Birefringence

<u>Outline</u>

- Polarized Light (Linear & Circular)
- Birefringent Materials
- Quarter-Wave Plate & Half-Wave Plate

Reading: Ch 8.5 in Kong and Shen

True / False

1. The plasma frequency ω_p is the frequency above which a material becomes a plasma.

____

2. The magnitude of the $\vec{E}$ -field of this wave is 1

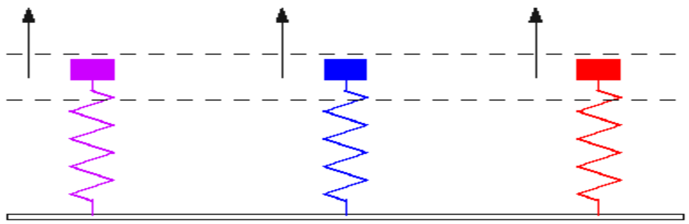
____

$$\vec{E} = (\hat{x} + \hat{y})e^{j(\omega t - kz)}$$

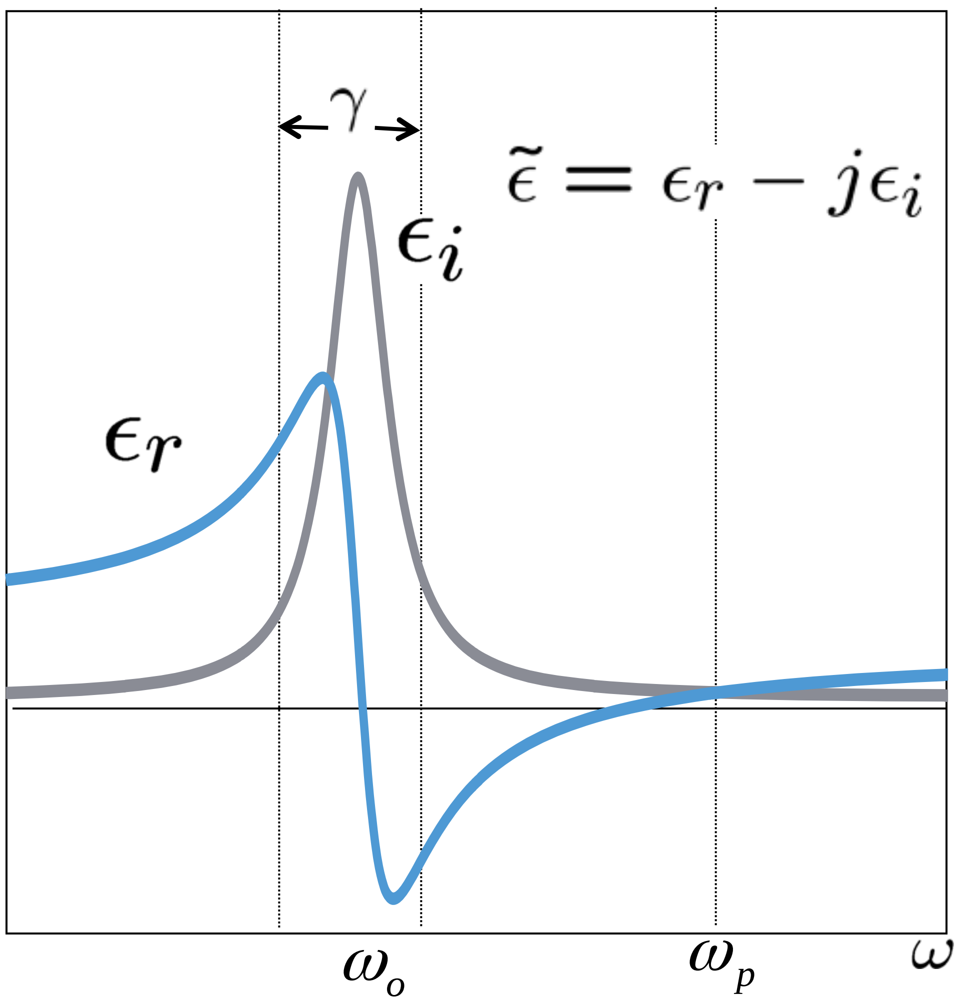
3. The wave above is polarized 45^{o} with respect to the x-axis.

____

Microscopic Lorentz Oscillator Model



$$\epsilon = \epsilon_o \left(1 + \frac{\omega_p^2}{\omega_o^2 - \omega^2 + j\omega\gamma} \right)$$



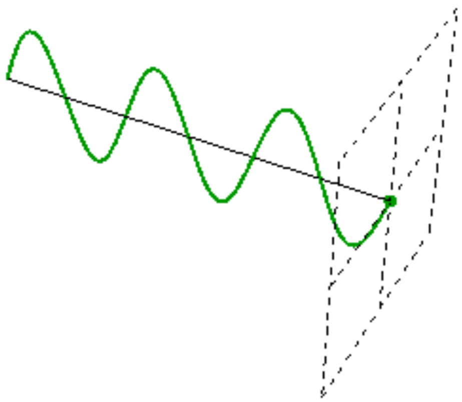
$$\omega_p^2 = \frac{Nq^2}{\epsilon_o m}$$

