EM Reflection & Transmission in Layered Media

Reading - Shen and Kong – Ch. 4

Outline

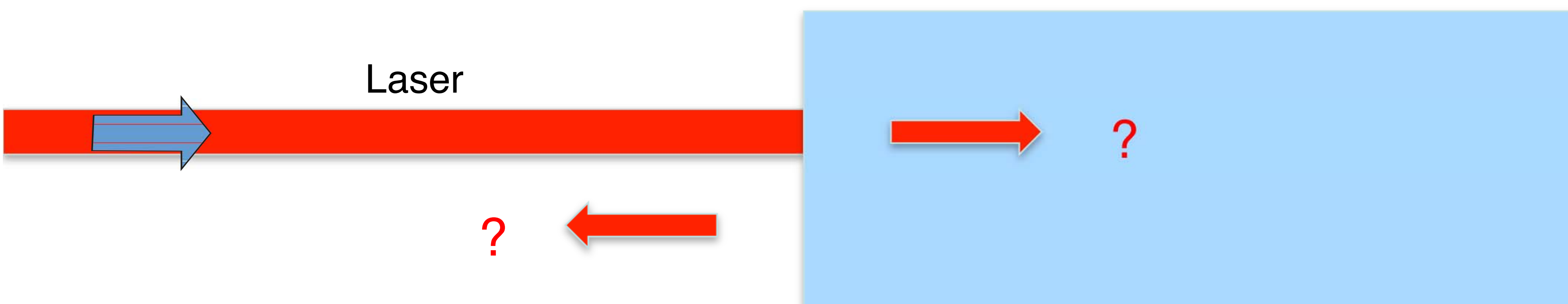
- Review of Reflection and Transmission
- Reflection and Transmission in Layered Media
- Anti-Reflection Coatings
- Optical Resonators
- Use of Gain

TRUE or FALSE

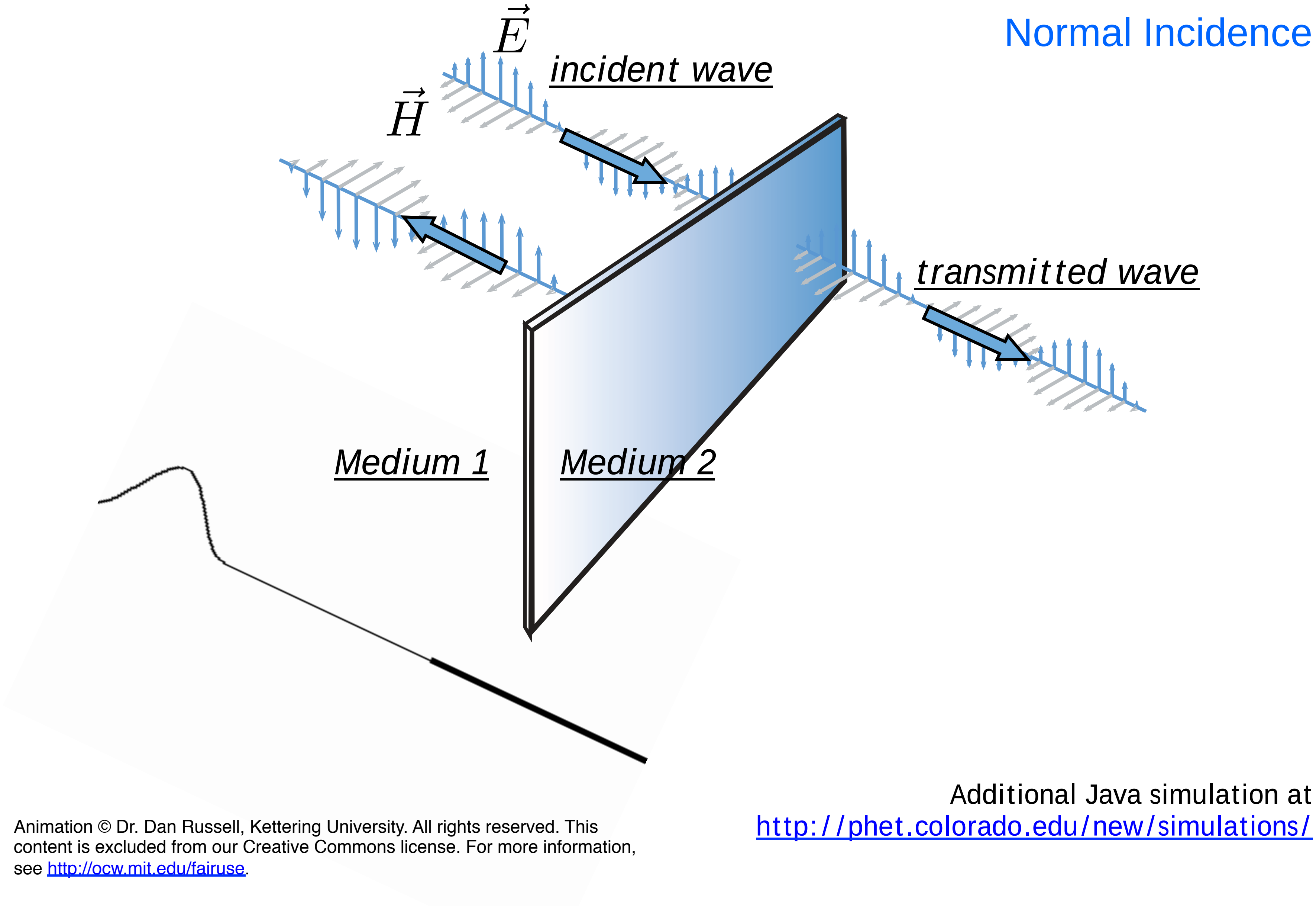
$$r = \frac{E_o^r}{E_o^i} = \frac{n_1 - n_2}{n_1 + n_2}$$

$$t = \frac{E_o^t}{E_o^i} = \frac{2\,n_1}{n_1 + n_2}$$

1. The refractive index of glass is approximately $n = 1.5$ for visible frequencies. If we shine a 1 mW laser on glass, more than 0.5 mW of the light will be transmitted. ____

