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6.002 Circuits and Electronics, Spring 2007
Transcript – Lecture 3

Let's get started. Can you hear me back there? Loud and clear. OK. Let's get started. Before I begin, just a couple of announcements. Brad Buren is one of our students here and he needs a note-taker.

It's a paid position. So if you are interested you can stop by after class and see him. He's sitting right here out there, OK? Second, just a reminder that 6.002 does have prerequisites. And the prerequisites are 8.02 and 18.03.

So with that let me start off with the usual. Do a quick review of what we've done so far. So we started out life looking at the laws of physics and Maxwell's equations and so on. And those were way too hard so we said let's make life easy for ourselves.

So we chose to play in this playground in which we said we shall adhere to the lumped matter discipline. OK? The LMD. So we are in that playground. So this entire course, and for that matter large parts of EECS are within that playground, within which the lumped matter discipline applies.

So as soon as we jumped into the playground, the LMD playground, we could take Maxwell's equations and abstract them out into two very, very simple rules. And the very simple rules were KVL and KCL.

KVL simply said that I can sum the voltages in any loop in a circuit and the result then would be zero. Similarly, I can sum the currents that enter or exit any node and the sum will also be zero. So what you can now do is, if you feel like, you can go around and brag.

Oh, yeah, we use Maxwell's equations in everyday life and, yeah, it's good stuff. And the key is that this is really an encapsulation of Maxwell's equations within this playground that we are in. So I talked about the first method of circuit analysis in the last lecture.

And that method simply took the, wrote KVL for all the loops, wrote KCL for all the nodes and wrote element vi relationships. And together gave you a big bunch of equations. And you sat down and grunged through the equations and you solved for branch voltages and currents.

So we reviewed a second method of circuit analysis. And I'll simply call it circuit composition. The basic idea behind this method was to learn some simple rules of how resistors add and conductances add and so on and so forth and look at a circuit and simplify the circuit by making series simplifications when the resistors are in series and so on and so forth, and compose it and play around with it till we end up with the current, the voltages that we are looking for.

This is the intuitive method. And so a section in Chapter 2, I believe, of the course notes discusses several examples using this method and attempts to make a little bit formal the intuitive approach that is applied in this method.

So we then looked at the node method. And the node method was simply a particular way of applying KVL and KCL. Node method, remember? We took a ground node. Then we labeled the nodes of the remaining voltages with respect to that ground.

Then we wrote KCL for each of the nodes. And when we wrote KCL for each of the nodes, remember, KVL was implicit in this expression that we used for each of the currents that were exiting each node.

So if Ej was a node voltage, then Ej minus Ei multiplied by the conductance Gi was the current that was going through one of those, I should call it Gij. This is a conductance that connects nodes i and j.

That gave us the KVL that fed into the same system. So these are three methods. The node method, by the way, is sort of the workhorse of the 6.002 industry. And for that matter for all of the circuits industry.

When in doubt, apply the mode method, you'll be OK. That applies to linear circuits, nonlinear circuits, what have you. What I'm going to do today is go through two more methods. So notice that the first few lectures of this course, the first three lectures simply comprise transitioning you from the world of physics to the world of EECS.

And then two lectures on giving you a bag of tricks. So we start you off with the sort of tools, your mallets and chisels and so on and so forth. And these five methods are your tools. We'll look at two methods today.

One method is called the method of superposition and the second method is called the Thevenin method. And these methods apply only to linear circuits. So we look at the subset of circuits that are linear, and these two methods apply to only those circuits.

These are methods that combined with intuition really enables you to solve very interesting circuits very, very quickly. So let me do an example using a usual node method. And then jump into introducing the superposition methods and Thevenin methods using that same example.

So let me draw you an example circuit here. So, again, I'm using this example, I will use this example to introduce the method of superposition and the Thevenin method. So what I'm going to do is start off the usual way and analyze the circuit using a method that you know now, the node method.

And what I'll do is write down the node equations for this by applying the node method. So if you recall the node method. I choose a ground node. I'm going to choose this node. It's got both the voltage source connected to it, and it's also got many other edges impinging on it.

