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6.002 Circuits and Electronics, Spring 2007  
Transcript – Lecture 19

All right. Good morning all. Today we embark on another new chapter in what we do. And the topic is going to be -- We will talk about this thing called an Operational Amplifier. Before I get into the lecture, I want to point out a couple of things.

One is that you are going to hear about two big words in today's lecture. Two big and incredibly important words. And I want to mention those words to you right now so that when I come to them in lecture you can say OK, I better pay really close attention, these are important words.

All right. One of them is abstraction. The second one is feedback. Two incredibly important concepts. Abstraction, you have been seeing a couple times during this course, once in the beginning where we abstracted out Maxwell's equations by focusing on a smaller playground and simply using KVL, KCL in place of those equations.

A big abstraction. It turns out that almost all of EECS is based upon abstractions at various levels. In the first lecture, I also showed you the layer upon layer of abstraction that we built to be able to build interesting systems.

The second big thing is feedback. And I am going to relate this to anti-lock breaks in cars. And so, you can wait and see how we do that. It's an incredibly important concept. Before we dive into the amplifier abstraction, let me first talk about something that you know.

Start with something that you know and then lead up into the operational amplifier and its circuits. You know about the MOSFET amplifier. The MOSFET amplifier that you know about looked like this. It was based on a MOSFET.

There was a  $V_S$  supply. There was a  $v_I$  input, a  $v_O$  output and, as I said, a  $V_S$  supply. So, this was a MOSFET circuit that you've seen before. One way of viewing this is that this circuit has three major ports.

This here is the input port with voltage  $v_I$ . This here, between the drain terminal and the ground, is the output port. I take the output between the drain terminal and ground. And, finally, we have a third port, which is this one.

It is called the power port. I apply  $V_S$  between this terminal here and the ground terminal. And that gives us the power port. This device here was a three port device. Input port or control port, output port and a power port.

And so we looked at the circuit and did a whole bunch of analyses of it. Then what I can do at this point, now that you've seen this, it's often times interesting to think about abstracting this out into some kind of a building block.

Much like in software, you write a procedure and you abstract out the internal details of the procedure in the procedure declaration and in the call that you make. In the same way, we can take this little device here and abstract that out into the following abstraction.

We could abstract that out as a device that looks like this. I have my input port, I have my output port and I have my power port. So, I can apply  $V_S$  here. Notice that I've taken these six terminals here, one, two, three, four, five and six, and put a box around it.

And just exposed the terminals to you. And I need to tell you a little bit more about the internal properties, but suffice it to say that you can begin working with this little block. An even simpler version of this for many applications might just look like this,  $v_I$  and  $v_O$  where there is a ground that is shared among them that is implicit in this picture.

And  $v_I$  and  $v_O$  can simply be the node voltages at these nodes. This is a progressively more abstract representation of this amplifier. What we can do is, provided we know, we can abstract out the relevant properties of this block and expose them outside.

And the relevant properties might well be that, let's say here the properties may be that  $I_{in}$  is always zero. I can also express to you the gain of this amplifier. I may also be able to tell you the Thevenin equivalent for the output.

There are some properties that I can give you that will let you use this building block abstractly. Today, what we will do is introduce a

powerful abstraction of a type of amplifier. This is called the operational amplifier or "op amp" for short.

What I am going to do is give you a slightly more involved building block than the one I have shown you there. But suffice it to say that the idea is going to be the same. This building block looks like this.

This building block has an input port. This building block also has a port in which to connect power or the power port. And the way I am going to connect power, I am going to connect a plus  $V_S$  supply here.

That is going to be my ground node. And I am going to connect a minus  $V_S$  supply to this node here. So, these voltages are both  $V_S$ . I want to apply a plus  $V_S$  here and a negative  $V_S$  out here. And I am going to take the output between the ground node and the output node of the operational amplifier and call that a  $v_O$ .

This is the output port. So, input port and output port and a power port. Think of this as a pattern where I have an input port across which I connect the input. I have a power port across which I connect a plus  $V_S$ , minus  $V_S$  supply, and then I take the output terminal and take a ground terminal, which is defined by external components of my circuitry, and use this as my reference node.

Remember ground is just a reference node. I am going to use this as a reference node. These two are equal in magnitude. And take this as my output. And when I do something like this, I can build an even simpler, so this is an abstract differential input amplifier.