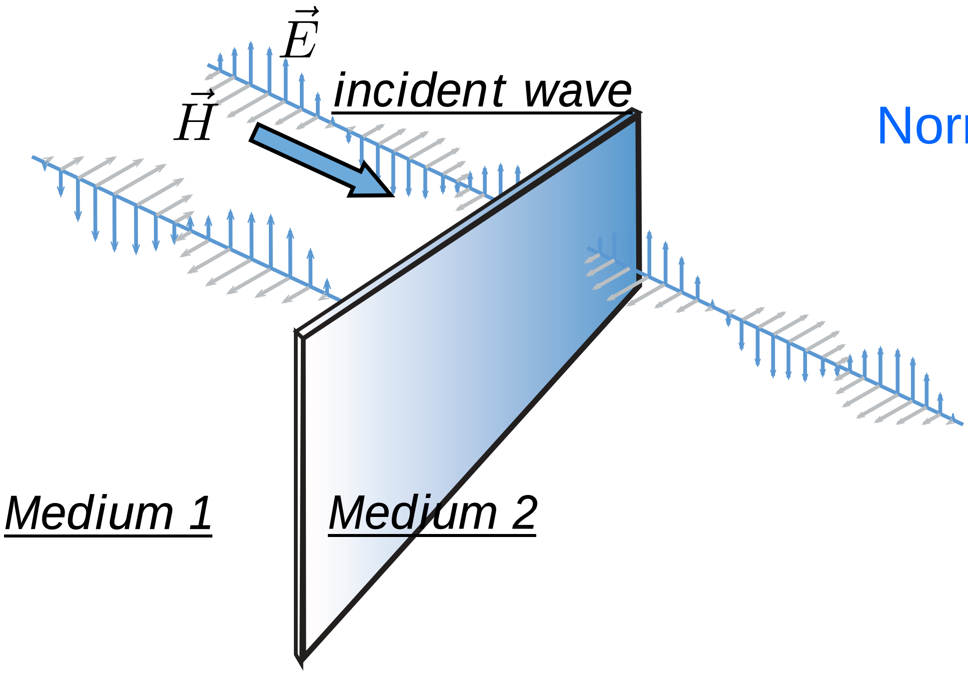


Reflection & Transmission of EM Waves at Boundaries



Additional Java simulation at
http://phet.colorado.edu/new/simulations/

Incident EM Waves at Boundaries



Normal Incidence

Incident Wave

Known

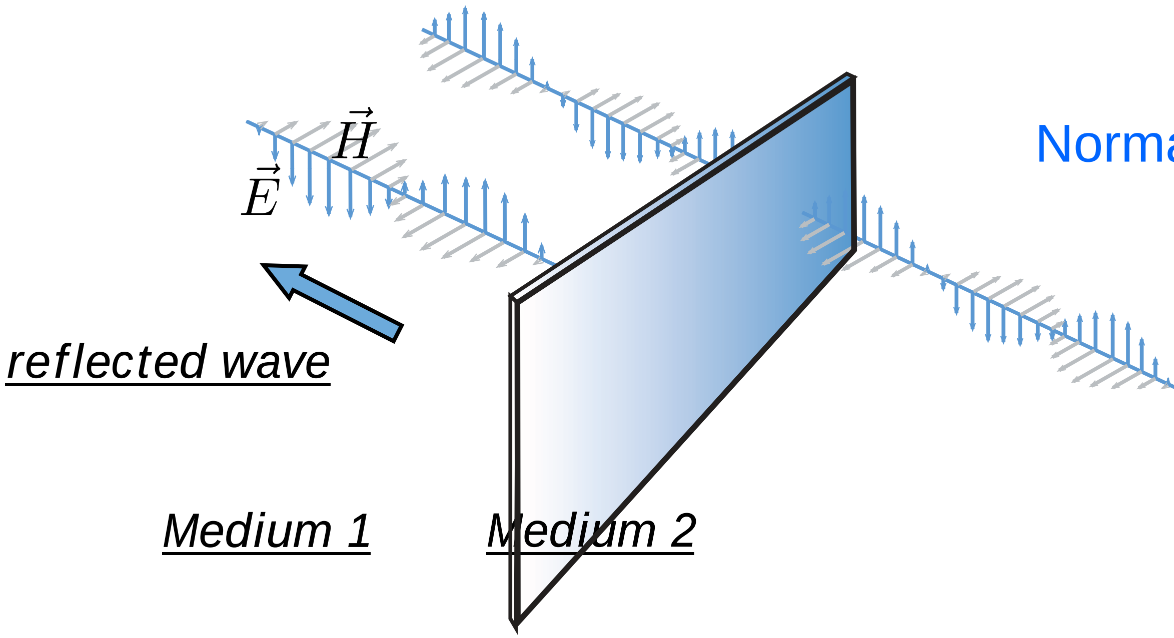
$$\vec{E}_i = \hat{x} E_o^i e^{-jk_1 z}$$

$$\vec{H}_i = \frac{1}{\eta_1} \hat{z} \times \vec{E}_i = \hat{y} \frac{1}{\eta_1} E_o^i e^{-jk_1 z}$$

$$k_1 = \omega \sqrt{\epsilon_1 \mu_1}$$

$$\eta_1 = \sqrt{\frac{\mu_1}{\epsilon_1}}$$

Reflected EM Waves at Boundaries



Normal Incidence

Reflected Wave

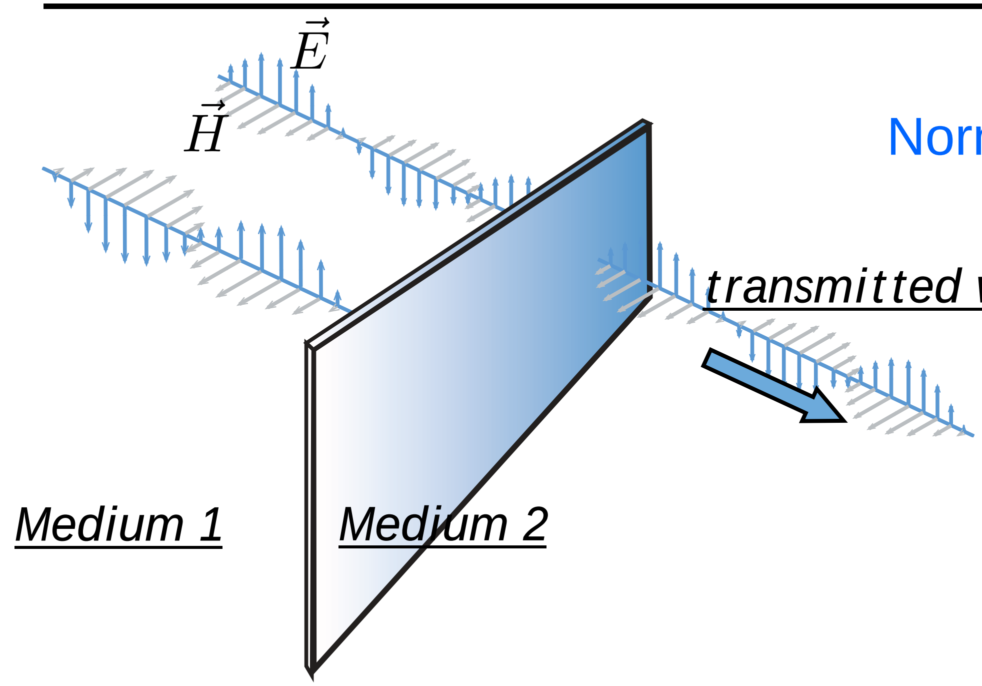
Unknown

$$\vec{E}_r = \hat{x} E_o^r e^{+jk_1 z}$$

$$\vec{H}_r = \frac{1}{\eta_1}(-\hat{z}) \times \vec{E}_r = -\hat{y}\frac{E_o^r}{\eta_1} e^{+jk_1 z}$$

DEFINE REFLECTION COEFFICIENT AS $r = \frac{E_o^r}{E_o^i}$

Transmitted EM Waves at Boundaries



Normal Incidence

Transmitted Wave Unknown

$$\vec{E}_r = \hat{x} E_o^t e^{-jk_2 z}$$

$$\vec{H}_t = \frac{1}{\eta_2} \hat{z} \times \vec{E}_t = \hat{y} \frac{E_o^t}{\eta_2} e^{-jk_2 z}$$