So I'm going to choose that as my ground node and I'm going to label the other nodes with their voltages. So this is an unknown. I'll label it as e. I guess we just have one unknown e. And I know the voltage of this node, and that is simply V.

Since it's V, there's a voltage source between the ground node and that node. So what I can do next is that I can write down the node equation for this node and then go from there. So let me go ahead and do that.

So let me sum up the currents going outside, going outwards. So I have e minus v divide by R1, I have e minus zero divide by R2, and I have minus i equals zero. This is a node equation. The first thing I want you to observe is that this equation is linear in V and i.

What I mean by linear is that you don't see terms like Vi or V-squared and things like that. It's some constant times V plus some constant times i equals some other constant. So that's quite nice.

So I'm going to rearrange the terms in the following manner. I'll move the known sources to the right-hand side and collect the coefficients of e on this side, so I get one by R1 plus one by R2 over here.

So stare at this for a moment and notice again here I have e, my unknown node voltage, there is some constant multiplier, and that equals some function of V summed up with some function of i. And, again, notice that this is a linear combination of V and i.

No multiplication terms and so on and so forth. This is a pretty standard form in which we will represent equations quite often. And just to label it, this is often labeled G as the conductance matrix.

Of course this is e, our unknown node voltages, and this is a linear sum of sources. So this is a very standard way that we will represent equations. We did that last week as well, or rather on Tuesday where I took a conductance matrix, multiplied that by a column vector of unknown node voltages and equated that to some linear combination of my source voltages.

The reason the circuit is linear is that I have only linear elements in the circuit. I don't have any nonlinear elements. And because of that I can rewrite this in the following manner. I'm just going to express e as a function of V and i and bring it over to this side.

So it's some function of i. So I get R1 R2 divide by R1 plus R2. And I bring R1 R2 to this side. That's what I get. So stare at this for a few seconds, very common form. My unknown node voltage is equal to this stuff on the right-hand side.

The stuff on the right-hand side has a term multiplying the source voltage V and some other term multiplying the current I. And if I were to put this in sort of symbol-like form my unknown node voltage is some constant times V1 plus some constant times, is of the form constant times the source current, constant times the source voltage and so on.

The units of As and Vs are different because in this case A has no units because V is a voltage. And so is e. In this case V has units of resistance. So that V times i gives me a voltage. So stare at this equation for a few seconds and this should help us build up some insight that will allow us to write down the answer almost by inspection.

I'm going to show you a method now, in a few minutes, which will allow you to write down the answer e just by starring at the circuit without having to go through node equations and so on. The more and more methods I teach you, the more you will be able to do a lot of this completely by yourselves.

In this particular example it's a relatively simple circuit but these methods would be particularly useful when you have more complicated situations. But before I go on let me spend a few minutes pontificating on linearity.

So that's a linear circuit. And this equation gives me the unknown node voltage e as a linear sum of source voltages and source currents. Linearity implies two properties, the property of homogeneity and also gives vice to the property of superposition.

Let's do homogeneity first. What this says is if I have a circuit, some circuit and I feed it some sort of inputs, A, then let's say my output is S. If you're feeling hungry think of these as apples and the circuit converts them into applesauce.

So what homogeneity says is that what I can do is if I take each of my apples and instead of feeding it an entire apple what if I give it three-quarters of an apple? Say I multiple all my inputs by some constant alpha, three-quarters.

What that says is that at the output instead of getting one full bottle of applesauce I'm going to get three-quarters of a bottle of apple sauce. So if I proportionately reduce all the inputs and if this is a linear circuit then so shall my output be reduced in the same proportion.

So that's homogeneity. Next, let's look at superposition. The property of superposition says the following. The same kind of circuit. If I feed it apples then I get applesauce. I take the same circuit, and this time around if I feed the circuit a different set of inputs, say blueberries.

And let's say my output, oops, let me do it this way. So as my output I get blueberry sauce, if such exists. So apples applesauce, blueberries give me blueberry sauce. Then what I'm going to get if I mix up the two, so let's say I take my circuit, the same circuit with a set of inputs and in this example one output.