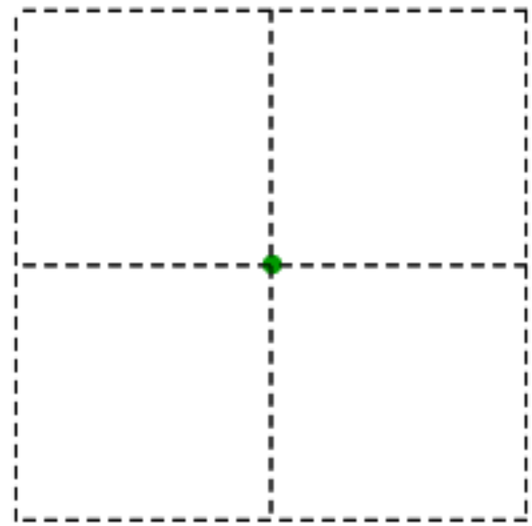
$$\omega_o^2 = \frac{k_{spring}}{m}$$

Sinusoidal Uniform Plane Waves

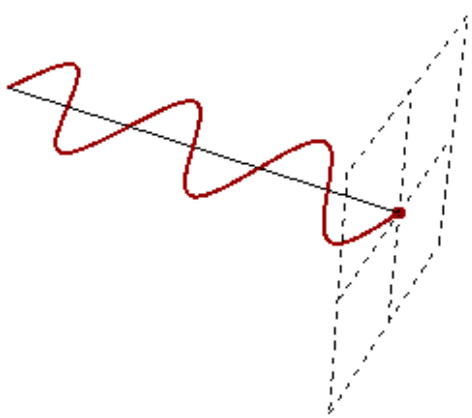
$$E_y = A_1 cos(\omega t - kz)$$

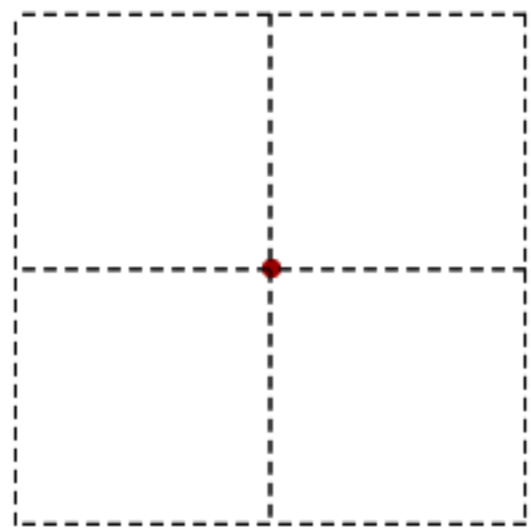
$\widehat{y}$



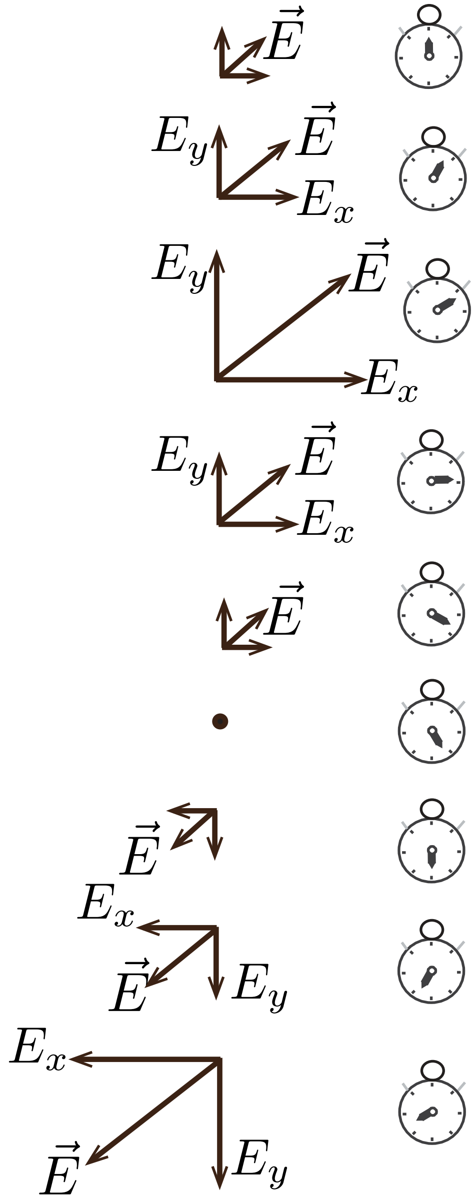


$$E_x = A_2 cos(\omega t - kz)$$



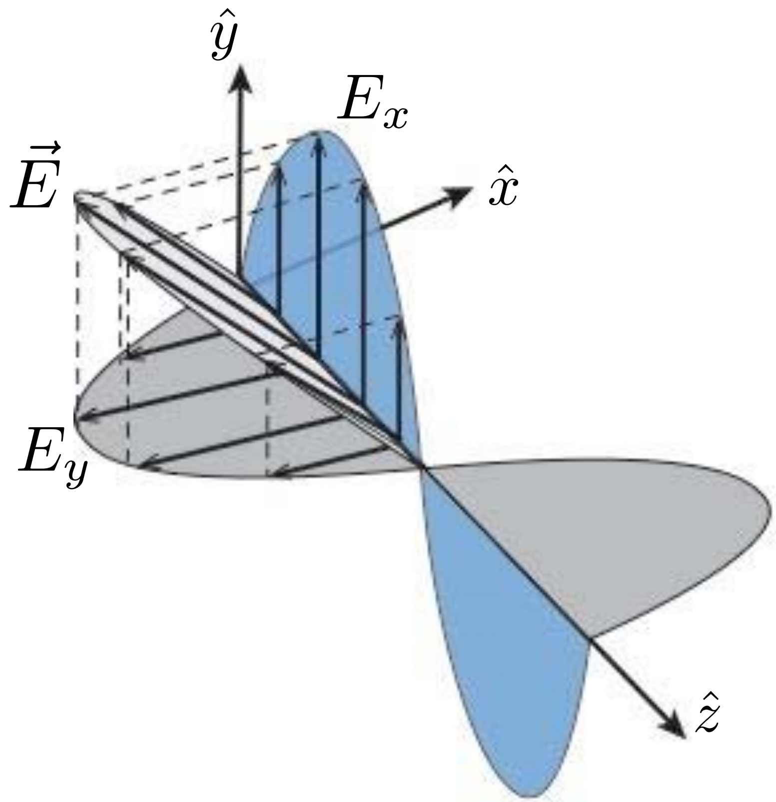


45° Polarization



$$E_x(z,t) = \hat{x}Re\left(\tilde{E}_o e^{j(\omega t - kz)}\right)$$

$$E_y(z,t) = \hat{y}Re\left(\tilde{E}_o e^{j(\omega t - kz)}\right)$$



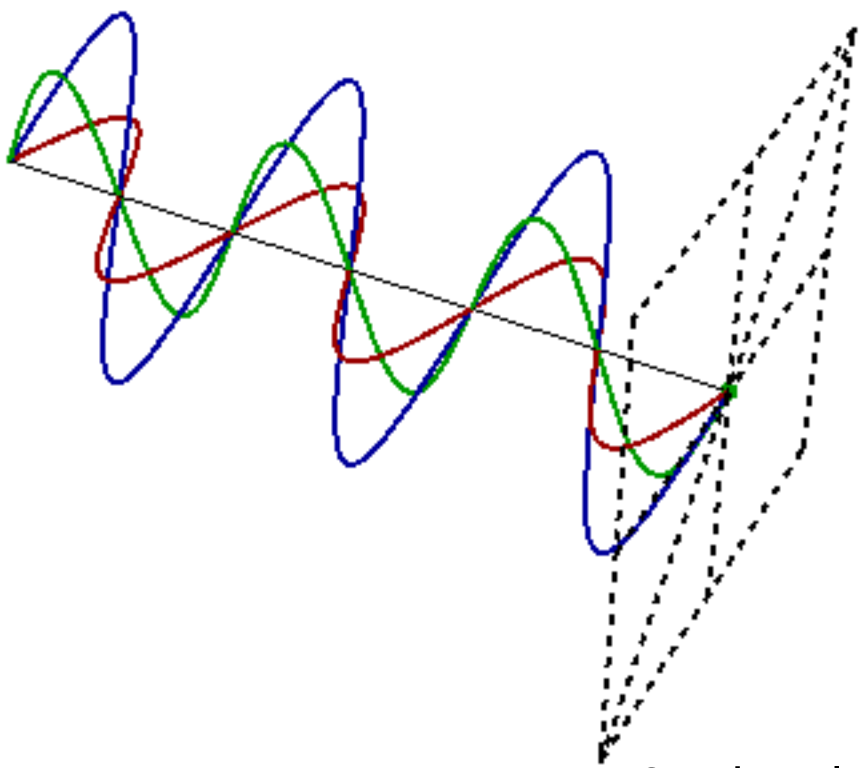
The complex amplitude, $\tilde{E}_o$,is the same for both components.

