All right, good morning. So today, we are going to talk about what is both a basic device in itself, the amplifier, and it also serves as a real key example of both nonlinear analysis and small signal analysis. So, today, dependent sources and amplifiers.

So, let me first spend a few seconds just pointing out to you some of the key points from our previous lectures.

I also want to point out that each chapter in the course notes has a summary at the end of it. And if you take a quick scan of the summary at the end of each chapter, it highlights the major takeaway points from each chapter.

It stresses what's important, and if you have to remember a few things, what are those things to remember?

So, to quickly review, we talked about a few primitive elements: resistors, voltage sources, and so on. And by now, you should have the facility to play around with these device elements.

And then we talked about the Node method, and this is kind of the workhorse of 6.002. When in doubt, use the Node method. OK, and this will work both for linear circuits and nonlinear circuits.

OK, so if you see a problem, or if you see a situation in real life that requires analysis, then as a first step, you should try to think of whether you could apply some of the key intuitive shortcut methods, superposition.

One of my favorites, the Thevenin method, the Norton method, or the method that involves composition, that is very quickly analyzing circuits that have resistors in series and parallel.

OK, so if you can apply one of these quick, intuitive, shortcut methods, go do so.

If you can't, then usually you can resort to the Node method irrespective of whether the circuit is linear or nonlinear. So the last week was focused on the nonlinear method or nonlinear circuits, and we spent the first lecture talking about a straightforward application of the Node method, which gave us a bunch of nonlinear equations that we had to solve.

In the last lecture, we talked about the small signal trick. What we said is if you look at the whole space of nonlinear circuits, then within that space, if we focus on small variations, small perturbations about an operating point, then even the behavior of nonlinear circuits in that small regime would be linear.

So small signal method.

And as an example, I showed you how I could take a highly nonlinear device like the garage door opener LED, and using that, build a pretty nice transmitter that would transmit music. And as long as we kept the signal small, and operated the device in a region where its transfer curve was relatively smooth, and I biased, or set the operating point appropriately, I would get a linear, small signal response. OK.

So today, we're going to do a couple things.

We're going to look at dependent sources.

And the reading for this is section 2.6 of your course notes. And, the dependent source will be a new element in your tool chest.

We will also do amplifiers, and amplifiers are in section 7.1 and section 7.2 of your course notes.

So, before I begin with dependent sources, I'm just a huge believer in motivating things with real world examples. OK, so let me start by motivating: why we need an amplifier?

Why do we need to do things like this?

Or why do we even bother? And, spend a few minutes really getting you to appreciate that amplification is fundamental.

OK, it's as foundational to life as high fat potato chips and stuff like that. So, let's do some basic examples here. So first, let me talk about, why do we need to amplify signals.

Why amplify? Why do we care about building an amplifier? So, an amplifier, think of a little box, and apply some sort of small input. And I get a larger output.

In this example, this may be a voltage with a swing of 10 mV, and in this case, the output might be another voltage with a swing of, say, 100 mV. And commonly, the amplifier, in addition to an input and an output, input port and output port, may also contain the power port, OK, so that I can apply a power supply to the amplifier because commonly as an amplifier signal, I'm looking for a power gain as well, an increase in the power provided by the output. So, that's an abstract definition of an amplifier, and let's take a look at an example of why we may need this. So let's say I have a small, useful signal, and let's say the signal has 1 mV peak to peak. And, I'm looking to transmit the signal over a wire to some other point.

But let's say that in this environment, I get a bunch of noise that is in a noisy environment.

And in this environment, let's assume that some noise may get superimposed. And if I have a 1 mV signal, and 10 mV of noise, then what I end up with at the output is something that looks like this.

And it's really hard to distinguish my 1 mV signal from that large amount of noise. On the other hand, if I do the following, if I took the signal and passed the signal to an amplifier, and I amplified the signal to be a much larger version of the same signal, let's say in this particular situation 100 mV peak to peak signal.