Let's say I mix up my inputs and some of my inputs in the following way, here I feed an A1 plus B1 and here A2 plus B2 and so on then at the output I am going to get a mush of apple sauce and blueberry sauce.

All this says is that if I apply just apples I get applesauce. If I apply just blueberries I get blueberry sauce. Then if I were to figure out how this blender would have worked had I fed in the combinations of apples and blueberries, then for the purposes of understanding that blender all I could have done was taken by two outputs and just mixed them up together myself and that's exactly what I'd get.

So if I sum up the inputs my outputs would also be the sum of the outputs with the inputs applied by themselves. So let me take this here and munge around with hit for a few seconds and get something interesting out of it.

So notice two inputs, two inputs, outputs. In your notes I've given you another template for the next set of scribbles I'm going to make here. So use the next set of templates on page three. What I'm going to do here is something very simple, set one output to zero and feed a voltage V1.

So that's feed a voltage V1 and set the other output to zero. And let's say I get Y1 as an output. And in this case I set the first voltage to zero and feed a different voltage V2 on the second input.

And let's say my output is Y2. This is just a particular application of the superposition principle I just outlined. Apply V1 set one output to zero. Apply V2 set the original output to zero. Then what I'm going to find is that the answer will simply look like this, just replace for As and Bs what I just did and we get V1 and zero here and we get zero and V2 here.

And as my output I'm going to get exactly the sum Y1 plus Y2. This is simply a particular application of superposition where what I'm saying is the following. If you look at this circuit here effectively what have I done? Effectively what I've done is apply the voltage V1 on one input and a voltage V2 on the other input.

V1 here. V2 here. And the output is Y1 plus Y2. What I'm saying is look backwards now. What I'm saying is that the whole components of the output Y1 plus Y2 could individually be derived in the following manner.

I could get the component Y1 by simply applying one of the voltages and setting the other to zero. I can get the other component Y2 by setting yet another input to zero and applying the voltage V2 to get Y2.

And sum then up and that's my answer. This will become a lot clearer with an example. Again, remember if I have a bunch of inputs applied to a circuit, V1, V2 and so on, and I get some output then what this is saying is that I can alternatively find out the answer by applying just one voltage, setting all the others to zero, measuring the output, apply a second voltage, set all inputs to zero, measure the output and sum of applesauce and blueberry sauce and there you get the answer.

Let's do an example. And before we go into that I talked about setting voltage sources and current sources to zero. First of all, what does it mean to set a voltage source to zero? This is the same as this.

Setting a voltage source to zero is simply replacing the voltage source with a short, and setting a current source to zero simply implies an open circuit. So when I say zero that source, if it's a voltage source short it, if it's a current source open it.

I can take any two nodes in the world and measure the potential difference across them. So there may be some potential difference across these set by the circuit that I haven't shown you on this side.

There might be some other circuit that is controlling the voltage of these two nodes. The same with the short. What's V going to be? But there is a V. It's zero. So that's method four, method of superposition.

And this method says that the output of a circuit -- Again, remember I'm focusing on linear circuits. Remember, I have this playground where LMD applies. And within that playground I'm playing in the south goal area.

In the south goal area, in that subset of the playground circuits are linear. So in that part of the playground superposition applies because there circuits are linear. So the output of a circuit is determined by summing up the responses to each source acting alone.

Now, in this statement here this source stands for independent source. I haven't talked about independent versus dependent sources. We'll talk about dependent sources a few weeks from today. And just so you don't get confused, for dependent sources you will be looking at

Section 3.3.3 of your course notes to see how superposition works with dependent sources.

But remember we haven't covered dependent sources yet. We will be covering them about two weeks from now. So let's go back to our example and apply the method of superposition to an example. So the method says sum up the outputs of each of the sub-circuits where I'm applying one source acting alone.

So let me just do this here. Let me start with the circuit. And let me start with shutting I off. So I have voltage V -- I have R2. And I'm shutting I off. So I have replaced this with an open circuit.