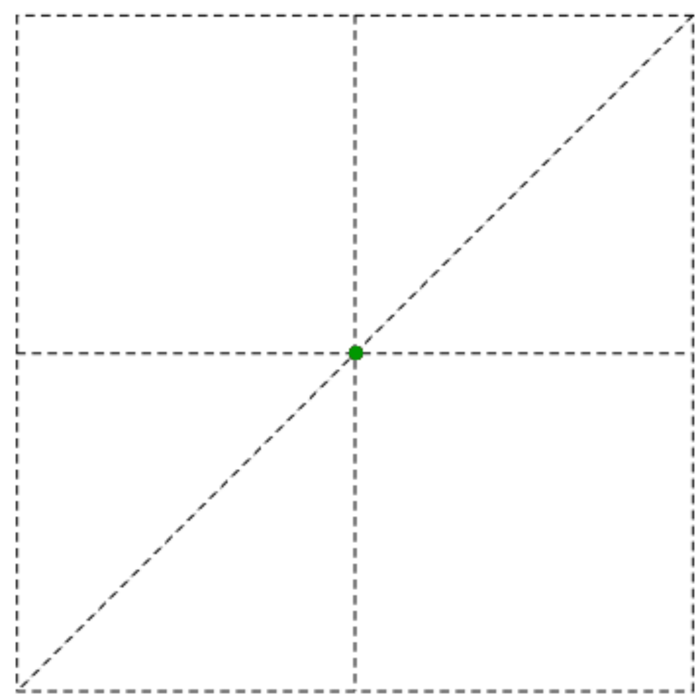
Therefore E_x and E_y are always in phase.

Where is the magnetic field?

Superposition of Sinusoidal Uniform Plane Waves

$$\overline{E} = A\,(cos(\omega t - kz)\,\hat{y} + cos(\omega t - kz)\,\hat{x})$$





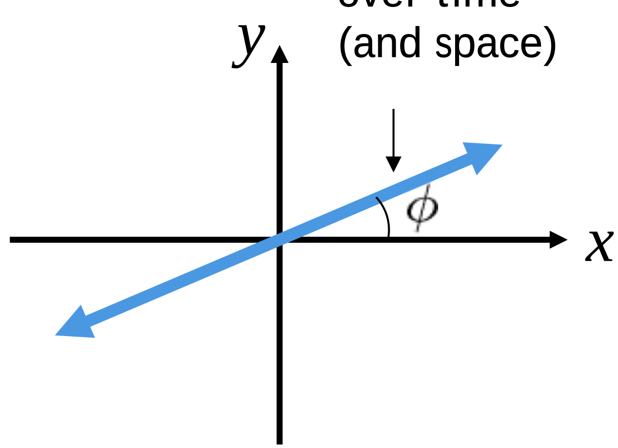
Can it only be at 45^{o} ?

Arbitrary-Angle Linear Polarization

$$E_x(z,t) = \hat{x}\, Re\{\tilde{E}_o\, cos(\phi)\, exp[j(\omega t - kz)]\}$$

$$E_y(z,t) = \hat{y}\, Re\{\tilde{E}_o\, sin(\phi)\, exp[j(\omega t - kz)]\}$$

Here, the *y*-component is in phase
with the *x*-component,
but has different magnitude.



Arbitrary-Angle Linear Polarization

$$E_x(z,t) = \hat{x}\, Re\{\tilde{E}_o\, cos(\phi)\, exp[j(\omega t - kz)]\}$$

$$E_y(z,t) = \hat{y}\, Re\{\tilde{E}_o\, sin(\phi)\, exp[j(\omega t - kz)]\}$$

Specifically:

0° linear (*x*) polarization: $E_y/E_x = 0$

90° linear (*y*) polarization: $E_y/E_x = \infty$

45° linear polarization: $E_y/E_x = 1$

Arbitrary linear polarization: $E_y/E_x = constant$

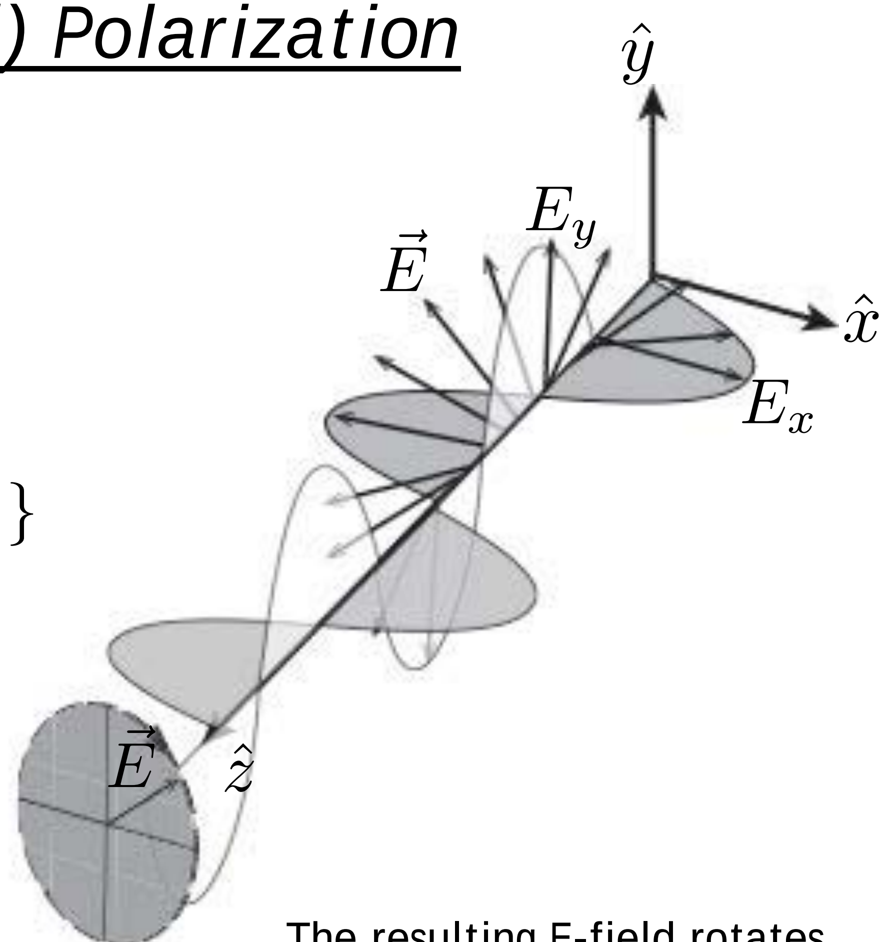
$$\frac{E_y(z,t)}{E_x(z,t)} = \frac{sin(\phi)}{cos(\phi)} = tan(\phi)$$

Circular (or Helical) Polarization

$$E_x(z,t) = \hat{x}\tilde{E}_o sin(\omega t - kz)$$
$$E_y(z,t) = \hat{y}\tilde{E}_o cos(\omega t - kz)$$

... or, more generally,

$$E_x(z,t) = \hat{x}Re\{-j\tilde{E}_o e^{j(\omega t - kz)}\}$$
$$E_y(z,t) = \hat{y}Re\{j\tilde{E}_o e^{j(\omega t - kz)}\}$$



The complex amplitude of the *x*-component is ***-j*** times the complex amplitude of the *y*-component.

E_x and E_y are always
90° out of phase

The resulting E-field rotates **counterclockwise** around the propagation-vector (looking along *z-axis*).