OK, so I magnified the signal by a factor of 100.

OK, let's say it's a linear amplifier, I linearly amplified signal to be 100 mV, then in that case, if I had a noise on top of this, it's going to be less discernible. The signal will look like this.

OK, my 10 mV noise would add on to it.

But, this is still pretty decent.

I can still recognize the input.

And so, this is one application of amplification.

If I need to send something from point A to point B as an analog signal, then an amplified signal is less prone to noise attacks than a small signal.

Not surprisingly, a large number of devices that are used in everyday life have amplifiers built into them.

So, get a little cell phone, and virtually every single cell phone contains an amplifier. By the way, this is an all digital cell phone. It's a Kyocera, I forget the number now. It's completely digital.

OK, although they say it's completely digital, it turns out that a significant fraction of the circuitry is analog, in particular, so digital is sort of a marketing term to say that there's something special about this. But remember, there's a bunch of analog stuff.

So, here's my little antenna from the cell phone.

OK, and typically the first thing that happens to a signal as it comes out of the antenna in your cell phone is, look at cell phone circuits, or cell phone systems would be something that looks like this, OK, this, and may have a label LNA. If someone were to take a guess at what LNA might stand for? What's that?

Linear amplifier. That's pretty good.

So that's LNA. Close enough.

A is correct. It's amplifier.

What does L and N stand for? Low noise.

OK, so this stands for low noise amplifier.

So, I get a really rinky dinky small signal here, and then the low noise amplifier amplifies a signal.

And in real cell phones, and for that matter, in your 802.11b, or 802.11a, or 802.11g wireless cards, same thing. Antenna, low noise amplifier, and then you may have a bunch of processing.

And commonly, you have a bunch of analog processing. And then, you convert the analog to a digital signal. OK, I recall last week I asked somebody in class here, how would we transmit the signal from point A to point B without it being impacted way too much by noise, and he said, oh, go digital. Good point.

OK, so if I go digital, I can transfer the signal without noise being a real factor.

But the analog to digital converters need the signal strengths to be a given value before it can chop it up into digital levels. OK, so an amplifier is very fundamental. OK, and so in this case, what may be a signal of a few tens of microvolts to be amplified to some large enough value that it can be further processed. So, that's application of amplification in the analog domain.

Let me talk about amplification in the digital domain.

So, that's in the analog domain.

This amplification is in the domain that I have both analog and digital. OK, and now let me talk about amplification in the digital domain, OK?

I'm going to argue that amplification is absolutely foundational to the digital domain.

OK, the digital abstraction would not occur if I did not have basic amplification. OK, and the next minute and 37 seconds I will prove that to you, OK?

So, let's do so. So, let's suppose I have a very simple digital system, and the system simply contains a pair of inverters. So, if I send a one here, it's a zero here and a one here, which is a very simple, trivial, digital system. And here's the input.

Here's the output. And we said that for digital systems of this sort to work, they have to follow a static discipline. OK, our signals and our circuits must follow a discipline for them all to work together. And, the discipline we described comprised of signals adhering to certain voltage thresholds so that all the components in the system could agree on what comprised a zero, and what comprised a one, OK? So the way we did that was we said that you would have a threshold called VIH, V input high, and another threshold called VIL, V input low. OK, and we said that this circuit must recognize signals that are higher than VIH, 3 V for example as a one, and simultaneously, any signal that has a voltage level less than VIL, say, two volts, should be recognized as a zero.

That was the input constraint. On the output, it had a similar set of constraints, where we had tougher constraints on devices, where we said that the output had to satisfy a output low constraint, output high constraint. What this said is that for this circuit to be called a good digital circuit that satisfies the static discipline, signals that were ones here should be recognized as such. And if I am producing a one as an output, then the signal level should be higher than VOH.

Similarly, if the signal's a zero, then it should be less than VOL. So as an example, this may be 2 V, this may be 3 V, and this may be 4 V, and this may be 1 V.