So I is zero. Let me call the node voltage eV to reflect that component of the node voltage that arises due to V acting alone. And you should look at this pattern here and very quickly be able to write the answer for patterns like this voltage, the two resistors.

That's called a resistive divider. It will appear again and again and again. And eV is simply V times R2 divided by R1 plus R2. That's still my ground node. So the voltage here is simply this voltage divided by the two resistors to give you the current multiplied by R2 to give you the voltage across this R.

Remember this pattern. You apply voltage divider patterns probably more times than any other pattern that you might imagine. So that's with the V acting alone. Now, let me do I acting alone. So for I acting alone -- And what I do this time around is replace this with a short, replace the voltage source to the short.

And let me call this voltage eI for the component of the voltage due to the current I. And eI, in this case, is simply given by yet another pattern here, the current across a pair or resistors is simply the effective resistance multiplied by the current so it's i and the effective resistance is R1, R2 or R1 plus R2.

That's eI. That's a component that node due to the current I. Now, so the method says that. Then take these components, sum them up and there you have the answer. So E is simply ev plus ei. The components of V and I acting alone, just simply V times R2 divided by R1 plus R2 plus R1, R2.

There we go. Fortunately, the fates have been kind to us and the answer is the same as the answer we obtained with the node method. No surprise here. So this is actually an incredibly simple method.

So you can take a very complex circuit. What have you really done here? You can take a very complex circuit and you can solve a very complex circuit by breaking it down into many simple individual sub problems.

You will do this in EECS time and time and time again. Whether it's in software systems or hardware systems or what have you, you're often times building complicated systems. Remember doom on this side? And the way and when you put these things together, let's say a large software system, is you don't write the whole piece of software starting main and grunge down.

You build a lot of little components and tie the components together. In the same manner here you take a big circuit and you find its behavior for each source acting alone. Lots of little inky dinky simple little circuits.

And you will see examples in your homework where you're given a big circuit or because it set all the Is to zero and the other Vs to zero the whole circuit almost vanishes and all that you're left with is a little resistor or two.

So this is the very, very powerful method. I'd like to do a little demonstration for you. And what I'm going to show you is the demo is a vat of water. Actually, I'll tell you what it is in a second.

But assume it is salt water for now. I'll apply two voltages. In this case I'm going to apply a sinusoid. That's not very good. A sinusoid and a triangular wave. And what I'm going to do is measure the response at this site.

Now, this is a vat of salt water. And I'm going to tell you it behaves like a linear system. If you view each little particle, or each little cubic-centimeter or whatever of water, it'll behave like little resistor.

So this vat of salt water behaves like big distributed resistor in the following manner. And so on. This of this big mesh of little resistors, but it's all resistors. It's a linear circuit. So I'm going to apply two voltages, a triangular and a sinusoid, and we're going to observe the output.

And what do you expect to see there? You will see the superposition of the two, which is you'll see a sinusoid. And then you'll see the jagged triangular thing articulating the sinusoid pattern. What I'm going to do right now, don't put any water yet.

This is the vat of nothing right now. It's all empty. Can we show the screen on this side? The oscilloscope screen? OK. Oh, there you go. So this is the screen of the oscilloscope now. Notice that I have a sinusoid and I have a triangular wave and the output is zero.

And the reason is there is nothing in this vat. It's empty. So previously when I taught this course I would get saltwater and pour saltwater. Then we discovered a much better source of water that conducted electricity like one real mean fluid.

Cambridge water. It just works very pleasantly. It just conducts electricity like nothing at all. And I've been thinking of using Charles River water next time and see what happens, although there we'd probably get some biological organisms doing strange things at you.

But go ahead. Our friendly demonstration expert, Lorenzo, will pour some water into the vat. And you should begin seeing the output being a superposition of the two. So as he pours, there you go, do you see that? So you do see the sinusoidal articulation and the jagged wave form.

And just to have some more fun, what I can do is increase one of the voltages. And you'll see -- Now you know what would have happened if I had used Charles River water. So my output keeps increasing as I increase the corresponding wave form.

I could do this, this is fun. So let me pause there and go onto the next topic. So that little demonstration showed you that even something as simple as this physical entity vat of water behaves like a linear system, and we can model that linear system as a set of resistors.