If projected on a constant z plane the E-field vector would rotate **clockwise !!!**

Right vs. Left Circular (or Helical) Polarization

$$E_x(z,t) = -\hat{x}\tilde{E}_o sin(\omega t - kz)$$
$$E_y(z,t) = \hat{y}\tilde{E}_o cos(\omega t - kz)$$

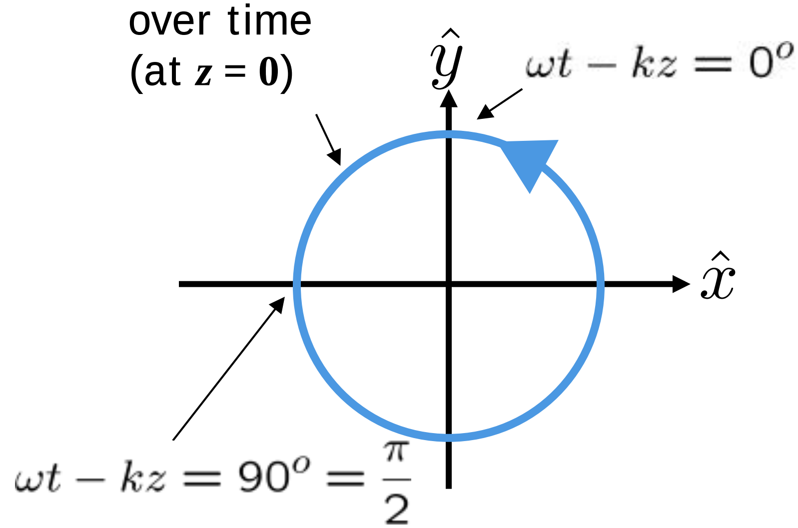
... or, more generally,

$$E_x(z,t) = \hat{x}Re\{+j\tilde{E}_o e^{j(\omega t - kz)}\}$$
$$E_y(z,t) = \hat{y}Re\{j\tilde{E}_o e^{j(\omega t - kz)}\}$$

Here, the complex amplitude of the x-component is **+*j*** times the complex amplitude of the y-component.

So the components are always **90˚ out of phase, but in the other direction**

E-field variation over time (at $z = 0$)



The resulting E-field rotates **clockwise** around the propagation-vector (looking along *z-axis*).

If projected on a constant z plane the E-field vector would rotate **counterclockwise !!!**

Unequal arbitrary-relative-phase components yield ***elliptical polarization***

$$E_x(z,t) = \hat{x}E_{ox}cos(\omega t - kz)$$

$$E_y(z,t) = \hat{y}E_{oy}cos(\omega t - kz - \theta)$$

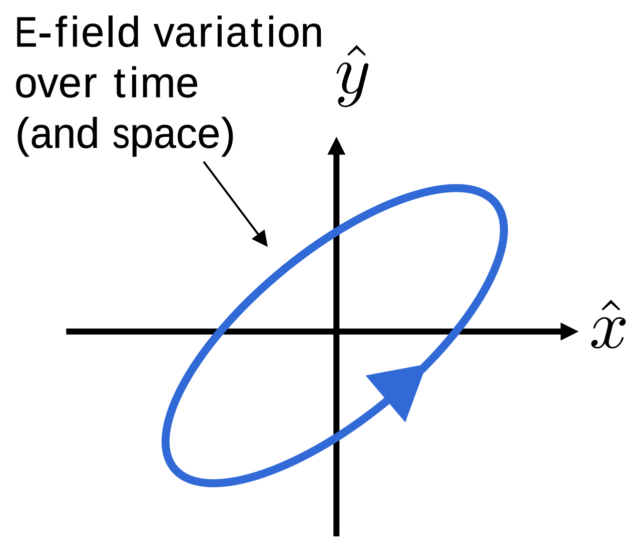
where $E_{ox} \neq E_{oy}$

... or, more generally,

$$E_x(z,t) = \hat{x}Re\{E_{ox}e^{j(\omega t - kz)}\}$$

$$E_y(z,t) = \hat{y}Re\{E_{oy}e^{j(\omega t - kz - \theta)}\}$$

... where $\tilde{E}_{ox}$ *and* $\tilde{E}_{oy}$ are arbitrary complex amplitudes

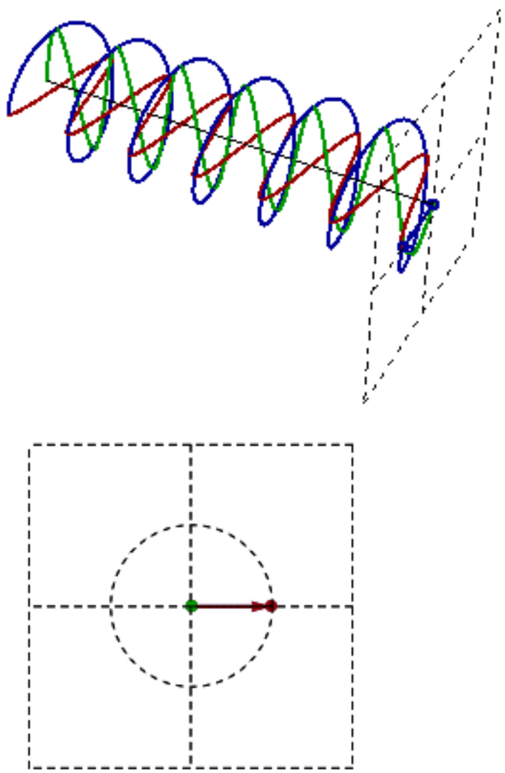


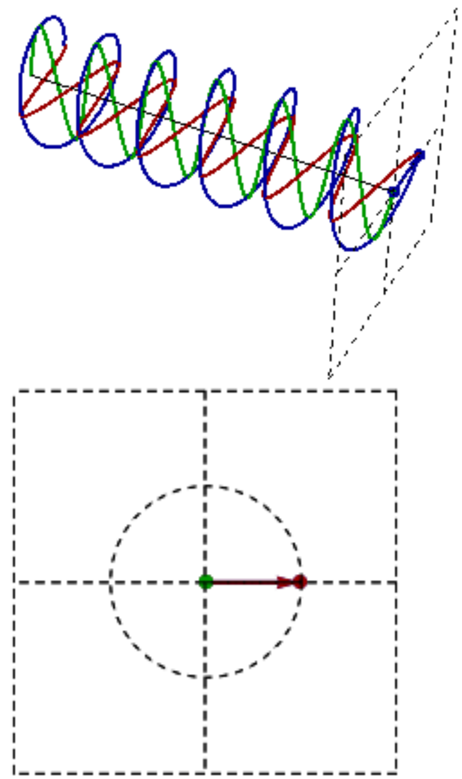
The resulting E-field can rotate clockwise or counter-clockwise around the k-vector (looking along k).

Sinusoidal Uniform Plane Waves

$$\overline{E} = A\left(sin(\omega t - kz)\,\widehat{y} \pm cos(\omega t - kz)\,\widehat{x}\right)$$