OK, so input, I should recognize 2 V and less as a zero, but at the output I have to produce a very, very low value, 1 V.

So, I have some noise margin. So as an example, say if I made a plot of the input/output, so I get my VIL here and VIH here.

This is time. This would comprise a valid digital signal: zero, one, zero, one, and so on. OK, now, I had a tougher set of constraints at the output. I would have VOL, VOH. So, at the output, OK, I'm required to stretch the ones and zeros to be further apart from each other so that I get noise margin, and the corresponding signal for our little circuit there would look like so. Right, if this is a valid input, then this would be the corresponding, valid output. OK, and need I say more?

OK, you can see that, intuitively, look, there's amplification happening here, and the reason is that VOL is chosen to be less than VIL, and VOH is higher than VIH. So therefore, the signal has to be stretched. The signal has to be amplified.

OK, and what's the minimum amplification needed for the system to work? The minimum amplification is if I had a signal that looked like this.

OK, that barely skimmed the VIL, VIH level.

OK, so if signal were this high peak to peak, VIH minus VIL, and what's the absolute minimum signal at the output? It would look something like this. OK, barely skimming VOL and VOH, OK, so the corresponding output level would be VOH minus VOL. OK, so this is the absolute minimum amplification that my digital circuit has to provide.

OK, and notice, VOH is larger than VIH.

VOL is smaller than VIL. Therefore, this quantity needs to be greater than one. OK, so I've shown you both a simple, graphical, intuitive explanation, and this is a slightly more formal proof that even the digital circuit really requires to have amplification built into it, if it is to satisfy valid static disciplines.

Yes? Yes.

The question is, is that the same as gain?

Good question. Yes, the term amplification has many, many variants. You could say gain.

You could say amplification. You could say increase in signal strength, and so on and so forth.

And in fact, when talking about low noise amplifiers, people sometimes talk about having the low noise, high gain amplifier at the input stage.

OK, so let me pause there in terms of motivation.

So, I believe I've motivated every which way: pure analog, analog/digital, and digital. OK, so I've covered every single base here. And so, we need amplification.

OK, so let's look at how to build a fundamental, primitive device called the amplifier.

Before we do that, however, let me take a quick detour. It will be convenient for me, as I show you how to build an amplifier, to introduce a new device, a new element, called the dependent source.

OK, let me introduce a new device for your arsenal of devices, along with resistors, You learned about a MOSFET, a switch, voltage source, current source, and now a dependent source. So, a dependent source looks like this, OK, has an output port, and has a control port. So, a dependent source in its simplest form has two ports: an input port and an output port. Remember, a port is a convenient pairing of terminals, and I apply signals to such terminal pairs. But this is a abstract diagram for a dependent source, and to get a little bit more specific, let me show you an example of a dependent source.

So, let's say, here's my input, and I label the terminal variables for the input.

VC is the voltage applied to the input, and IC is the current into this terminal here. And, here is the symbol for the dependent source. Much like a current source or a voltage source has a circle around it, the corresponding symbol for a dependent source is like so.

So this example, for instance, is a dependent, current source.

I can apply the corresponding output variables, I0, OK, and I can say that the current, I, is some function.

In this example, I've designed the example that the current through the current source, I, is some function of the input voltage or the control voltage, VC.

OK, so notice that the current through a current source, the current through this current source, I, is some function of another variable.

OK, in this example, it's the voltage across its control port. Not surprisingly, this device is called a voltage controlled current source -- -- or a VCCS. So, in like manner I can also devise other forms of sources. You can think of this is a device where a voltage controls an output current.

You can think of all other combinations, current controlling current, voltage controlling voltage, current controlling voltage, and so on.

So, another example, I give you another dependent source, and in this situation, my output current is controlled by an input current, VC.

IC rather. And I claim that I for this one is some function of a current, IC.