DEFINE TRANSMISSION COEFFICIENT AS $t = \frac{E_o^t}{E_o^i}$

$$k_2 = \omega\sqrt{\epsilon_2 \mu_2}$$

$$\eta_2 = \sqrt{\frac{\mu_2}{\epsilon_2}}$$

Reflection & Transmission of EM Waves at Boundaries

$$\vec{E}_1 = \vec{E}_i + \vec{E}_r = \hat{x}\left(E_o^i\, e^{-jk_1z} + E_o^r\, e^{+jk_1z}\right)$$

$$\vec{E}_2 = \vec{E}_t = \hat{x}E_o^t\, e^{-jk_2z}$$

Medium 1

Medium 2

$$\vec{H}_1 = \vec{H}_i + \vec{H}_r = \hat{y}\left(\frac{E_o^i}{\eta_1}\, e^{-jk_1z} - \frac{E_o^r}{\eta_1}\, e^{+jk_1z}\right)$$

$$\vec{H}_2 = \vec{H}_r = \hat{y}\frac{E_o^t}{\eta_2}\, e^{-jk_2z}$$

$$\overline{E}_{1(z=0)} = \overline{E}_{2(z=0)}$$

$$\overline{H}_{1(z=0)} = \overline{H}_{2(z=0)}$$

Reflectivity & Transmissivity of Waves

- Define the *reflection coefficient* as

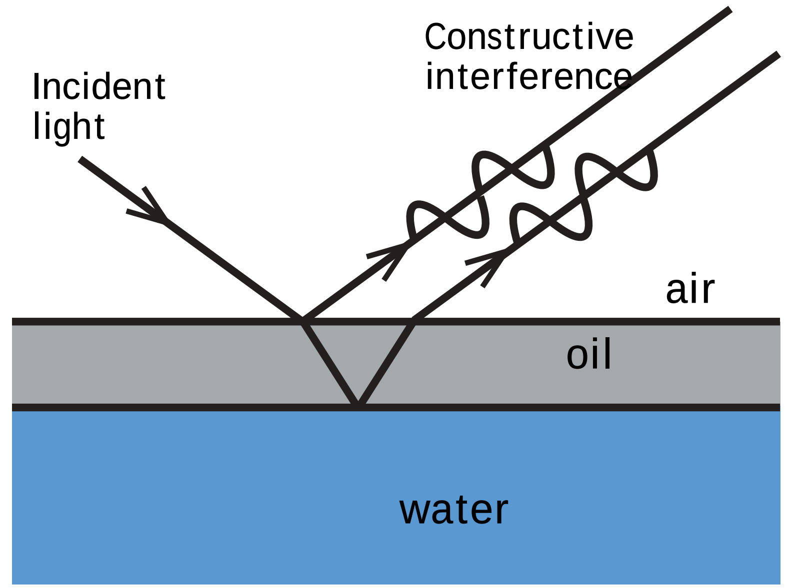
$$r = \frac{E_o^r}{E_o^i} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{n_1 - n_2}{n_1 + n_2}$$

- Define the *transmission coefficient* as

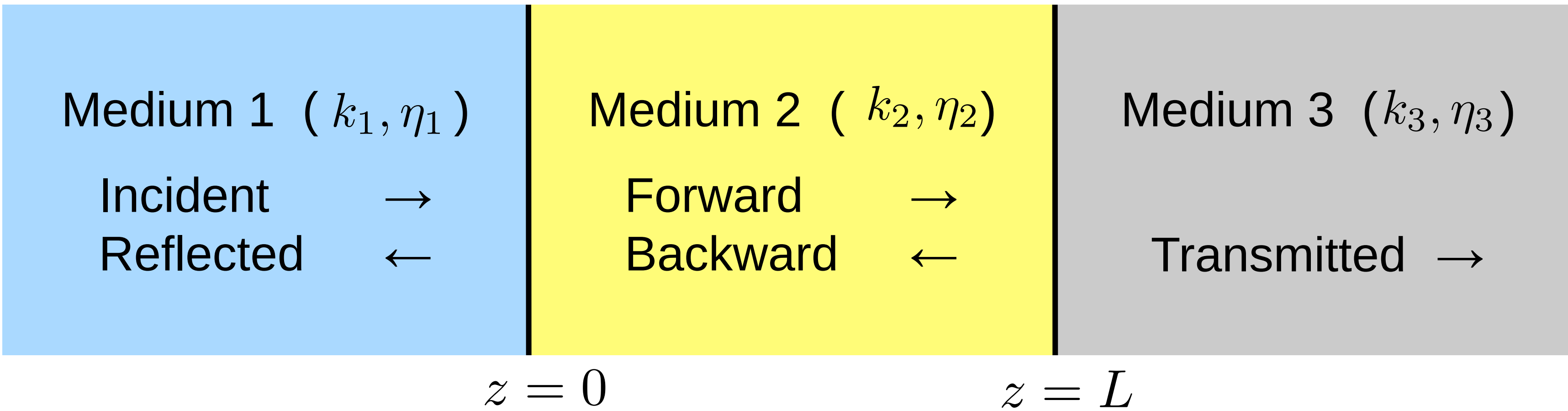
$$t = \frac{E_o^t}{E_o^i} = \frac{2\eta_2}{\eta_2 + \eta_1} = \frac{2\,n_1}{n_1 + n_2}$$

Thin Film Interference

Image by Yoko Nekonomania http://www.flickr.com/photos/nekonomania/4827035737/ on flickr



Reflection & Transmission in Layered Media



Incident: $E_i e^{-jk_1 z}$

Reflected: $E_r e^{+jk_1 z}$

Forward: $E_f e^{-jk_2 z}$

Backward: $E_b e^{+jk_2 (z-L)}$

Transmitted: $E_t e^{-jk_3 (z-L)}$

Omit $e^{j\omega t}$

$$H_\pm = \pm E_\pm / \eta$$

$$k \equiv \omega \sqrt{\epsilon \mu}$$

$$\eta \equiv \sqrt{\frac{\mu}{\epsilon}}$$

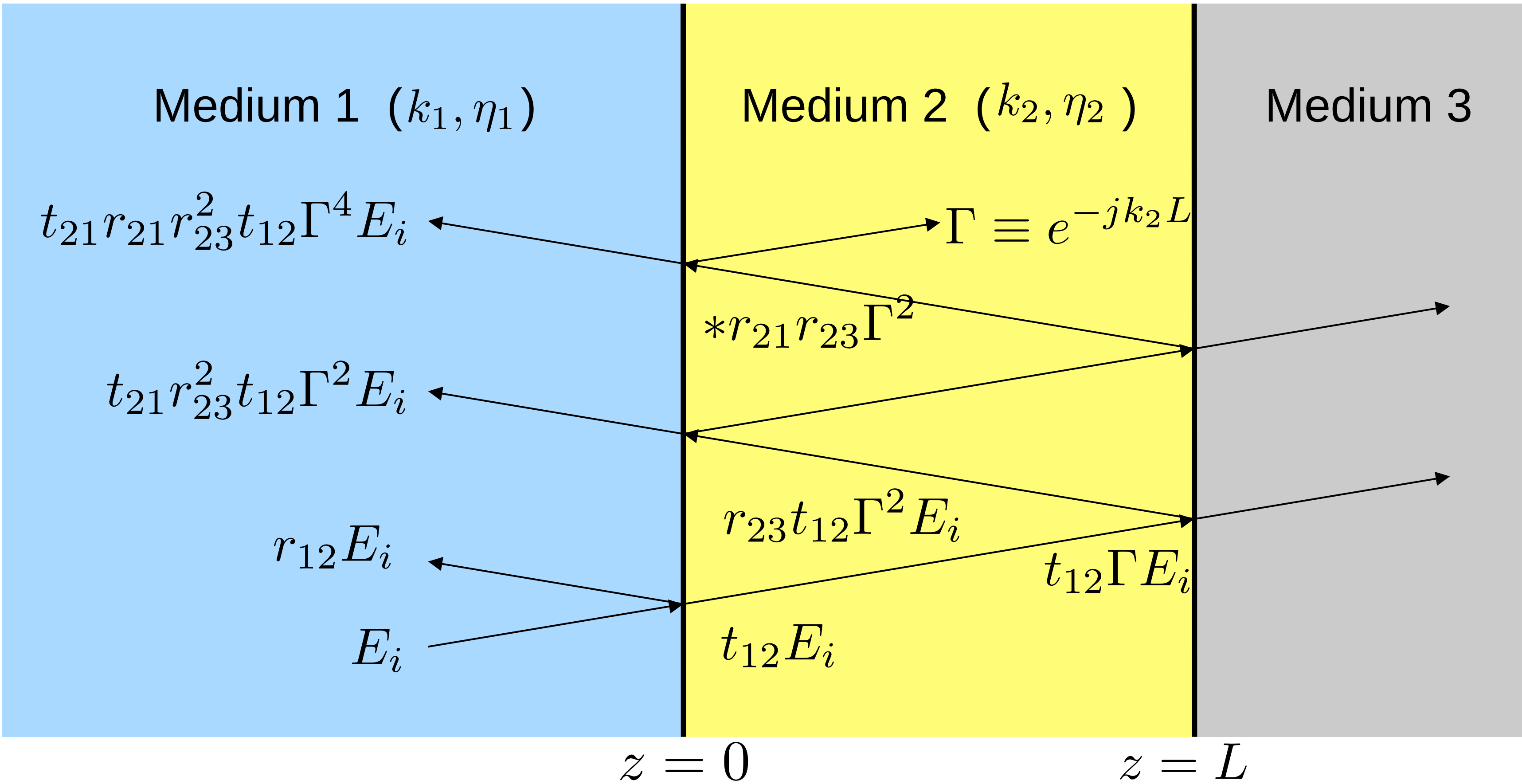
Reflection & Transmission in Layered Media