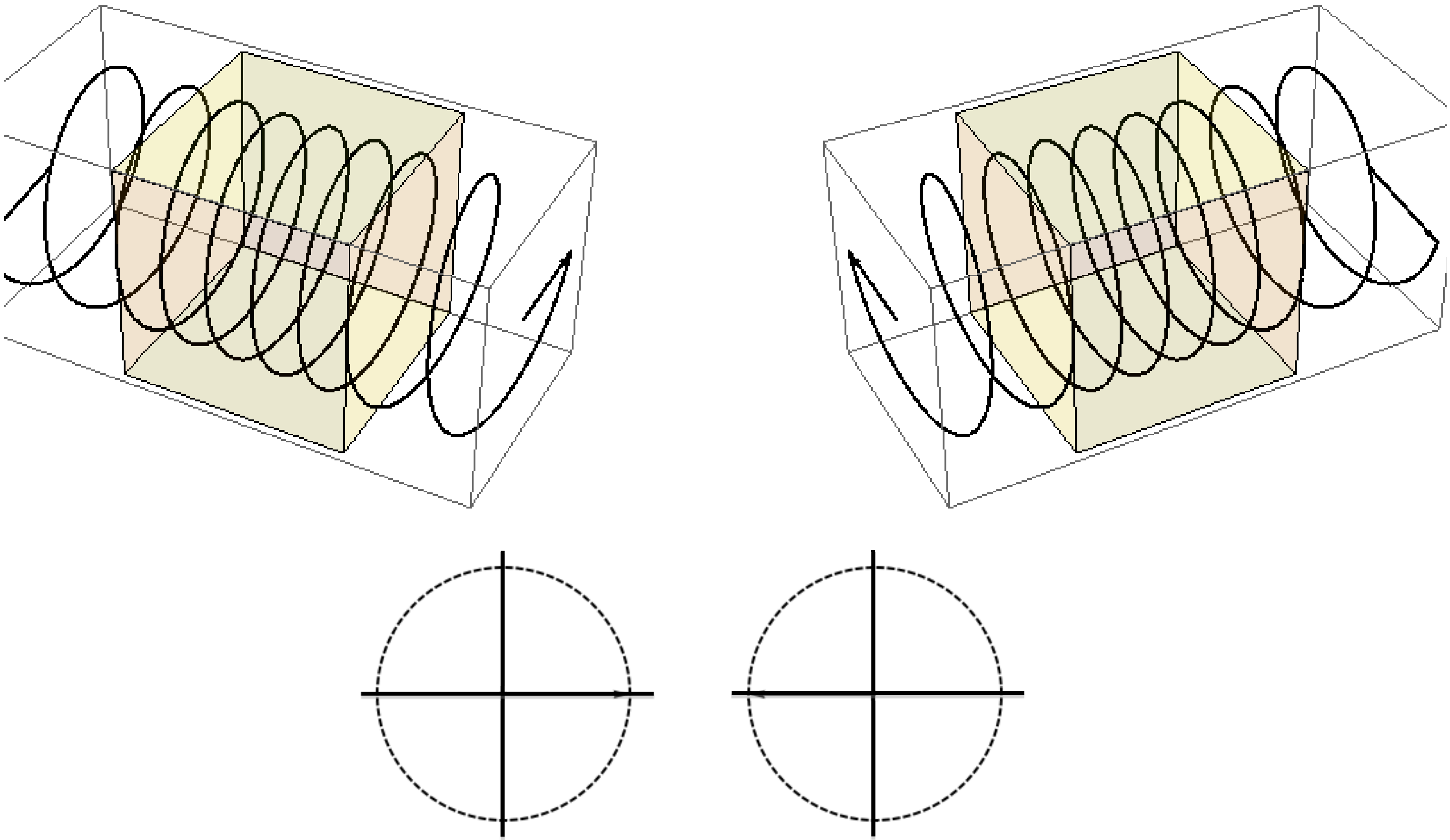
Left IEEE Definitions: Right





$$\vec{E}_L = E_o\left(\widehat{x} + \widehat{y}e^{+j\pi/2}\right)e^{-jkz} \qquad \vec{E}_R = E_o\left(\widehat{x} + \widehat{y}e^{-j\pi/2}\right)e^{-jkz}$$

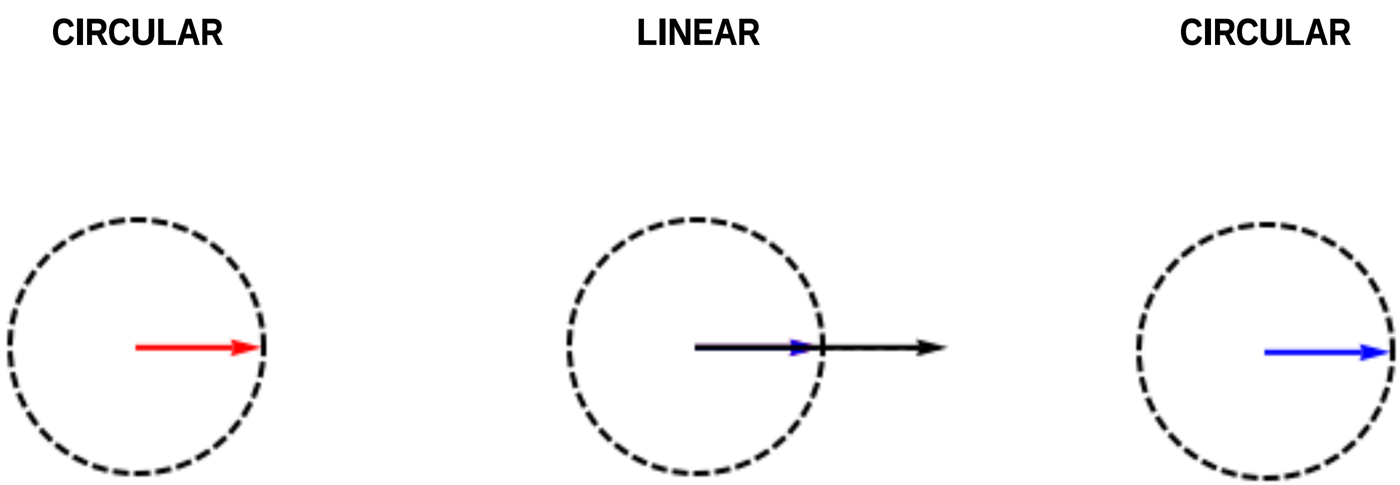
A linearly polarized wave can be represented as a sum of two circularly polarized waves



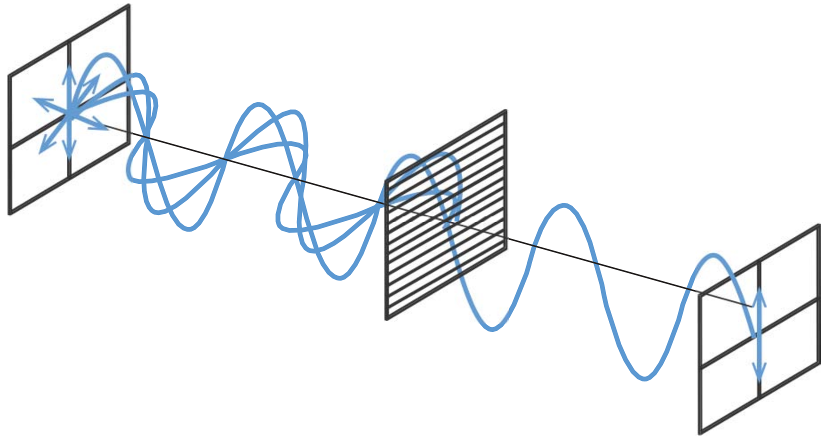
A linearly polarized wave can be represented as a sum of two circularly polarized waves

$$E_x(z,t) = \hat{x}\tilde{E}_o sin(\omega t - kz)$$
$$E_y(z,t) = \hat{y}\tilde{E}_o cos(\omega t - kz)$$

$$E_x(z,t) = -\hat{x}\tilde{E}_o sin(\omega t - kz)$$
$$E_y(z,t) = \hat{y}\tilde{E}_o cos(\omega t - kz)$$



Polarizers for Linear and Circular Polarizations

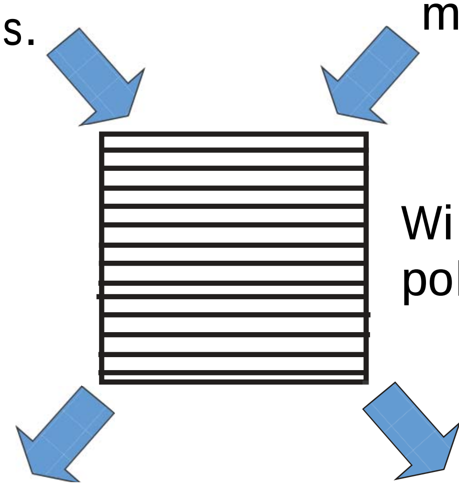


CASE 1:

Linearly polarized light with magnitude E_o oriented 45° with respect to the x-axis.