OK, it's another dependent source where the output current for its output port is related to the current, IC. And, this is a current controlled current source. OK, it's a current controlled current source. And, if I had lots of time on my hands, and I was wanting to kill time, I'd sit around drawing for you, other types of dependent sources. I would draw for you a current controlled voltage sourced, and I could also draw for you a voltage controlled voltage source.

OK, so that's an abstract diagram for such a source.

And so, let's do a few examples involving elements like this.

To begin, just so you can build up your intuition, let me start by doing a very simple circuit, involving an independent current source, OK, just so we can relate back to what we've been doing so far.

So, let's say I have some resistor, and I have a standard current source with current I nought.

This is an independent current source.

Remember the circle? And, some resistor, R, and let's say I care about the voltage across the resistor.

OK, so I have a current I nought flowing through it.

So, I can very quickly write down VR as, simply, I0 R. OK, it's the drop across the resistor when a current I nought flows through it.

OK, so this is what you've been used to doing.

Correspondingly, I can do an example with a dependent current source. And, as an example, I'll use a voltage controlled current source.

OK, a voltage controlled current source is a dependent current source whose output current depends on the voltage applied at the control port of the current source.

So let me build a little circuit.

OK, so here's my current. And let's say it's VC IC for the control port, and similarly, let's say my current I here is some function of the control port voltage. And let's say, to be specific, there is some K over VC, some function. OK, there are a variety of dependent sources that can be built, and here's a hypothetical device where the output current is mathematically related to the input in the following manner. So, let me build a circuit of the following form. So, let's add the resistor, R, and here's my circuit, OK?

And, as before, let me look to figuring out what VR is. So, notice that I have to supply some voltage at the input so that the output can depend on the input because right now I don't know what the input here.

So what I'll do is let me apply VR over here.

OK, so let me make this connection.

OK, let me make the connection from here to here.

What I've done is I've applied VR at the control port of the dependent current source. OK, and I often draw a circuit like this. This looks pretty messy.

I will often draw the circuit like so: R, VR.

OK, short form circuit drawing would look like this.

This is a complete drawing that I show you the explicit connections of the control port, but oftentimes, when the control port does not have any other impact in the circuit, you can eliminate, don't explicitly show the control port. Rather, you can simply show the dependence of the output current on whatever circuit variable you have in mind. So, you can draw the diamond like this, and see its current is some function of VR.

VR in this is case is K divided by VR, OK?

OK, so let's go ahead and analyze this little circuit here, and look at what this might give us.

Our goal, as before, is to find out the value, VR. So, in this case, let's apply the Node method to this node, and sum the currents into that node to be zero. OK, so sum the currents going into that node to be zero. The current going down is simply VR divided by R. OK, and that is equal to the current that is going out of the node.

And so that is equal to F of VR.

And I know that F of VR is given by K divided by VR.

OK, a simple application of the Node method.

So then, I collect VR's on the left hand side, and I get VR squared is K times R, OK, and VR is simply the square root of KR. There you go: I'm done. OK, I've gone ahead an applied the Node method to this, and when have to figure out the current here, I simply reflect the fact that it depends on VR like so, and I just go ahead and solve the circuit. Remember, the workhorse of the circuit industry, the Node method, when in doubt, apply it.

It simply works. And notice, this is a nonlinear circuit. OK, the dependence is nonlinear, and I get the response like so.

So, to plug in some numbers, supposing K was 10 to the minus 3 amperes per volt, and R was one kilo ohm, then I can plug the numbers in and the kilo here cancels with the 10 to the minus 3, and I get VR equals 1 V.

OK, this simply says, if I build a circuit like this, then this voltage here will be 1 V.

So, again, as long as you remember that the dependent source is simply another little circuit element, OK, and you usually draw just the output port for dependent sources, and reflect the way that the control affects the current, that'll suffice, and you get, through the application of the Node method, the variable you're interested in. Let's do another example, OK, of another fun current source, a voltage controlled current source, and look at it this way.