Apply boundary conditions …

- E at $z = 0 \rightarrow E_i + E_r = E_f + E_b$
- H at $z = 0 \rightarrow E_i/\eta_1 - E_r/\eta_1 = E_f/\eta_2 - E_b/\eta_2$
- E at $z = L \rightarrow E_f e^{-jk_2 L} + E_b e^{+jk_2 L} = E_t e^{-jk_3 L}$
- H at $z = L \rightarrow E_f e^{-jk_2 L}/\eta_2 - E_b e^{+jk_2 L}/\eta_2 = E_t e^{-jk_3 L}/\eta_3$
- … and solve for E_r, E_f, E_b and E_t as functions of E_i.

Could "easily" be extended to more layers.

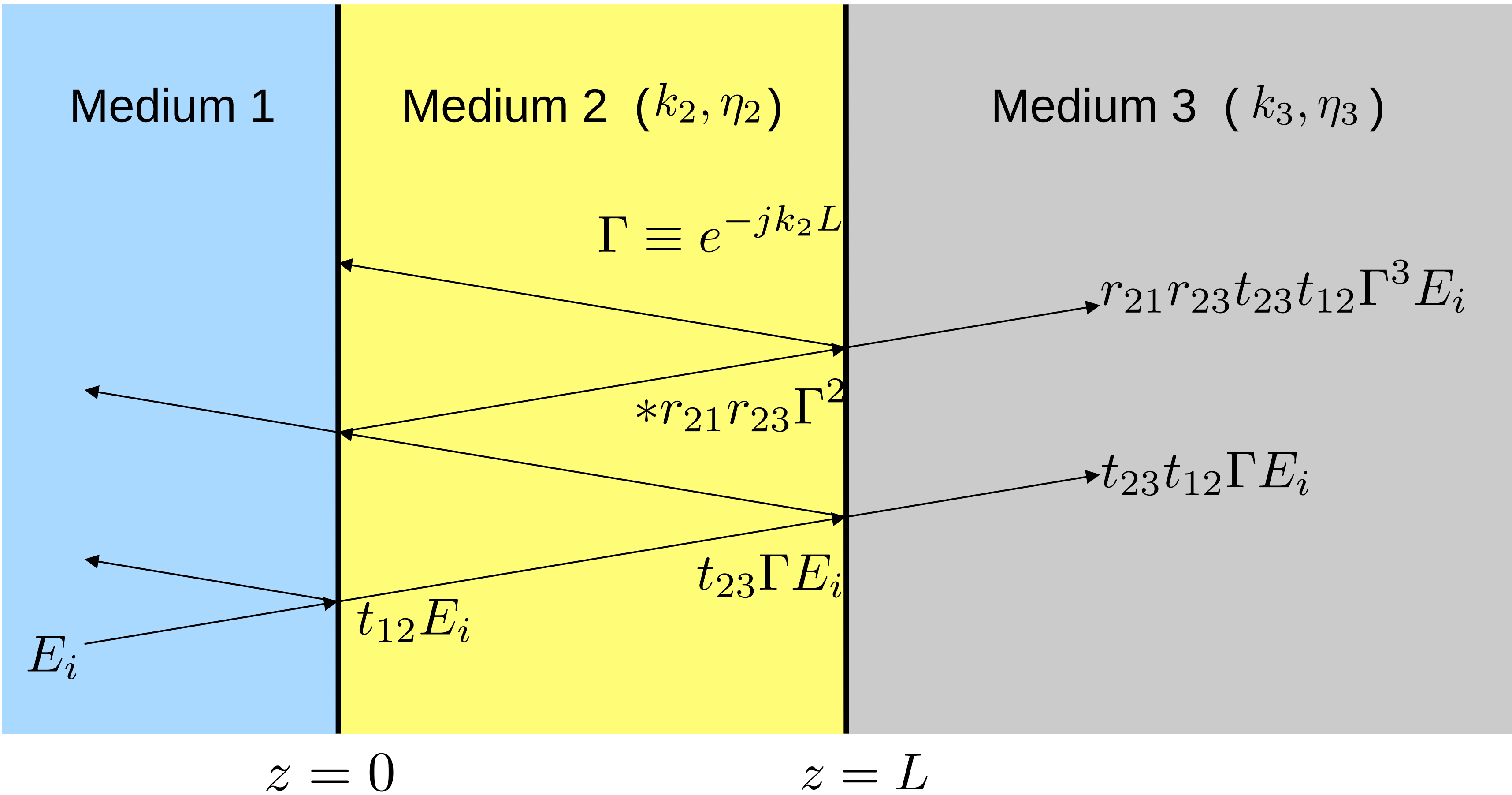
Reflection by Infinite Series



$$E_r = E_i(r_{21} + t_{21}r_{23}t_{12}\Gamma^2(1 + r_{21}r_{23}\Gamma^2 + r_{21}^2r_{23}^2\Gamma^4 ...))$$

$$= E_i(r_{21} + t_{21}r_{23}t_{12}\Gamma^2/(1 - r_{21}r_{23}\Gamma^2))$$

Transmission by Infinite Series



$$E_t = E_i(t_{23} t_{12} \Gamma(1 + r_{21} r_{23} \Gamma^2 + r_{21}^2 r_{23}^2 \Gamma^4 \ldots))$$
$$= E_i t_{23} t_{21} \Gamma / (1 - r_{21} r_{23} \Gamma^2))$$

Is Zero Reflection Possible?

One could solve for conditions under which ...

- $E_r = 0$ ... no reflected wave
- $|E_t|^2/\eta_3 = |E_i|^2/\eta_1$ ... transmitted wave carries incident power

and then determine conditions on L and η_2 for which there is no reflection, for example. This would yield the design of an anti-reflection coating.

Or, one could use generalized impedances ...