CASE 2:

Circularly polarized light with magnitude E_o .



Wire grid polarizer

What is the average power at the input and output?

Today's Culture Moment

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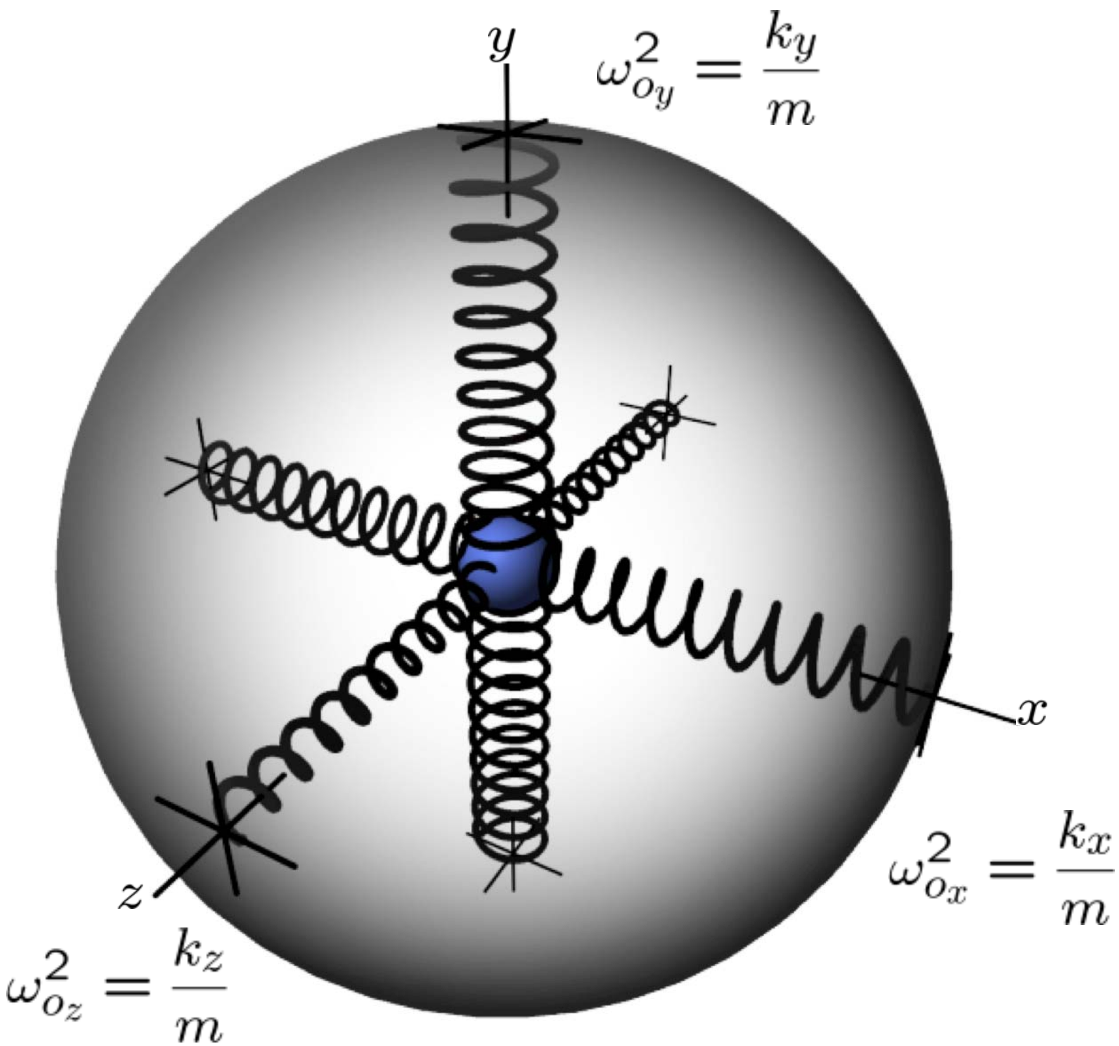
3D vision

Anisotropic Material

The molecular "spring constant" can be different for different directions



If $\omega_{o_x} = \omega_{o_z}$,
then the material has a single optics axis and is called **uniaxial** crystal

Microscopic Lorentz Oscillator Model

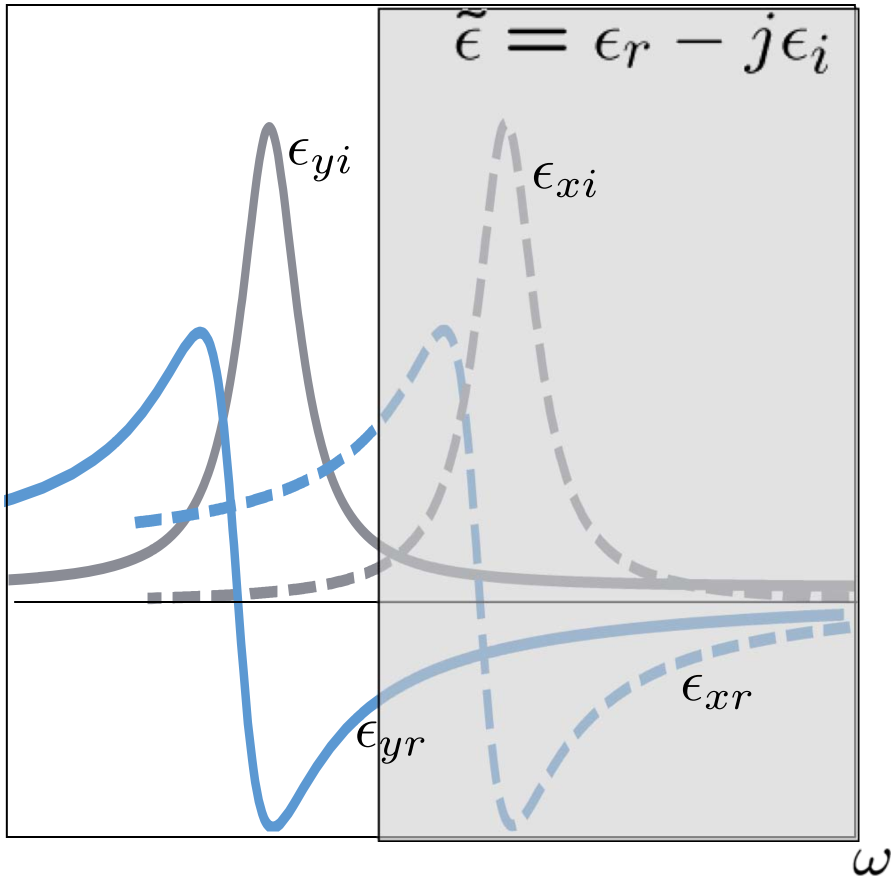
$$P(\omega) = \frac{\epsilon_o \omega_p^2}{\omega_o^2 - \omega^2 + j\omega\gamma} E(\omega) = (\tilde{\epsilon}(\omega) - 1)\epsilon_o E(\omega)$$

In the transparent regime

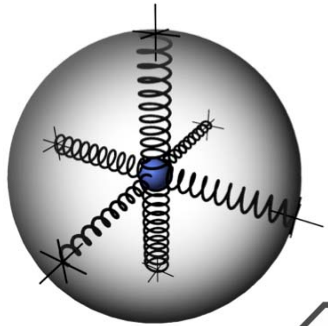
$\epsilon_{y_r} >> \epsilon_{y_i}$ ...