In other words, this amplifier is going to amplify whatever I apply at the input. A slightly more abstract representation of this looks like this.  $v_{OUT}$  and plus/minus  $v_{IN}$ . This is a slightly more abstract representation where, remember, we are going to draw this again and again, maybe at least 38 or 39 times in this course.

And, remember, each time you draw it, remember that there is an implicit power port, a plus/minus supply that is connected which we don't show. And I remember when I first learned about it a long time ago there was a confusion in me initially.

How does this work? Where is the power coming from? Just remember that power comes from a plus/minus supply, and we just don't show that in this abstraction. Now, the details, a lot of details are in Chapter 16 of your course notes.

That's the reading for that. The other thing is that there are some other key properties of this amplifier. And let me discuss those very quickly. First of all, I can draw a circuit model for the amplifier.

Make some room for myself here. And this is a circuit model for what we call the ideal operational amplifier. And the circuit model is going to look like this. This is an abstract device. And, in terms of analyzing how this behaves in a circuit, I am going to show you this abstract circuit that looks as follows.

Some input  $v$  is applied at these two terminals here. And this terminal is called my  $v$  plus terminal and this is called my  $v$  minus terminal, so this corresponds to these two terminals. I am telling you that the current going in is going to be zero, so  $i$  plus is going to be zero and  $i$  minus is going to be zero.

$i$  plus is the current in here and  $i$  minus is the current into the  $v$  minus terminal, and both these currents are going to be zero in this device here. The output is going to look like this. Let me just call it  $v_{OUT}$  to be consistent with this here.

And taken with ground as my reference. The output is simply  $Av$ . In other words, what I am doing is I am going to model this as a device that has a dependent source at its output. And the dependent source here is a voltage controlled voltage source.

It is a dependent source, it is a voltage controlled voltage source such that the output voltage is  $A$  times the voltage  $v$  across its input. This is actually very simple. Think of these three terminals I have shown you here.

I applied input across these. And the output is going to be  $A$  times whatever I applied. And  $A$  is going to tend towards infinity.  $A$  is going to be huge. And specific values for  $A$  might be a hundred thousand or a million or things of that sort.

Huge  $A$  in this abstract amplifier. In addition to that, the other properties are that it is going to have infinite input resistance. That means looking in this looks like an open circuit. The fact that this is open here implies the infinite input resistance across this port.

What about the output here? Remember, this is a voltage source. And we have a zero output resistance, which means that no matter how

the load affects this, as I apply a load this is going to behave like an ideal voltage source and keep holding the voltage constant based on whatever the function I establish here.

And  $A$  is virtually infinite. Let me pause there for a few seconds and just dwell on this so you just understand what the basic device is. Following this basic definition, I am just going to build a whole bunch of fun little circuits.

The analysis will be pretty straightforward, but this is a big conceptual leap here where there is some circuitry inside. Containing resistors, MOSFETs, a whole bunch of stuff in there. I am not telling you what is inside it.

Much like I could build an abstract amplifier, I could put an abstract box around the amplifier you saw earlier, I want to put a box around some circuitry. I am not telling you what the circuitry is.

And, if you are curious, you should look at page 581 of your course notes. There is an example solved. The example is for a differential amplifier. This is the small signal analysis chapter. That differential amplifier that's solved in that example is usually the first stage in an operational amplifier circuit.

That differential amplifier is the first stage at the input. And that differential amplifier, as the name implies, amplifies not a single voltage but amplifies a differential voltage. Note that this guy amplifies the voltage difference between these two terminals.

That's  $v$  here. And  $v$  is simply the same as  $v$  plus minus  $v$  minus. It's the node voltage here minus the node voltage here. That is what's amplified. It amplifies a difference. Therefore, it is called a difference amplifier or a differential amplifier.

And so that input stage is what is inside the op amp. It's got a bunch of other circuitry like level shifters and so on. And at the output it has got a buffer. At the output it has something that is reminiscent of the source follower circuit that you learned about in recitations, solved an example in the course notes and in your homework as well.

And you solved a variant of the source follower on your quiz as well in problem two. So, a circuit that looks like that appears at the output. Remember, for the source follower, the resistance looking in from the output was very, very small.

You have seen some of the pieces that go inside the amplifier, but we will deal with this as a building block and simply represent it using this abstract little circuit. To dwell on this a little longer, this little device here is the workhorse of the analog industry.