Today's Culture Moment

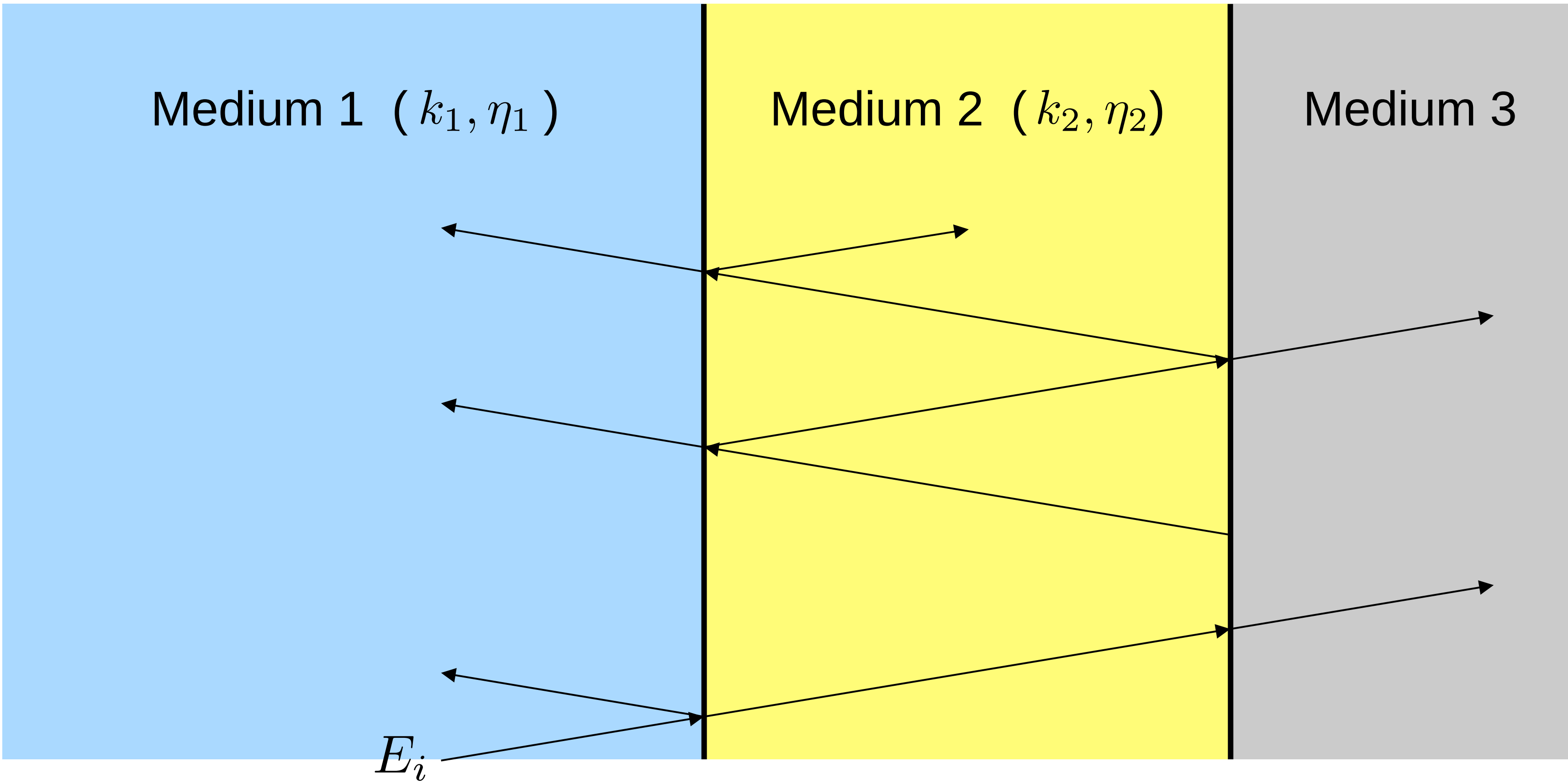
GPS

Image by ines saraiva http://www.flickr.com/photos/inessaraiva/4006000559/ on flickr

The Global Positioning System (GPS) is a constellation of 24 Earth-orbiting satellites. The orbits are arranged so that at any time, anywhere on Earth, there are at least four satellites "visible" in the sky. GPS operations depend on a very accurate time reference; each GPS satellite has atomic clocks on board.

Galileo – a global system being developed by the European Union and other partner countries, planned to be operational by 2014
Beidou – People's Republic of China's regional system, covering Asia and the West Pacific
COMPASS – People's Republic of China's global system, planned to be operational by 2020
GLONASS – Russia's global navigation system

Reflection and Transmission by an Infinite Series

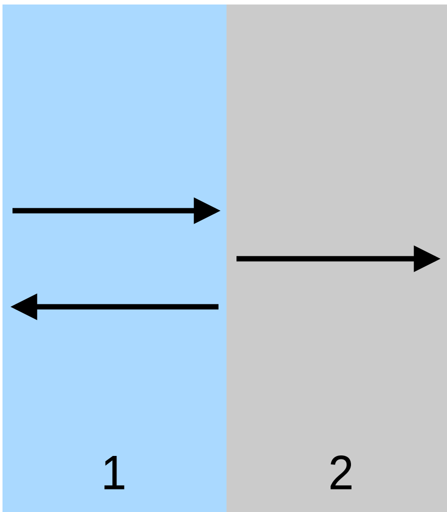


$$E_r = E_i(r_{12} + t_{21} r_{23} t_{12} \Gamma^2 / (1 - r_{21} r_{23} \Gamma^2)) \quad \Gamma \equiv e^{-jk_2 L}$$

$$E_t = E_i(t_{23} t_{12} \Gamma / (1 - r_{21} r_{23} \Gamma^2))$$

How do we get zero reflection?

Generalized Impedance



$z = 0$

Define a spatially-dependent impedance

$$\eta(z) = -\frac{E(z)}{H(z)}$$

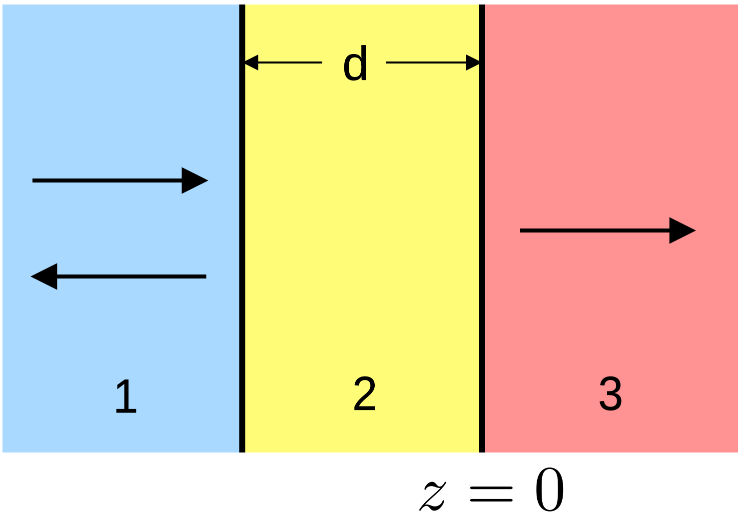
In region 1 ($z < 0$) we have

$$\eta_1(z) = \sqrt{\frac{\mu_1}{\varepsilon_1}}\,\frac{e^{-jkz} + re^{jkz}}{e^{-jkz} - re^{jkz}}$$

In region 2 ($z > 0$) we have

$$\eta_2(z) = \sqrt{\frac{\mu_2}{\varepsilon_2}}$$

Generalized Impedance



The incident wave in region 1 now sees an impedance of regions 2 and 3:

$$\eta(-d) = \sqrt{\frac{\mu_2}{\varepsilon_2}} \frac{e^{jk_2 d} + r_{23} e^{-jk_2 d}}{e^{jk_2 d} - r_{23} e^{-jk_2 d}}$$

Reflection of incident wave can be eliminated if we match impedance

$$\eta(-d) = \sqrt{\frac{\mu_1}{\varepsilon_1}}$$

Matching Impedances

We need

$$\sqrt{\frac{\mu_1}{\varepsilon_1}} = \sqrt{\frac{\mu_2}{\varepsilon_2}} \frac{e^{jk_2 d} + r_{23} e^{-jk_2 d}}{e^{jk_2 d} - r_{23} e^{-jk_2 d}} = \sqrt{\frac{\mu_2}{\varepsilon_2}} \frac{1 + r_{23} e^{-2jk_2 d}}{1 - r_{23} e^{-2jk_2 d}}$$

