6.011: Signals, Systems & Inference

Lec 2

Transforms

DT convolution to z-transform (and system function)

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k]$$

$$x[n] = z_0^n \,, \quad \text{all } n$$

$$y[n] = \underbrace{\left(\sum_{k=-\infty}^{\infty} h[k]{z_0}^{k} \right)}_{H(z_0)} z_0^n$$

DT convolution to DTFT (and frequency response)

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]x[n \quad k]$$

$$x[n] = \left(e^{j\Omega_0}\right)^n, \text{ all } n, \text{ and } \quad \pi < \Omega_0 \leq \pi$$

$$y[n] = \underbrace{\left(\sum_{k=-\infty}^{\infty} h[k]e^{\quad j\Omega_0 k}\right)}_{H(e^{j\Omega_0})} \left(e^{j\Omega_0}\right)^n$$

Using frequency response to specify response to sinusoidal inputs

$$x[n] = A\cos(\Omega_0 n + \theta)$$

$$y[n] = \left|H(e^{j\Omega_0})\right| A\cos\Big(\Omega_0 n + \theta + \angle H(e^{j\Omega_0})\Big)$$

Frequency response (DTFT of unit sample response)

$$H(e^{j\Omega}) = \sum_{k=-\infty}^{\infty} h[k]e^{\;\;j\Omega k}$$

When is the above infinite sum well-defined?

$$h[n] = \frac{1}{2\pi}\int_{\pi}^{\pi} H(e^{j\Omega})e^{j\Omega n}\,d\Omega$$

Spectral content of a signal (DTFT of the signal)

$$X(e^{j\Omega}) = \sum_{k=-\infty}^{\infty} x[k]e^{\quad j\Omega k}$$

$$x[n] = \frac{1}{2\pi}\int_{\pi}^{\pi} X(e^{j\Omega})e^{j\Omega n}\, d\Omega$$

Classes of DT signals that have DTFTs

Absolutely summable (finite "action", have continuous spectra)

– or not, but …

Square summable (finite "energy", spectra have discontinuities)

– or not, but …

Bounded (finite amplitude, spectra involve generalized functions like impulses)

Convolution in time to multiplication in frequency

Putting together frequency response, spectral content, and superposition, we find

$$y[n] = h * x[n]$$

in the time domain translates to

$$Y(e^{j\Omega}) = H(e^{j\Omega})X(e^{j\Omega})$$

in the frequency domain.

Exercise

Derive the DTFT of

$$r[n] = \sum_{k=-\infty}^{\infty} x[k]x[k-n]$$