Much like your primitive gate abstraction, your inverter and NAND gate and so on, much as your primitive inverter or NAND gate was from the foundations of the digital industry. Remember we learned how to build this little abstract device called a NAND gate or an inverter? We noticed that those form the foundations of very complicated microprocessors.

Those were the building blocks of the digital industry. In the same way, this little beast here is the building block of the analog industry. Just to give you an analogy from software, think of this abstract little device as a library routine from a library of functions when you program in C++ or whatever.

Can someone give me an example of an incredibly popular routine that we use all the time that may be called the workhorse of the software industry? Pardon? An abstraction, an abstract procedure. One example might be something like a printf.

Printf is an abstract name for a procedure that goes and does something for you. It is amazing how we take the lowly printf for granted. I stick my printf into my program, it includes the standard IO library and it goes and prints a value.

You won't believe how complicated the printf is. As you go into learning more advanced software subjects, implementing the printf is a nightmare. It is horrendously complicated. Just imagine. You give it a string and it has to go and print that on your terminal or on your Windows system or whatever.

Think of the complicated steps it has to go through. But, as far as you're concerned, it's simple. Just print out something and you're done. The same way. Think of this as the printf of the analog business.

It is really simple, and the analysis is going to be incredibly simple, it will be mind-bogglingly simple, but inside it, heavens forbid if you look inside it. Tell you what, go into to S-T-D-I-O dot in one of the library routines and just pore through printf.

The world's worst horrendous macros are in there. I mean it is just nasty. The same way inside the op amp, it is nasty. You don't want to go there. Much like in your C programming in your classes, you were able to use printf without fully knowing how it was implemented.

Probably some MIT god or some key graduate implemented it, but once it was implemented you just used it based on simple abstract rules as to how it behaved. You didn't have to know what was inside it to use it.

The same way with the operational amplifier. So, just think of printf when you see this and just imagine how simple it is going to be to use it. You may think that I spend way too much time, ten minutes dwelling on this abstract concept, but I like to dwell on things that I think are incredibly important.

The concept of abstraction is very important. And it's not just in software. The concept of abstraction pervades all of EECS. And if I were to give you a project to say go and ask every professor what is the one word that you think best describes all of EECS? Just pick one word.

Go ask every single professor you know. What is a single word? If you were to characterize all of EECS with just one word, what might that word be? In my mind, it is the A word, abstraction. It is all over.

If you do a grep on all the words used by all your professors in your four years here, I promise you the first one will be know. And the second one will be abstraction. Check it out. See if what I am saying is true or not.

It is all over the place. In 6.001, how many times do you think the word abstraction was used in 6.001? It's all over the map. It's the A word all over. Imagine your shock when you see it being used in 002 because the same concept applies.

We build more complicated systems by abstracting out the details of lesser objects, and then using those to build the more complicated systems. Abstraction is a very powerful mechanism of dealing with complexity.

Next step is how do I go about using the op amp? Let me show you how it looks on a scope. What I am going to do is apply input to the op



amp, I am going to look at the output, place the resistor  $R_L$  to ground and look at the output.

And here I am going to apply a plus  $V_S$  and out here a minus  $V_S$ . Again, remember that a plus  $V_S$  simply looks like this and a minus  $V_S$  simply looks like this. It's just an inverted  $V_S$  applied here so I get a minus  $V_S$  at this input.

First of all, what I would like to do is as I change  $v_{IN}$ , I am going to plot for you how  $v_{OUT}$  looks.  $v_{IN}$  and this is  $v_O$ . I am going to plot  $v_{IN}$  in terms of microvolts and  $v_O$  in volts.  $v_{IN}$  is going to have a very very small, the scale is going to be in microvolts because remember the gain of this is huge.

It's on the order of ten to the sixth. It's huge. Small changes in  $v_{IN}$  are going to cause massive changes in  $v_O$ . I have a very fine scale on the X axis. What is going to happen if I somehow magically make  $v_{IN}$  exactly zero? If I short these two terminals, if this was a completely ideal op amp, which it never is, if it's a completely ideal op amp, then my output should be zero.

As I increase my  $v_{IN}$  the output should be  $A$  times  $v_{IN}$ . For some small value of  $v_{IN}$ , small  $v$ , let's say one microvolt, the output should be one volt.  $A$  is a constant so this would look like a straight line.