For lossless material, ε and μ are real, so only choices are

$$e^{2jk_2 d} = \pm 1$$

Choose -1 and obtain … requires $d = \lambda / 4n_2$

$$\sqrt{\frac{\mu_1}{\varepsilon_1}} = \sqrt{\frac{\mu_2}{\varepsilon_2}} \frac{1 - r_{23}}{1 + r_{23}}$$

Matching Impedances

Consider impedance at $z = 0$

$$\sqrt{\frac{\mu_2}{\varepsilon_2}}\frac{1+r_{23}}{1-r_{23}} = \sqrt{\frac{\mu_3}{\varepsilon_3}} \quad \Rightarrow \quad \frac{1+r_{23}}{1-r_{23}} = \sqrt{\frac{\mu_3}{\varepsilon_3}}\sqrt{\frac{\varepsilon_2}{\mu_2}}$$

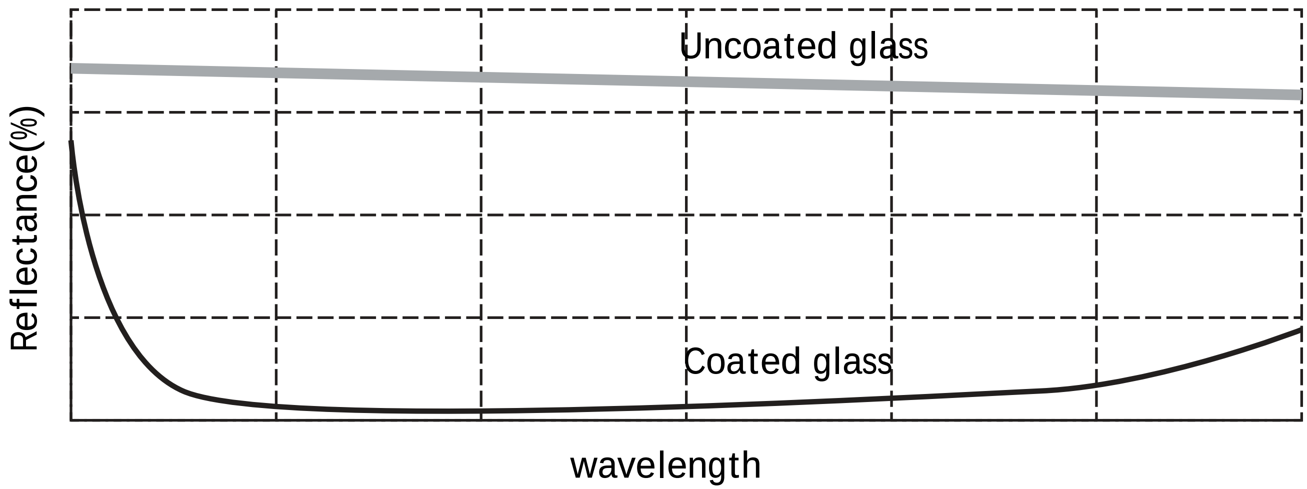
So, we can eliminate the reflection as long as

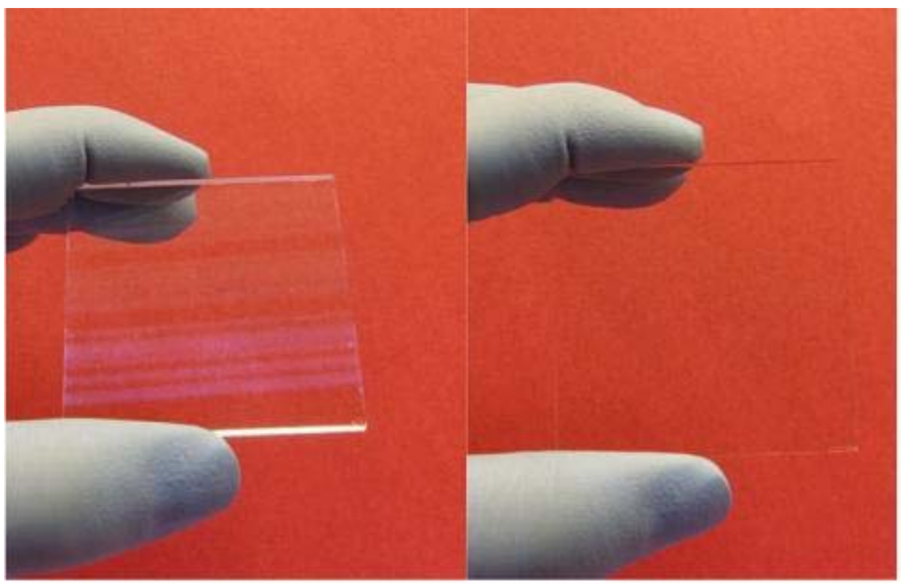
$$\sqrt{\frac{\mu_1}{\varepsilon_1}} = \sqrt{\frac{\mu_2}{\varepsilon_2}}\left(\sqrt{\frac{\mu_2}{\varepsilon_2}}\sqrt{\frac{\varepsilon_3}{\mu_3}}\right) \quad \Rightarrow \quad \frac{\mu_2}{\varepsilon_2} = \sqrt{\frac{\mu_1}{\varepsilon_1}\frac{\mu_3}{\varepsilon_3}}$$

