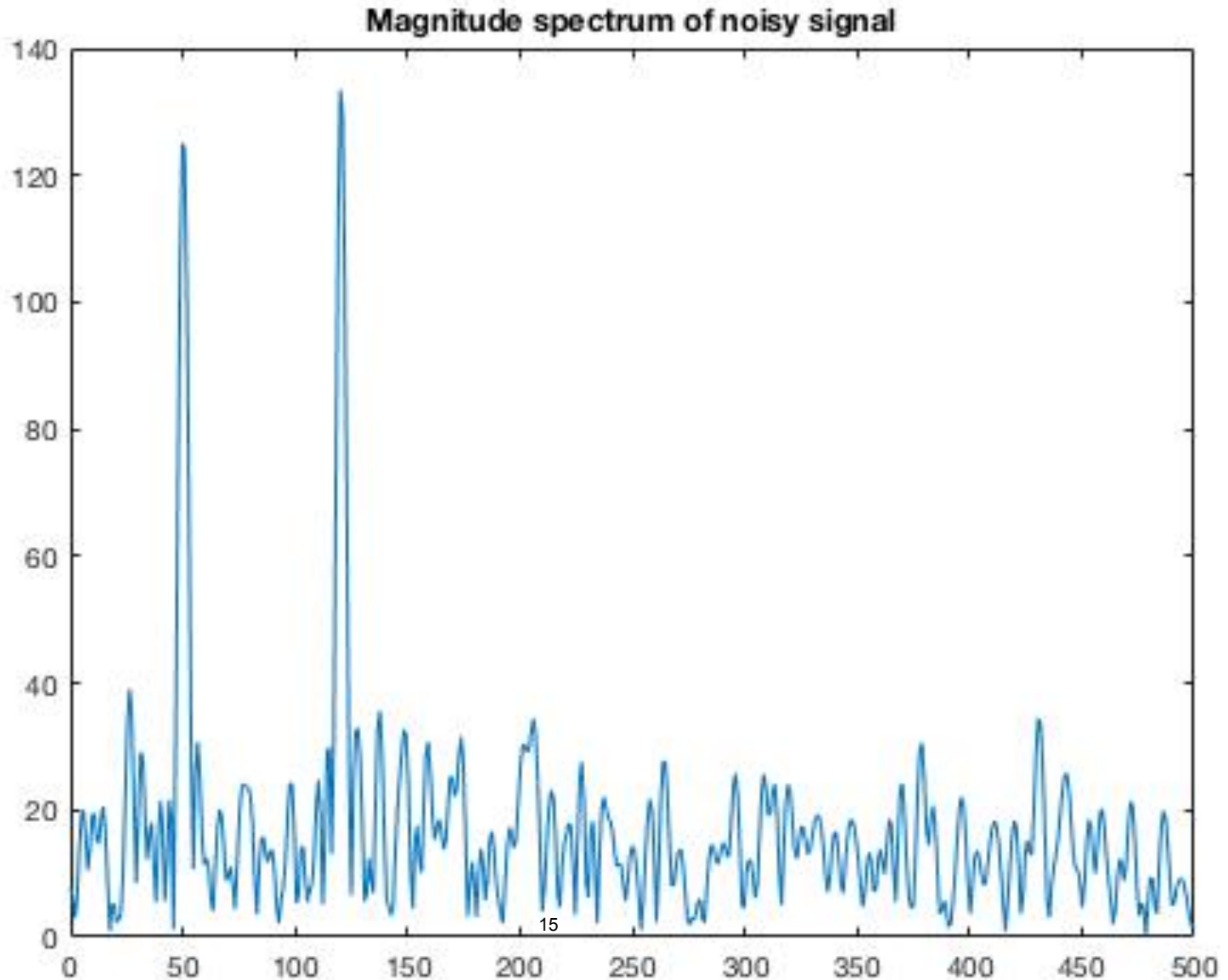
ESD of noise-free signal



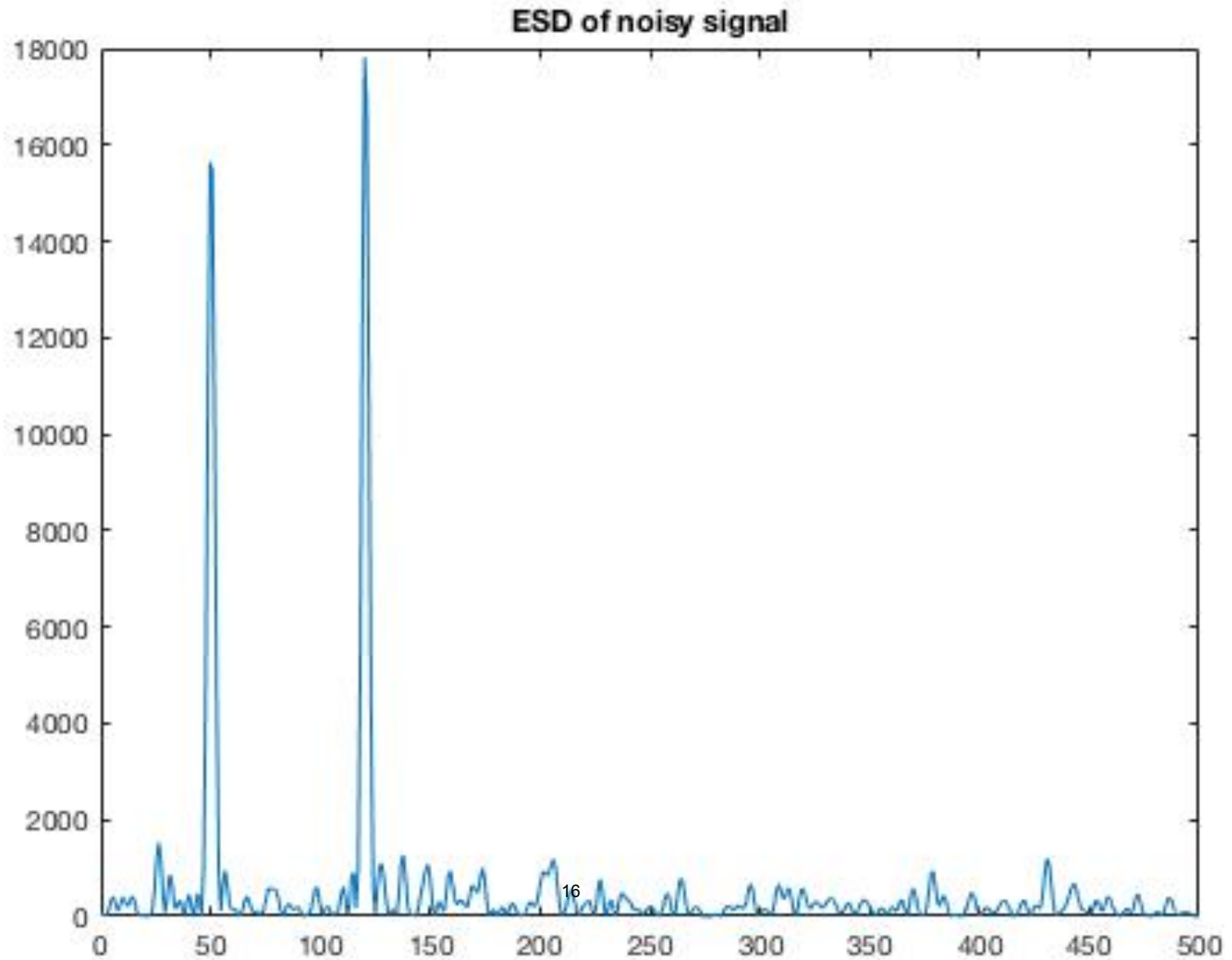