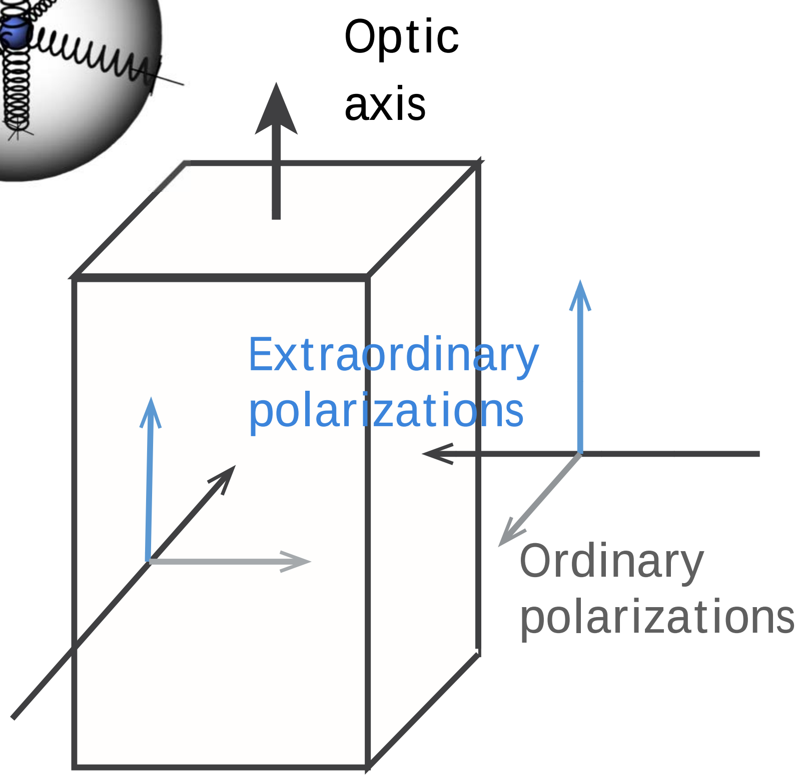
$$n_y = \sqrt{\frac{\epsilon_{y_r}}{\epsilon_o}}$$

$$n_x = \sqrt{\frac{\epsilon_{y_r}}{\epsilon_o}}$$



Uniaxial Crystal





Uniaxial crystals have one refractive index for light polarized along the **optic axis** (n_e)

and another for light polarized in either of the two directions perpendicular to it (n_o).

Light polarized along the optic axis is called the **extraordinary** ray,

and light polarized perpendicular to it is called the **ordinary** ray.

These polarization directions are the **crystal principal axes**.

Ordinary...

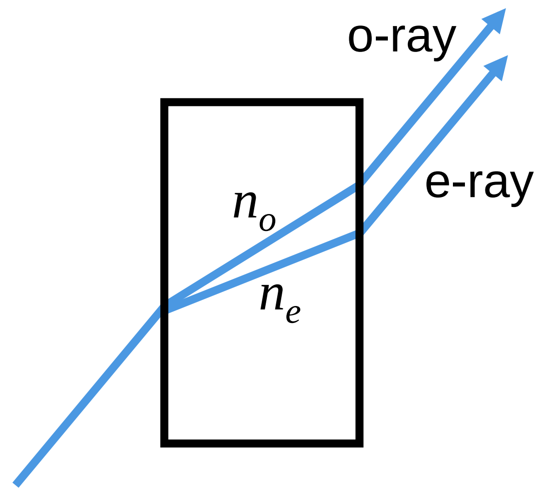
$$n_x = n_z \equiv n_o$$

Extraordinary...

$$n_y \equiv n_e$$

Birefringent Materials

Image by Arenamotanus http://www.flickr.com/photos/arenamontanus/2756010517/ on flickr



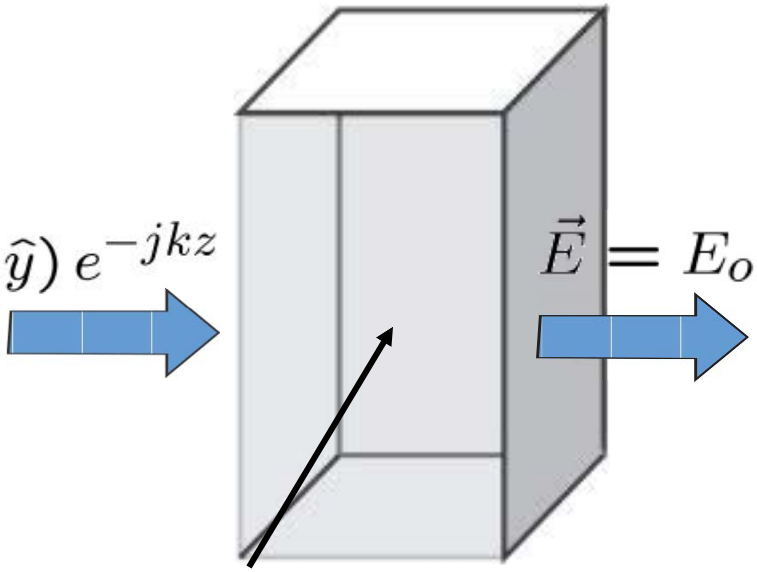
Crystal $\lambda = 583nm$	n_o	n_e
Tourmaline	1.669	1.638
Calcite	1.6584	1.4864
Quartz	1.5443	1.5534
Sodium nitrate	1.5854	1.3369
Ice	1.309	1.313
Rutile (TiO_2)	2.616	2.903

All transparent crystals with non-cubic lattice structure are birefringent.

Polarization Conversion

Linear to Circular

$$\vec{E} = E_o\left(\hat{x} + \hat{y}\right) e^{-jkz}$$



$$\vec{E} = E_o\left(\hat{x}e^{-jk_e d} + \hat{y}e^{-jk_o d}\right) e^{-jkz}$$

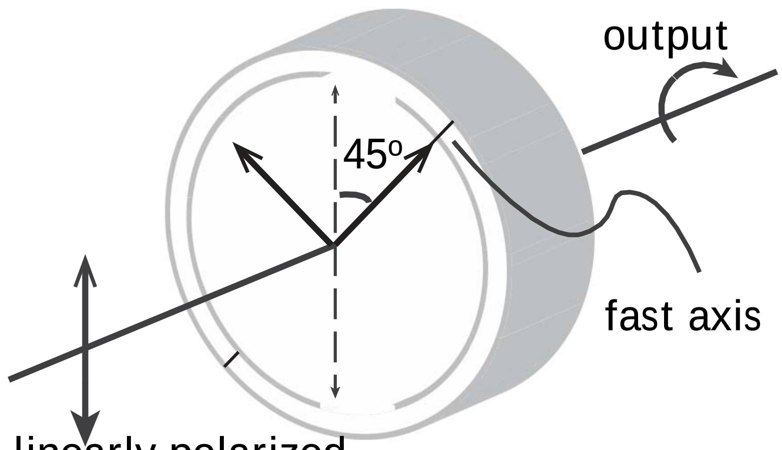
inside

$$\vec{E} = E_o\left(\hat{x}e^{-jk_e z} + \hat{y}e^{-jk_o z}\right)$$

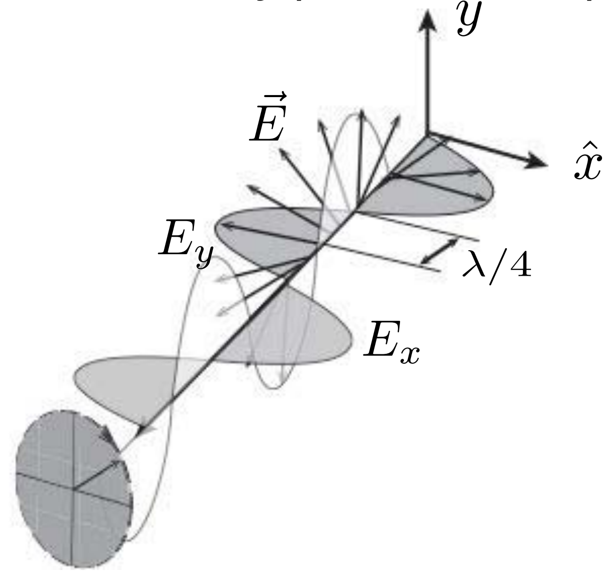
Polarization of output wave is determined by…