$$\eta_2 \cdot \eta_2 = \eta_1 \cdot \eta_3$$

$$(n_2)^2 = n_1 n_3$$

Anti-reflection Coating





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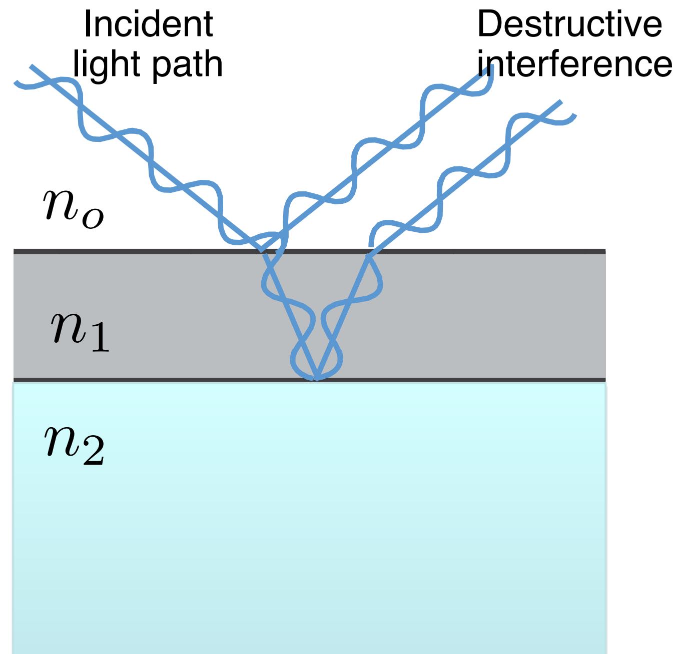
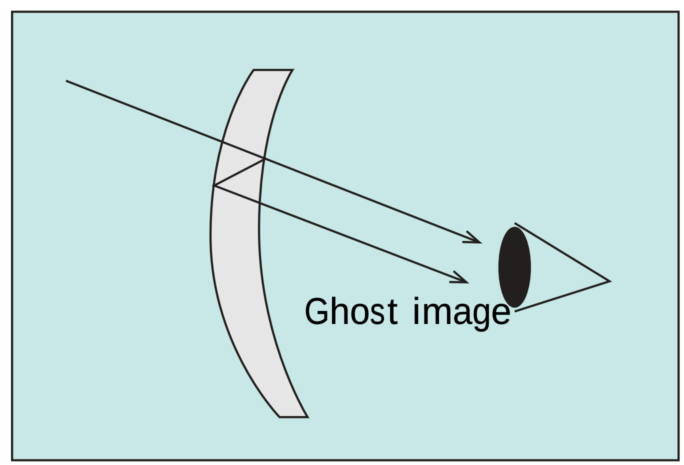
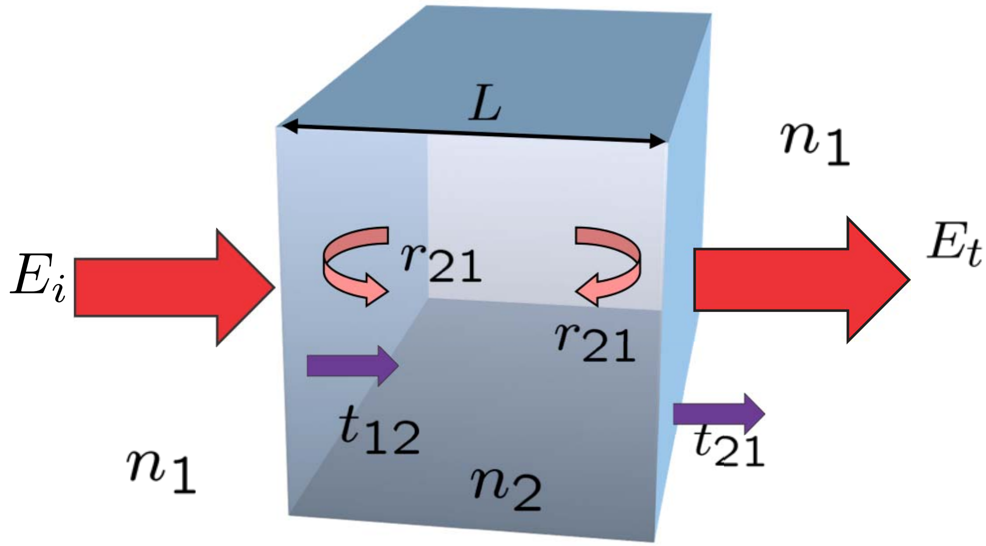
Everyday Anti-Reflection Coatings





Transmission Again

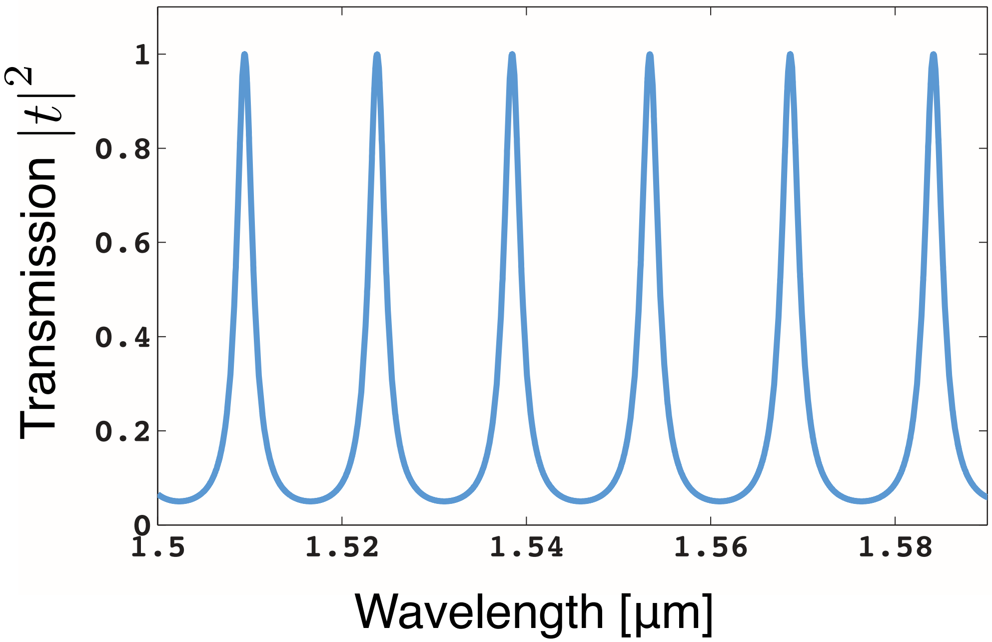


Transmitted Wave from a few slides ago

$$E_t = \frac{E_i t_{21} t_{12} e^{-jk_2 L}}{1 - r_{21} r_{21} e^{-j2k_2 L}}$$

Fabry-Perot Resonance

$$t = \frac{t_{12}t_{21}\, e^{-jkL}}{1 - r_{12}r_{21}\; e^{-2jkL}}$$

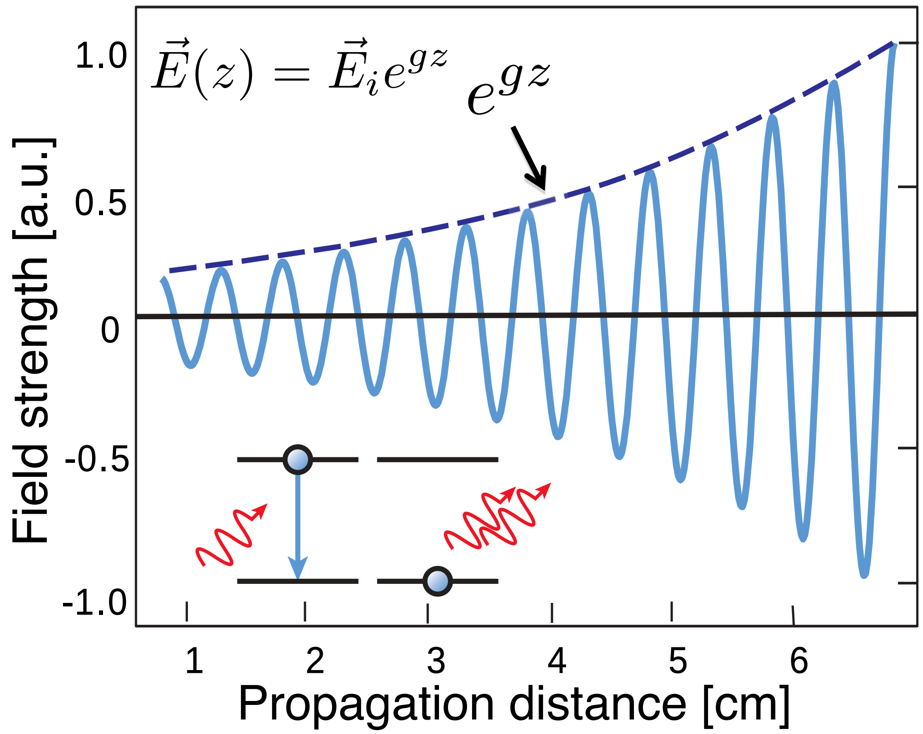


Fabry-Perot Resonance: $e^{-2jk_2L} = 1$ maximum transmission

$e^{-2jk_2L} = -1$ minimum reflection

Resonators with Internal Gain

What if it was possible to make a material with "negative absorption" so the field grew in magnitude as it passed through a material?