So, let's say I have a resistor, and I have a current source, a resistor, RL, and this goes to some, I apply a VS here. Remember this short form notation; that's simply applying a supply VS between that node and the ground. OK, and let us say the current IV through the device is some function of the current at its control port. OK, so I'm not going to show you that. But remember that the device already looks like this, that there is a control port here. I'm not showing that to you.

And let us say that I apply some voltage, VI, to the input port. The reason we often don't show the input port is for many practical dependent sources, the input has no other effect on the circuit.

So, for example, in this case, the input has infinite resistance looking in.

So therefore, if I apply a VI here, it doesn't draw any current from VI.

I simply apply the voltage, VI.

It doesn't affect the circuit in any other way except in terms of how it controls the current ID.

So let's say the current ID is some function of VI because VI is applied at the control port. OK, and as I pointed out before, I oftentimes, just for clarity, just to show this dependent source explicitly.

OK, so let's work the example. So as I said, I'm going to choose ID to be F of VI, and let's pick some specific parameters here. Let's say it's K by two VI minus one, both squared. OK, and let's say this is true for VI less than equal to one volt.

And let us also say that ID equals zero for VI less than one volt. OK, it's a dependent source, and it can have various forms of dependences on the input.

And, I just picked an example of some hypothetical, or as yet, hypothetical dependent source, the current through which is related to the input using a square law relation, VI minus one all squared as long as VI is greater than one. And if VI is less than one, then the current is simply zero, it shuts off.

So, I can go ahead and apply. So, let's say I want to find out V0 versus VI. So, I care about finding out V0. V0 is the voltage of this node with respect to ground. OK, so it's a slightly more complicated circuit than you saw up here, than you saw up there.

So, let's go ahead and do this example.

Start by applying the workhorse of the circuits business, the Node method, and let's start with doing this for VI. Let's first do it for VI greater than one, notice the behavior of this is different for different ranges of VI.

So let's first do it for VI greater than or equal to one and apply the Node method. Node method says sum the currents going into this node; we know the voltage at this node. It's VI.

We know the voltage at this node.

It's VS. OK, the only unknown is V nought. And so, let's go ahead and write the node equations for that node.

So, the current going up, let me simply equate the current going up to the current that has been supplied by this particular node here. And, that should equate that the two of them should sum to zero, the current going up plus the current going down should sum to zero.

So, I get V0 minus VS divided by R.

That's the current going up. Plus, the current going down must sum to zero, plus ID must sum to zero.

And ID is going to be K divided by two VI minus one all squared.

That must equal zero. Straightforward application of Node method, current going up plus the current going down at this node should equal zero because the total current leaving the node must be zero, OK?

So I can go ahead and simplify this, multiply it throughout by, I call this RL here. So, multiply it throughout by RL, and move all of this to the other side, so I get VS divided by RL, multiply it throughout by RL.

I get VS at this side. I take this term to the other side. This becomes a minus.

RL multiplies here, so I get KRL.

That's the expression I get. V nought is VS minus KRL divided by two times VI minus one all squared.

Let me put a box around this because I will be referring to this more times in 6.002 for a variety of reasons than probably any other equation on Earth. OK, this is the first time you saw it. You saw it here.

OK, mark it down. You'll smile every other time you look at it in quizzes, and you will find out why this comes up very often in 6.002. So, I'll just give you a few seconds to savor this big moment in your 6.002 life.

All right, OK, so it's pretty simple actually.

I mean, there's really not much.

A lot of this stuff is just a plain old, simple application of the Node method, and things just fall out.

It's just so simple. So, the V nought, I apply the Node method, I get V nought for this nonlinear circuit. I can also it for VI less than one. For VI less than one, when VI is less than one, what happens?

ID is zero. OK, since ID is zero, think of this as an open circuit.

OK, so there's no voltage drop across RL.