And let's say my supply voltages are 12 volts minus 12 volts, if this were an ideal amplifier and I didn't have to worry about the supply, this would just go on extending forever. But I have a plus 12 volt supply and a minus 12 volt supply.

My output cannot go past those limits. And so, therefore, my output kind of flattens out at these two points. And it is called hitting the rails. Output goes up and you hear a thunk sound and you hit the rails.

When you play with op amps in your next lab, if you listen really, really carefully you may hear it. So, this saturates out. Not surprisingly, this region where the output saturates at the supply is called the saturation region.

Remember, don't confuse it with-- It's not the same as your saturation in the MOSFET. It is a totally different thing. It is just happenstance that we call this saturation. And if you would like to think about it, you can think of it as the thunk region.

That's probably more appropriate to distinguish it from the saturation region in the MOSFET. And, not surprisingly, this one is called the active region. And it is in this region that we use the op amp.

Here it has hit the rails and is kind of dangling out there. It's not much use to us. It's in this active region that we use it because this is where the gain is seen. Now, it turns out that this is a very high gain device.

It is very skittish. This gain is kind of a really funny thing. It's dependent on a bunch of factors. This could be temperature dependent. This gain here and this curve is just completely skittish.

It could depend on temperature. It could depend on time of day. It could depend on what medication this amplifier is on. It could depend on its mood swings. Who knows what? This is kind of unstable.

And A in particular is highly unstable. It is going to be big, that's for sure, but it could be ten to the six, on a rainy day it might be two times ten to the six. If it feeling sleepy it may be point five times ten to the sixth.

It is big but I cannot rely on it. Let me show you an example. I want to show you this curve for this MOSFET, apply an input and plotting the output. What I will do is take a look at this curve. Then what I am going to do is use a heat gun to heat the op amp and you are going to see this vary all over the map.

If you still remember last week, some of you may remember that from some place in a similar situation where the gm for the MOSFETs you were given was also dependent on temperature and stuff like that.

It is a very common occurrence. And that is certainly the case for the MOSFET. Let's apply input. Let's do this. This is  $v_{IN}$  versus  $v_{OUT}$  for the amplifier. Notice that this is plus 12 volts, this is minus 12 volts.

It is about two volts per division. This axis here is in microvolts, I believe. For a very small change, for a few tens of microvolts, I have an incredibly high gain. Notice that this has an incredibly high gain here.

The gain is the slope of this line, almost a vertical line. What I am going to do next, is to have some fun, is I am going to heat the op amp. To show you that A is kind of really skittish and also the fact that

it doesn't quite hit zero, it does all kinds of weird things, I am going to heat the op amp.

And then let's take a look at how that curve fluctuates. What you saw there was that the op amp began to behave really weirdly as I heated it. Instead of doing this it sometimes did this really weirdly, like getting an offset from the center and so on.

And it does a bunch of other weird things, but we won't go into those details. It's not relevant for this course. But the point is that the gain and the offset at the input are dependent on temperature.

And we look for ways to make it less dependent on temperature. As the next step, what I would like to do is build a circuit. This is model equivalent of your Hello World program. We are going to use the printf and build a small program on the printf.

You don't have to worry about how printf is implemented, just that we can build very highly interesting circuits with this horrendously complicated function based on a simple abstraction of the device.

The circuit that we will build is called a noninverting amplifier. From now on, I am not going to show you the plus/minus VS. I am not going to show the power port, but it is in there. It's hidden under the abstraction layer.

This is my op amp. And I am going to build the following circuit. This is my v plus and this is my v minus. What I am going to do is for the v plus I shall apply a  $v_{IN}$ . Let me talk a little bit about ground as well.

Ground is commonly taken as the point at which I connect my VS and minus VS supply. It is kind of at the midpoint. And if VS and minus VS are very carefully tuned then the output is also going to be at that same ground reference when the input is zero.

So, the ground is defined as the point at which I connect my plus/minus VS supplies. I apply my  $v_{IN}$  out here. Then what I am going to do, here is my output  $v_O$ . I am going to have a resistive divider to ground here and label these R1 and R2.

And what I am going to do here is feed this back to the input, to the v minus input. I am going to sample the voltage here and feed that into here. So, this is my abstract model and this is my Hello World program.

What we are going to do is simply analyze how this little program behaves. So, my equivalent circuit model. The way to analyze these is after one or two of these examples, you will be able to directly analyze this just by looking at it, by inspection.