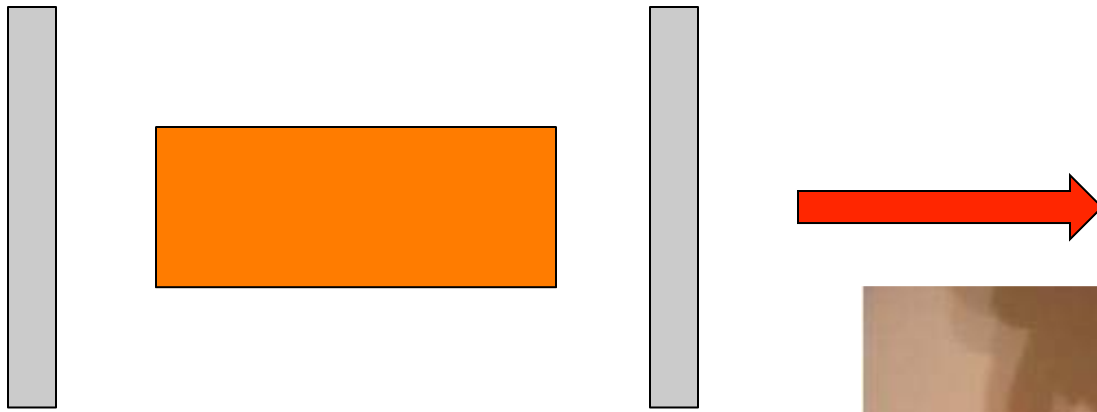


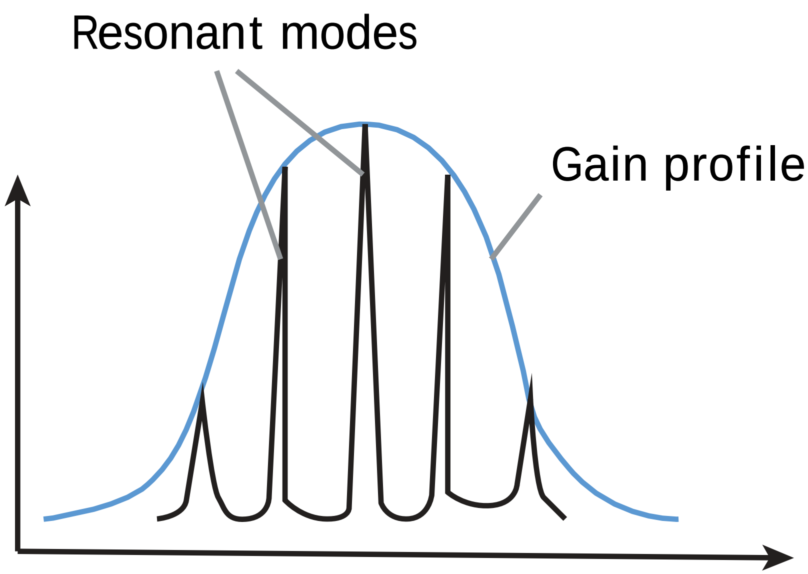
$$\frac{E_t}{E_i} = \frac{\tilde{t}_1 \tilde{t}_2 \, e^{-j\tilde{k}L}}{1 - \tilde{r}_1 \tilde{r}_2 \, e^{-2j\tilde{k}L}} = \frac{\tilde{t}_1 \tilde{t}_2 \, e^{-jk_r L} \, e^{-gL}}{1 - \tilde{r}_1 \tilde{r}_2 \, e^{-2jk_r L} \, e^{-2gL}}$$

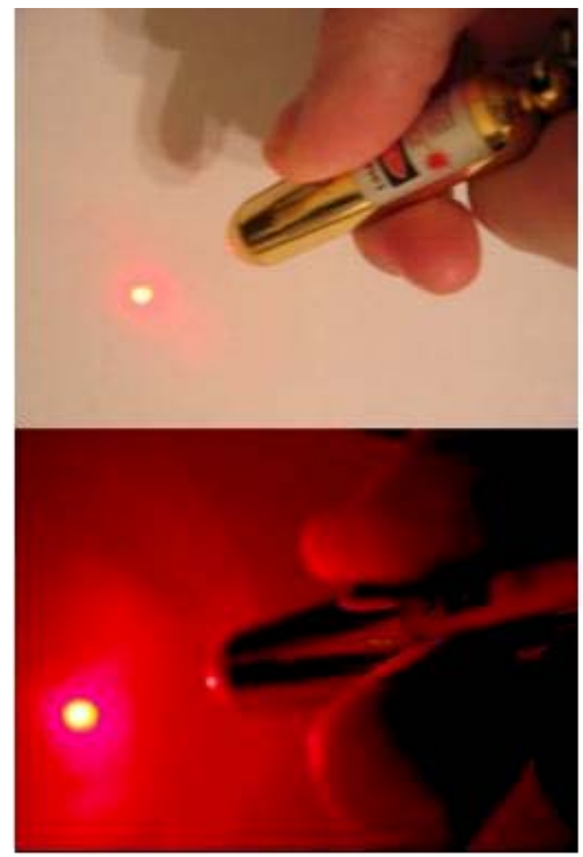
Resonance:

$$e^{2jkL} = 1$$

Laser Using Fabre-Perot Cavity







Key Takeaways

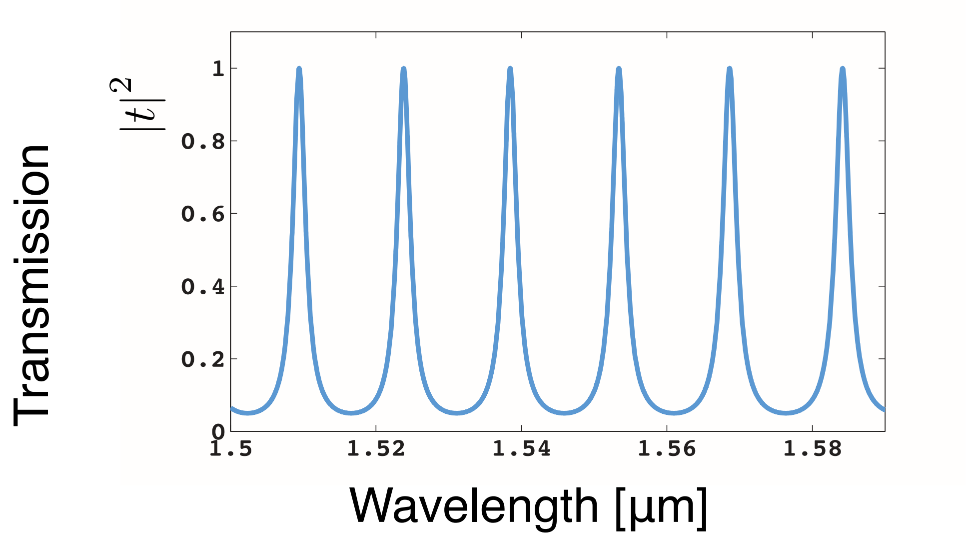
Reflection and Transmission by an Infinite Series

$$E_r = E_i(r_{21} + t_{21}r_{23}t_{12}\Gamma^2(1 + r_{21}r_{23}\Gamma^2 + r_{21}^2 r_{23}^2\Gamma^4 \ldots))$$
$$= E_i(r_{21} + t_{21}r_{23}t_{12}\Gamma^2/(1 - r_{21}r_{23}\Gamma^2))$$

$$E_t = E_i(t_{23}t_{12}\Gamma(1 + r_{21}r_{23}\Gamma^2 + r_{21}^2 r_{23}^2\Gamma^4 \ldots))$$
$$= E_i t_{23}t_{21}\Gamma/(1 - r_{21}r_{23}\Gamma^2))$$

Anti-reflective coatings by impedance matching:

$$d = \lambda/4n_2$$
$$(n_2)^2 = n_1 n_3$$



Fabry-Perot Resonance