Quiz 2 Review

Scope

- Quiz 1 material fair game but explicitly **not emphasized**
- 6 lectures on graphs, L09-L14, 2 Problem Sets, PS5-PS6

Graph Problems

- Graph reachability by BFS or DFS in $O(|E|)$ time
- Graph exploration/connected components via Full-BFS or Full-DFS
- Topological sort / Cycle detection via DFS
- Negative-weight cycle detection via Bellman-Ford
- Single Source Shortest Paths (SSSP)

Restrictions		SSSP Algorithm	
Graph	Weights	Name	Running Time $O(\cdot)$
DAG	Any	DAG Relaxation	$\|V\| + \|E\|$
General	Unweighted	BFS	$\|V\| + \|E\|$
General	Non-negative	Dijkstra	$\|V\| \log \|V\| + \|E\|$
General	Any	Bellman-Ford	$\|V\| \cdot \|E\|$

- All Pairs Shortest Paths (APSP)
  - Run a SSSP algorithm $|V|$ times
  - Johnson's solves APSP with negative weights in $O(|V|^2 \log |V| + |V||E|)$

Graph Problem Strategies

- Be sure to **explicitly describe a graph** in terms of problem parameters
- Convert problem into finding a shortest path, cycle, topo. sort, conn. comps., etc.
- May help to duplicate graph vertices to encode additional information
- May help to add auxiliary vertices/edges to graph
- May help to pre-process the graph (e.g., to remove part of the graph)

Graph Problem Common Mistakes

- Define your graphs! Specify vertices, edges, and weights clearly (and count them!)
  - (e.g., construct graph $G = (V, E)$ with a vertex for each...
    and a directed edge (u, v) with weight w for each...)
- State the problem you are solving, not just the algorithm you use to solve it
  - (e.g., solve SSSP from s by running DAG Relaxation...)
- Connect the graph problem you solve back to the original problem
  - (e.g., the weight of a path from s to t in G corresponds to the sum of tolls paid along a driving route, so a path of minimum weight corresponds to a route minimizing tolls)

Problem 1. Counting Blobs (S18 Quiz 2)

An **image** is a 2D grid of black and white square pixels where each white pixel is contained in a **blob**. Two white pixels are in the same blob if they share an edge of the grid. Black pixels are not contained in blobs. Given an $n \times m$ array representing an image, describe an $O(nm)$-time algorithm to count the number of blobs in the image.

Problem 2. Unicycles (S18 Quiz 2)

Given a **connected** undirected graph $G = (V, E)$ with strictly positive weights $w : E \to \mathbb{Z}^+$ where $|E| = |V|$, describe an $O(|V|)$-time algorithm to determine a path from vertex s to vertex t of minimum weight.

Problem 3. Doh!-nut (S18 Quiz 2)

Momer has just finished work at the FingSprield power plant at location p, and needs to drive to his home at location h. But along the way, if his driving route ever comes within driving distance k of a doughnut shop, he will stop and eat doughnuts, and his wife, Harge, will be angry. Momer knows the layout of FingSprield, which can be modeled as a set of n locations, with two-way roads of known driving distance connecting some pairs of locations (you may assume that no location is incident to more than five roads), as well as the locations of the d doughnut shops in the city. Describe an $O(n \log n)$-time algorithm to find the shortest driving route from the power plant back home that avoids driving within driving distance k of a doughnut shop (or determine no such route exists).

Problem 4. Long Shortest Paths

Given directed graph $G = (V, E)$ having arbitrary edge weights $w : E \to \mathbb{Z}$ and two vertices $s, t \in V$, describe an $O(|V|^3)$-time algorithm to find the minimum weight of any path from s to t containing **at least** $|V|$ edges.

6.006 Introduction to Algorithms

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