But, much as we did for the other pieces, let me grunge through drawing the equivalent circuit and grinding through the analysis, and then show you the much simpler way of doing it. And even here, even with this grinding analysis, it is going to be pretty simple in any case.

So, I will replace the op amp with its equivalent circuit model. Its equivalent circuit was  $v_+$  plus,  $v_-$  minus. So, that was the equivalent circuit model of the operational amplifier, just this piece.

I draw that for you. Then what I am going to do is I connect my  $v_+$  in here. And, remember, I have an  $R_1$ ,  $R_2$  resistive divider here. And this one gets connected to this terminal there. I also know that  $i_+$  is zero.

I also know that  $i_-$  is zero. All I've done is simply replaced the amplifier with its equivalent circuit. Let's go ahead and analyze that circuit now. Let's go ahead and analyze that circuit. And it's going to be pretty simple, actually.

What I am going to show you is the hard way of doing it. I will show you a much easier way, but the hard way itself is pathetically easy. What I want to do is find  $v_O$  in terms of  $v_{IN}$ . And there will be a bunch of other factors thrown in, including things like  $R_1$  and  $R_2$ ,  $A$  and stuff like that.

Let's go and analyze it.  $v_O$ , let's look at that circuit. By the way, let me take 30 seconds and make a little speech at this point. When you see circuits like this, and I saw this happen in quiz two as well, for some reason, when you see a new kind of circuit, don't completely go berserk or freeze or whatever.

There is just no reason to. You know the node method. The node method is the workhorse of our business. When in doubt apply the node method. It will simply work. Don't freeze. Don't think oh, man, I need to apply a pattern that I know already.

I must have seen this somewhere. When in doubt boom, apply the node method. This circuit here, all I have here is one unknown node

voltage. I know the voltage of  $v$  plus, I need to compute the voltage  $v_O$ .

There are two unknowns,  $v_O$  is an unknown and the voltage here at  $v$  minus is another unknown. This is a very simple circuit involving a dependent voltage controlled voltage source, and you need to find out  $v_O$  and  $v$  minus using the node method.

Just apply it. It's simple. Don't freeze. Just look at it and say I can do it and apply the node method. It will simply work. So, let's do that. What I can do here is  $v_O$  is  $A$  times  $v$  plus minus  $v$  minus.

This is actually really simple. And then, if I take  $v$  plus here, I know  $v$  plus is simply  $v_{IN}$  so I will just make that substitution right away. So,  $v$  plus is simply  $v_{IN}$ . What is  $v$  minus?  $v$  minus here is  $v_O$  -- What is  $v$  plus? I'm sorry,  $v$  minus.

$v$  minus is simply the voltage that is between  $R_1$  and  $R_2$ . Notice that no current flows in to the  $v$  minus node. There is no current flowing in. Voltage at  $v$  minus is simply the voltage given by the resistive divider, which is  $v_O$  times  $R_2$  divided by  $R_1$  plus  $R_2$ .

Stare at that for another second. The voltage at this node here is simply given by the resistive divider. Because no current is flowing in this direction. And no current flows in because I am telling you there is no current there based on my abstraction.

I am telling you  $i$  minus is zero. That voltage is simply the voltage at this resistive divider. And so I can simplify it further and write this as  $v_O$ . So I get, there is a one here. And I move this thing over to this side so I get one plus  $A$  times  $R_2$  divided by  $R_1$  plus  $R_2$ .

And that is equal to  $A v_{IN}$ . And simplifying it some more, I get  $v_O$  is  $A v_{IN}$  divided by one plus  $A R_2$  divided by  $R_1$  plus  $R_2$ . Notice how simple this is, and this is the hard method. All I have done is analyze the circuit using the basic circuit analysis principle that you learned the first week of the course, and I have the output for you.

I just noted very carefully what the relationships were between the various elements in the abstraction. Notice here that I am told that  $A$  is extremely large.  $A$  is on the order of ten to the six and so on.

And suppose it is the case that, let me write that down again.  $v_O$  is  $A v_{IN}$ , one plus  $A R_2$ ,  $R_2$ . Suppose  $R_1$  and  $R_2$  are more or less

comparable and  $A$  is ten to the six, it's a huge number, so this whole number is much, much greater than one.