+ unit-intensity white Gaussian noise



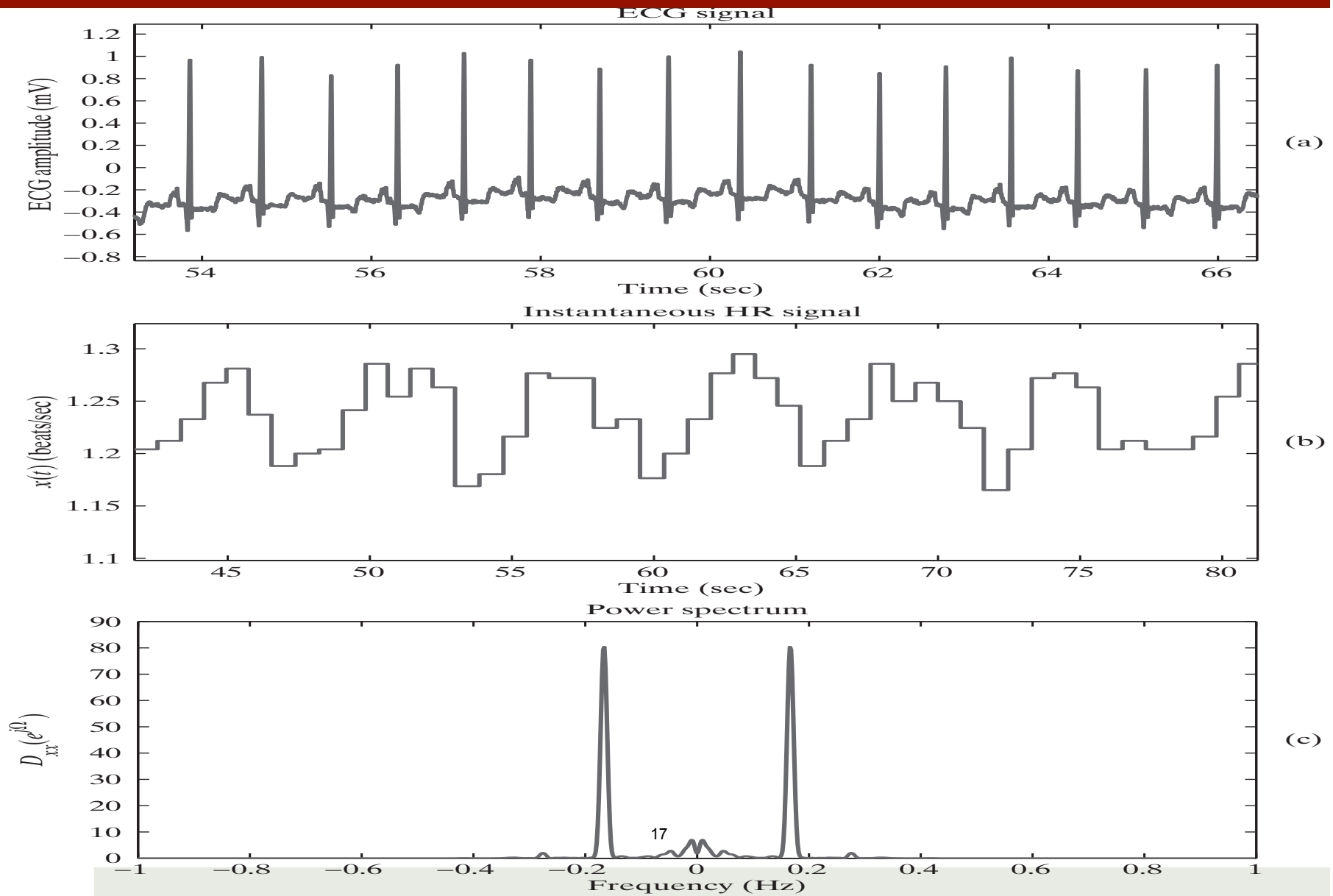
Magnitude spectrum of noisy signal



ESD of noisy signal



Heart rate variability



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