Unbeknownst to you, right now, in the past ten seconds I introduced a new concept. It's called subliminal advertising. So one of the things we do in EE a lot is model real systems. So often times if I wanted to look at the behavior of salt, behavior of a vat of water, I can model it as a set of resistors for certain kinds of activities.

Just hold that thought for some time later in your careers. All right. That's method four, the superposition method. Remember, it is methods like this that will make your life really, really, really easy.

If you find that you are having to do a lot of grunging homework or something, just step back and think superposition, think Thevenin or think composition rule. There must be a simpler way usually. Let's do the next method.

This is called the Thevenin method. To derive this method let me start by applying superposition to some circuit. So let's say I have some arbitrary network N. Assume it's a linear network and the network has a whole bunch of goodies in it.

It has a bunch of resistors, it has a bunch of voltage sources, and it has a bunch of current sources. Many current sources. Many voltage sources. Many resistors. Some jumbled voltage sources, current sources and resistors.

And I look at two nodes in this network. Here are two nodes in the network, two points in the network were elements connect. I'm looking at those two nodes and all I want to do is the following. I want to figure out if I take a rinky-dinky little current source and apply it there, all I want to figure out is what is V and what is I.

There is this mongo box out here, a black box of resistors, voltage source and current sources, too many to count. I pick two nodes, apply a current source, and all I care about is what is the voltage that I will measure by applying it here.

Notice the current here will be I because the current here is I. And I apply it here. I want to measure what the voltage is. Now, with the insight you've obtained from superposition, you should be able to jump up and state the form of the answer.

So by superposition we know the following. We know that the effect of the circuit will be the same as the sum of components being added up. Sum of component, sum of component, a bunch of components added up.

Each component will be the response of one source acting alone. So if I can figure out the effect of one source acting alone and put that down here, and do the same thing for all the sources, that's what I will get.

So for the source Vm it's a linear circuit. So I know that my answer is going to be, in the final answer is going to be a Vm term and it's going to be multiplied by some alpha M term. I know that. It's a linear circuit so I know that the answer shall have a term Vm multiplied by some constant.

Simple, I know that. Similarly, the same is true for, oh, this is the term Vm. And what I can do is I can measure just this effect by setting all the other sources to zero. So I can set all the other current sources to zero and all voltage sources, except for this one, and I can get that answer.

So, similarly, for every voltage source I am going to get a term. So for every single voltage source, M1, M2, M3 and so on I'm going to get such a term and they're all going to sum up. Similarly, I'm going to get a term for In.

And I know there will be an In term, and I know it's going to be some constant beta multiplying In. In this example of ours here, in this example, remember alpha was this and beta was this constant here.

There's some constant beta, some constant alpha. And because I have a whole bunch of current sources there's going to be such a term for each one of them. And each one of these terms, Vm, In will be the voltage I would see here if I set all the other Vms to zero and I set all the other current sources, except for that one to zero.

What am I missing? Is that it? The response here, V here. Am I missing anything here? Is that it? Now, don't all yell at once. What am I missing? Current source i, exactly. So if I have a current source i then there's an effect of this current as well.

And so I write down i there, too. It's going to be some constant multiplying I. And that constant is going to look like a resistor, right, because this circuit contains current sources, voltage sources and resistors.

If I've shorted all my voltage sources and opened all my current sources, what's left in here? Just a whole caboodle full of Rs. It's just going to look like some resistance R. And that's what I get here.

So this is what V is going to look like and that's a form. So let's take a look at these components. Let's focus on the easy part first. What does this look like? This component looks like an I, it looks like a current and has some resistance.

What is that resistance given by? Supposing I gave you this network and this currency source and I asked you tell me R. How would you measure R? What you would do is open all the current sources, short all the voltage sources, put a ohmmeter in there and measure the resistance R.

That's R. OK, so we understand this term. What about this term here? Can someone tell me the units of this term here, this big thing here? Voltage. This is a voltage. This is a voltage. iR is a voltage.