If it is much huger than one, what I can do is I can then write this as follows. I can say that this is more or less equal to  $A v_{IN}$  divided by  $A R_2$  divided by  $R_1$  plus  $R_2$ . I am ignoring the one here. As soon as I do that, notice I can cancel out  $A$  and I get  $v_O$  to be approximately equal to  $v_{IN}$  times  $R_1$  plus  $R_2$  divided by  $R_2$ .

Notice now that when the gain is very large the output is a function of the input multiplied by some number. The beauty of this thing here is that when  $A$  is very large, or this expression is very large,  $A$  cancels out and there is no  $A$  in this relationship.

This means that even though the basic amplifier was very skittish, the output here relates to the input based on components that I have control over. These are soldiers in my army. I control them. So, to give you a sense of some numbers here, suppose  $A$  was ten to the six.

And I choose  $R_1$  to be  $9R$ . And  $R$  to be some  $R$ . Then  $v_O$  is ten to the sixth  $v_{IN}$  divided by one plus ten to the six  $R$  divided by  $9R$  plus  $R$ . So, that is ten to the six  $v_{IN}$  divided by one plus ten to the six divided by ten.

All right. If I ignore the one here, the ten to the six and ten to the six cancel out, this ends up giving me  $10v_{IN}$ . So, I get a really nice amplifier whose output is simply ten times the input and determined solely by some resistor values.

Let me show you another quick demo this time and show you the amplifier again, but with resistors connected like that. And then I show you that I want to heat the amplifier to the wazoo, the op amp to the wazoo, but  $v_O$  is going to be absolutely rock solid.

Let's try that out. This time around, this is the transfer function, the  $v_O$  versus  $v_{IN}$ . And notice that this time around I have similar scales on the X and Y axes, and this has a slope of 10. This is the point where the amplifier saturates at plus 12 volts, and this is minus 12 volts, and this point here is a zero.

So, this is  $v_{IN}$ ,  $v_{OUT}$ , plus 12, minus 12 and this slope is 10. What I am going to do now is heat the op amp to the wazoo and this ain't going to change because it's my external resistors that control it independent of the value of  $A$ , provided  $A$  continues to be very large.

I am just articulating the  $v_{OUT}$ ,  $v_{IN}$  curve. And let me start heating the op amp. Notice that it's pretty stable. It doesn't change because it is independent of the amplifier values. What I have done now is by connecting these resistors in this way, I have a nice amplifier with a gain of ten.

The question you may ask yourselves is why? There is this little sucker in there that wants to shoot things up by ten to the sixth. Wants to knock things off the one rail or the negative rail. Why is it that it's behaving like a docile lamb here and giving us a nice little factor of ten gain no matter what I do to it? Why is it doing that? What is the intuition behind it? I will draw something on the board, but for the next ten seconds I want you think about it.

See if you can come up with some insight as to why is it doing that. Why is it exactly ten? Why isn't the ten to the sixth kind of killing me somehow? Why am I getting exactly ten no matter what happens? See if you can come up with some intuition and then I will show you how it works.

I will redraw the circuit in the meantime. Let me see if I can give you some intuition. This is my circuit, and let's say this is  $R$  and this is  $R$ . As an example, let's assume that the input is 5 volts,  $v_{IN}$  is 5 volts.

If  $R$  and  $R$  are equal, what should the output be? It's  $R$  and  $R$ , so it's  $R_1$  plus  $R_2$  divided by  $R_2$ , right? It's  $2R$  divided by  $R$ , so it has a gain of two. My amplifier has a gain of two because  $R_1$  plus  $R_2$  divided by  $R_2$ , which is my gain, is  $R$  plus  $R$  divided by  $R$  equals two.

So, this will be 10 volts. If that is 10 volts this is going to be 5 volts, correct? This  $R$  and  $R$ , voltage divider, this is five, so I get 5 volts here. This is  $v$  plus. This is  $v$  minus. I get  $R$  and  $R$ , 5 volts here, that's how the circuit looks.

Now let's understand what is going on. And listen very carefully. This is going to be a key insight that I hope you will carry with you for the rest of your lives. This is really, really key. What you are going to see is, I think, the third big ah-ha moment in 6.002.

Like small signal analysis, like the frequency domain stuff we saw, I think this is the third big one in the next 30 or 40 seconds, things that are completely either not necessarily intuitive but are just spectacular in terms of what they can do for you.