$$\frac{E_y}{E_x} = \frac{e^{-jk_e d}}{e^{-jk_o d}} = e^{-j(k_o - k_e)d}$$

Quarter-Wave Plate



Circularly polarized output



$$\frac{E_y}{E_x} = \frac{e^{-jk_e d}}{e^{-jk_o d}} = e^{-j(k_o - k_e)d}$$

Example:

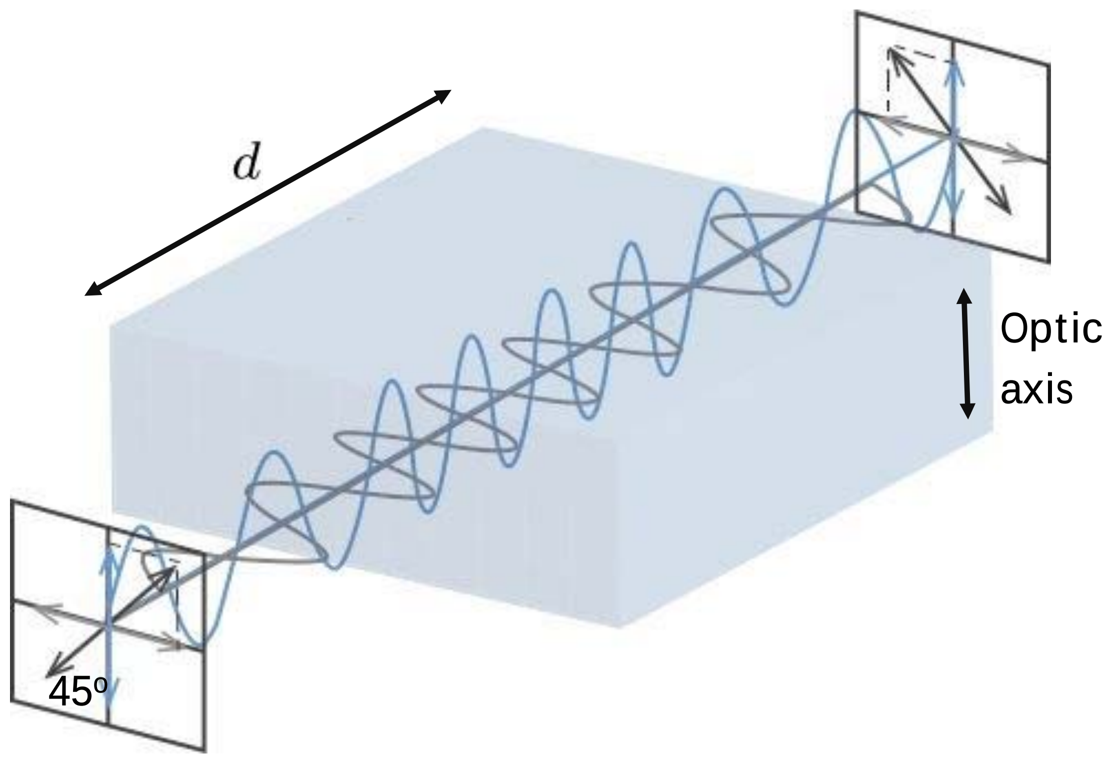
If we are to make quarter-wave plate using calcite (n_o = 1.6584, n_e = 1.4864), for incident light wavelength of = 590 nm, how thick would the plate be ?

$d_{\text{calcite QWP}}$ = (590nm/4) / (n_o-n_e) = 858 nm

Half-Wave Plate

The phase difference between the waves linearly polarized parallel and perpendicular to the optic axis is a half cycle

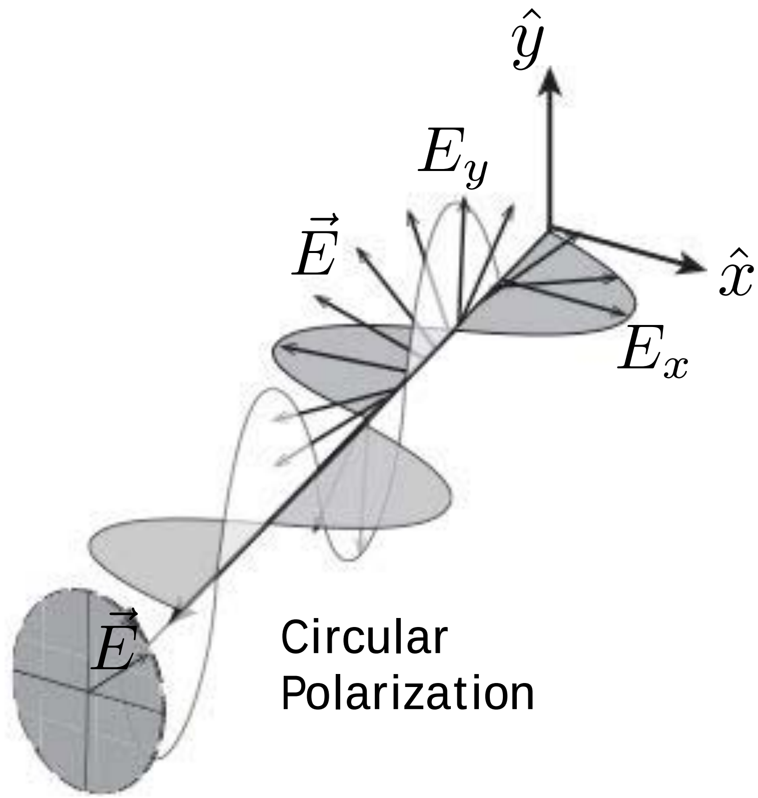
$$\vec{E} = E_o\,(\hat{x} - \hat{y})\,e^{-jk(z-d)}$$



$$\vec{E} = E_o\,(\hat{x} + \hat{y})\,e^{-jkz}$$

LINEAR IN → LINEAR OUT

Key Takeaways



EM Waves can be linearly, circularly, or elliptically polarized.

A circularly polarized wave can be represented as a sum of two linearly polarized waves having $\pi/2$ *phase shift.*

A linearly polarized wave can be represented as a sum of two circularly polarized waves.

In the general case, waves are elliptically polarized.

Waveplates can be made from birefringent materials:

Quarter wave plate: $\lambda/4 = (n_o - n_e)d$ (gives $\pi/2$ phase shift)
Half wave plate: $\lambda/2 = (n_o - n_e)d$ (gives π phase shift)