So this does behave like a voltage. And it behaves like some voltage V. So notice that as far as this current I is concerned the rest of the universe looks like a resistor and a voltage source behaving in some manner.

And let me just call it Vth for now, and you'll know why in a second. The voltage has a form, some voltage plus Ri. So, in other words, as far as this I is concerned this whole network here N full of all the nice stuff is indistinguishable to this I here.

So my I is sitting out there injecting a current into two nodes. If I am i, I'm looking at this, this network looks no different than a voltage source in series with the resistor R. Notice that the equation for this simple circuit is this, so I is given by V minus Vth divided by R.

Just remember. It's a circuit. In other words, Agarwal sitting here cannot tell the difference if I'm measuring the voltage here between a circuit that looks like a Vth in series to the resistor or this huge mess of voltage sources and current sources and so on.

Now, we will talk about Vth and R. R is called the resistance of the network as seen from the port with all the sources shut off. And similarly Vth, what is Vth? Vth is the open circuit voltage. In other words, if I apply the voltage here this is the response of all the current sources and all the voltage sources acting together.

So it's as if I took this out and simply measured my V here as if I didn't exist, correct? Because this is the component of i. So if I opened i and measured V, I would get that big term on the left-hand side.

That's my Vth. So that inspires the next method called the Thevenin method. In this method what I'm going to do is take some circuit, I'm on Page 9, with a mess of stuff. It's a big mess of stuff.

And if I care to look at its impact on something else that I add from the outside then as far as the outside world is concerned this is indistinguishable from a circuit that looks like this. So what I can do is if I want to figure out what's happening here then, for the purpose of my analysis, this simple network here with R and Vth becomes a surrogate for this entire mess.

So for the purpose of finding out the behavior at this point, I can take this huge mess and replace it with its Thevenin surrogate or Thevenin equivalent. This is called the Thevenin equivalent of this big network.

Let me do an example that will make the method completely clear. Again, remember in EECS, most of our lives are about how can we make things so simple as being able to be analyzed by inspection? And so this is a method that takes you further down that path.

So let me use the same circuit that I've been using before, my voltage V, R1, R2. This is an R. I'm 55 minutes fast so we have another three or four minutes. So this is my circuit. And let's say all I care about is finding out i1.

That's all I care about. And what I'm going to do is I'm going to box this up and see if I can replace that with its Thevenin equivalent. So I'm going to box that up. What I'm saying is that I'm going to box it up and replace it with this Thevenin equivalent.

I don't know what Vth and R are at this point. I'm just calling it Rth for fun. I don't know what these two values are, but if I knew what these two values were I can determine I really trivially as follows.

I can get i1 as simply V minus Vth divided by R1 plus Rth. So if I knew Vth and Rth, I can write down i1 by inspection in that manner. So next, finally, how do I get Vth and Rth? You get Rth by looking at this network and shutting off all the voltage sources and measuring the resistance there.

So I short my voltage source, that's R1. Oops, wrong way. I need to look this way. So looking this way, that's what I get. So what's Rth? Rth is simply R2. So I have opened my current source. Similarly, for

Vth, remember all I want to do is look at the two nodes, step back, put a voltmeter there, measure the voltage, that's my open circuit voltage.

So the way I do it is I take the circuit and simply measure the voltage there. That's R2. That's my current capital I. And I simply want to measure the open circuit voltage here, which is what? Just simply if I stand back and I kind of gingerly measure the voltage here without disturbing anything, I simply get IR2.

So Vth is IR2 and Rth is R2 and here is the formula for the current in this branch when I apply a voltage source and a resistor R1 to this little circuit here. OK, let's pause and let me summarize this in about ten seconds.

I had this circuit here. I wanted to find out i1. So what I said I'd do is take this complicated mess, well, it's not a complicated mess but assume it is, and replace with it a resistance Rth got by turning off all the sources.

And the voltage in series, Vth, which I get simply by pulling this thing out, taking my input, this part out and simply measuring the open circuit voltage out there, Vth. And then I replaced the whole network with this new network that they call the Thevenin network, and voila, I get the answer in a second.