And, this voltage V nought is equal to VS.

So, I like to see things in pictures.

I'm not an equations kind of person.

I'm much more of a graphical person.

So, let me draw a little graph to show how V nought, to see the form of V nought, and then let's study that little system a little bit more carefully.

So, this is page seven, and we plot V nought versus VI for you. And let's take a look at how this really simple circuit looks.

This has got nothing. It's got an RL resistor connected to a supply, and a dependent current source, and I apply some voltage VI at the input.

It's a very, very simple circuit.

So, let's see. So as long as VI is less than one, the output stays at VS. OK, that makes intuitive sense, right? As long as the current here is zero, this is like an open circuit here.

If this is an open circuit, then effectively, V nought is simply the voltage VS.

V nought simply appears here. If you want to grunge through KVL and KCL, go ahead. VS minus RL times the current is V nought, and the current is zero so it's, yes. So, this is simply VS.

When VI goes above one volt, fun stuff begins to happen.

OK, when V nought goes above one volt, then this equation applies because VI is greater than one.

This equation applies. And, when VI is a one, one minus one is zero. This term cancels out, so this is VS. OK, phew!

So, I start off here. As VI increases, what happens now? As VI increases, this term here becomes increasingly negative, OK, subtracting from VS. OK, so I get some behavior like this. V nought begins to drop.

And it makes intuitive sense, right?

As ID begins to increase, the voltage here will begin to drop because I'm drawing more and more current through RL.

I'm dropping more and more across RL.

So more and more drops across RL, so V nought begins to drop too. So, it looks something like this. I'll show you a little demo, but my claim is that you have just seen an amplifier.

Whoa. You just saw an amplifier.

So, I snuck an amplifier by you, OK?

So, I just snuck an amplifier past you.

I'll show you why in a second. So, let's take a look at this waveform here. Let's not worry about what happens way down here. We'll talk about that a little later. But, look at this curve here.

I claim there is amplification in the following sense.

Focus on some change in the input voltage, delta VI, OK, and for that change in input voltage, I get some change in the output voltage.

OK, for some change in the input voltage, delta VI, I get some change in the output voltage.

And guess what? In this, at least the way I have drawn it, delta V nought divided by delta VI, if I can find regions of the curve where this is greater than one, then I have amplification. OK, so what's that saying?

What that's saying is that if I apply some voltage here, OK, and I change that voltage by a small amount from, let's say, 2 V to 2.1. OK, I am going to find the output voltage. Let's say I go from 2 V to 2.1 here. OK, abstractly out there, I might have an output that goes from three to, let's say, two V perhaps. OK, so for a 0.1 change here, I'm going to get a bigger drop here, so from 3 V to 2 V, giving me an amplification in this little circuit.

OK, so we'll see this again and again, and you'll really understand it. So, I have a small change in the input, and I have a corresponding larger change in the output. So, I've shown you an amplifier. I haven't shown you a linear amplifier. There's an extra charge for that. OK, that'll happen later.

OK, all I've shown you so far is an amplifier, and this happens to be a crummy amplifier.

It's a nonlinear amplifier because, notice, this is not linear. It's a nice little curve, and so it's not linear. But, I promised you an amplifier, and I'm cheap, and that's all you get for now.

OK, we'll see linear stuff later, but for now, I have a little amplifier. So, let's do some real numbers, and plot some numbers down, and also look at a demo.

So, let's do an example. Let's say VS is 10 V, that the K is two milliamps per V squared, and let's say RL is five kilo-ohms, OK?

So, let me substitute these values into that equation, and I get V nought is, VS is ten.

So, it's ten minus, KRL divided by two.

So, K is two milliamps. Two milliamps times five kilo-ohms is ten divided by two gives me five, and VI minus one squared. That's what I have.

I just plug in a bunch of numbers, and that's what I get.

So, what I'll do is let me just do a little table for you, and plot using real numbers, simply plot those values for you.