Let's see. Let's suppose that because I am heating it, let's suppose that  $A$  suddenly tends to increase. It wants to increase because I have heated it.  $A$  is saying I want to get out this mold here and starts to break through its shackles here.

Let's say, as a Gedanken experiment, that it tries to shoot up this to 12 volts. It tries to push it up higher. This is just a Gedanken experiment. The up arrow says that the increase in  $A$  is trying to push up  $v_O$  momentarily.

Let's see what happens. It is trying to push up  $v_O$  momentarily, so let's say this goes to 12 hypothetically. If that goes to 12, what should this volt node go to? Six, exactly. This goes to 6 volts.

If that goes to six, what does  $v_{\text{minus}}$  go to? 6 volts again. So,  $v_{\text{minus}}$  goes to 6 volts. Now at the input I have 5 volts at  $v_{\text{plus}}$  and 6 volts at  $v_{\text{minus}}$ , so where should the output go? The output should go down because the voltage of the negative terminal is higher.

And so the output is  $A$  times  $v_{\text{plus}}$  minus  $v_{\text{minus}}$ . And because this has gone down, this has gone up here it is going to try to pull the output down. That is going to pull the output down let's say to 9 volts or something.

Cachunk, there is a big battle going on here.  $A$  has gone up, it has boosted it up to 12, but the moment that goes to 12, this goes to 6, this goes to 6, and the op amp output has to go down to 9 volts now because this input is higher here.

If this goes to 9, this goes to 4.5. If that goes to 4.5, this goes to 4.5. What happens now? If this goes to 4.5, what happens? It wants to go back up. Can't it make up its mind? This guy wants to go back up now because  $v_{\text{plus}}$  is higher than  $v_{\text{minus}}$ .

What am I seeing here? This whole circuit here behaves like my little son, my 9-year-old. If say do this, he wants to do the exact opposite. So, there is a trick in how you make them do things for you.

Look at this. Because of this arrangement of the circuit when  $A$  tries to push the output up, the rest of the circuit tries to pull it back down to where it used to be. If the circuit tries not to follow the true path, the rest of the circuit tries to whack it into shape so it follows a true path.



And what's happening is because, in this arrangement, I have fed back a portion of the output to the negative input. I have fed back some of the output to the negative input. And by providing this feedback of a portion of the output to the negative input, I have arranged it in a way that I have something called negative feedback.

What negative feedback does is that if this wanted to go wild and crazy, the circuit provides it with some negative feedback like you just saw. Feedback, a big word. If you take a poll of all the EECS faculty, I suspect that feedback would rank at least as the ninth or tenth most important word in the EECS.

If abstract is number one, I think this would rank like a nine or a ten or something. So, that's the reason why it worked. In the last couple of minutes, let me give you some insight, based on something that you know, on how feedback works.

This is a road here. Let's look at anti lock breaks. This is my tire. And let's say I have a set of disk brakes here. As the car is moving forward, if I apply the brakes the tire stops rolling, but if I apply the brakes too hard it can lock up the tire and the whole car can skid.

The way anti lock breaks work is as follows. There is a controller that sits here. And there is a little person looking at the wheel and seeing is it turning. So, this is a feedback. And it is saying is it turning? Yes.

Or, is it not turning? No. All this person watching the tire is doing is saying is it turning or is it not turning. That is it. That is a negative feedback. And so, if it is no and if it is yes.

If it is yes then what this does is it applies the brakes even more strongly. It is turning so I can apply more brakes. But if it says oops, it stopped turning, what it does is it simply releases, the controller releases the brakes.

And when the controller releases the brakes this one tends to loosen up a little bit and the tire starts turning again. So, this way you are constantly keeping the tire in its region of critical friction so that it is constantly moving.

And static friction applies to how hard you can brake and it doesn't start skidding. In fact, if you take your car out, and I don't say you do this. Let's say go onto the Charles River in the dead of winter and you drive on the lake and you slam your anti lock brakes on, on an icy

patch, you will notice that there is a constant sound that looks like something is vibrating in there.

That is exactly what is happening. Oops, the tire is locked. Release the brakes. The wheel is turning. Jam the brakes on. That is exactly what is happening. The same way as out there, you notice that oops, the output is going up, pull it down, oops, it's going down, pull it up.

So, there is constant negative feedback that is keeping the output stable. A very important concept. And I will ask your recitation instructors to cover the very simple method that is on page 9.