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6.002 Circuits and Electronics, Spring 2007
Transcript – Lecture 7

OK, good morning all. So before we begin, I just thought I'd show you a little news item that I happened to read that was very relevant to what we covered recently in 6.002. So you recall when we did the digital section a few days ago last Thursday, we talked about a switch.

We talked about the MOSFET switch, which when turned on and off, by input signals could help build gates which would then be combined in tens of millions of quantities and go into chips like the Pentium 4 and AMD Athlon 64, and so on it so forth.

So I just saw this news item that I came across, and this says they are rethinking the basic construction of the products. It talks about the semiconductor manufacturers like AMD, Intel, and others that build digital chips.

They are rethinking the basic construction of the products down to the architecture of the transistor. That's a MOS transistor, and the on/off switch inside the chip. OK, now this might imply that there is a single switch inside the chip, but no, there's tens of millions of transistors, or tens of millions of switches inside a chip.

And pretty much any advancement that can be made to the basic transistor can have a 10 million to 20 million times effect because there are that many of them on a single chip. So I thought that was very appropriate.

OK. Let's dive into a quick review. So this week, we had begun nonlinear analysis, and I just thought I'd blast through a few animations that I've created, trying to give you more insight into the behavior of some of the things that we have done.

Now first of all, as I did the last time, let me try to put it in perspective most of what you've learned thus far, and what we will be learning today. So the past week, we have been focusing on nonlinear analysis.

And as I pointed out, here is how this fits into the big picture. So, we had our 6.002 world, at what we said is that we are engineers. We are going to devise our own playground in which to play with our own rules.

And that's our playground. That's what we're going to learn about in 002, and for that matter, the rest of EECS at MIT. It's all within this playground here. And this is the playground with lumped circuit abstraction, and good old KVL, KCl, node method, your basic composition rules apply within this playground that directly come from Maxwell's equations because you have made the lumped matter discipline assumptions.

OK, so then we said a large part of the playground is linear, and some other much more intuitive techniques apply within the linear portion of that playground, techniques like the superposition, Thevenin and Norton.

In most exercises, and quizzes, and experiments, and so on that you do in real life, you can pretty much apply these simple techniques. Very rarely do you have to go into the node method for circuits that are more complicated than single source and a couple of elements.

And then, there's the nonlinear part. Remember, the reason I showed this is that this is the same playground. OK, linear and nonlinear are part of the same playground. OK, even nonlinear elements are lumped circuit elements, and they follow KVL, KCl, the node equation, and so on.

And then, last week we spent some time talking about the digital abstraction. So we focused on a smaller region of the playground. And the assumptions we made in there were even tighter. We said that it is part of the playground we shall only deal with binary values.

We'll digitize or lump values into highs and lows, and that's where our circuits are going to be. And these circuits, when looked at as a whole, were nonlinear. So, this is a simple NAND gate circuit.

And this is the input/output characteristic. So, for example, if I hold B at zero, and I apply a zero to one transition at A, then this is the output that I will see at C. So notice, this is decidedly nonlinear.

Then I said that, look, suppose we had to fix the input values at a given set. OK, so let's say, for example, I fix A at one, and B at one. OK, and then look at the circuit in this situation. What do I find? What I find is that the entire digital set of circuits that we were looking at move over into the linear space for a given set of switch settings, OK? So, when I set A 1 and B 1, A equal to one and B equal to one, my NAND gate becomes like this.

OK, it's a simple resistive network with a voltage source, VS. So, for a fixed set of inputs, for a given set of inputs, if I don't change my inputs, then my circuit looks like a linear circuit, and my good old linear analysis techniques apply.

So that was last week. And this week, we are looking at the nonlinear space. And we looked at a couple of techniques in the nonlinear space, analytical techniques and graphical techniques. And then, I showed you an example.

OK, I showed you an example circuit that was something that I would like to build involving the light emitting expo dweeb, my little garage door opener thingamajig, and I wanted to transmit music over that light beam.

I also showed you that it was highly distorted because it was in the nonlinear space. So, today what I'm going to do is introduce a new part of the playground. There's a new part of the playground, and I'll show you a technique whereby by focusing on this part of the playground and disciplining ourselves in the kind of inputs we apply to circuits, I'm going to show you that certain kinds of nonlinear circuits also move over, when used in a particular way, also move into the linear analysis domain.

OK, so let me leave that for now and go back into quickly reviewing the motivating example of music that I had taken last time. OK, so here was a little example. So I have a music source, VI, and I apply that.

This device that I call the, lightheartedly, the Light Emitting Expo Dweeb has a current, VD, across it, or a voltage, VD, across it, and a current ID through it. And the light intensity, I said, was proportional to the current.

And because of that, I was able to get the light to impinge on a receiving device, which produced a current that was proportional to the intensity of light falling on it. And that signal would then be amplified somehow.

We haven't talked about all of this stuff yet. This will happen next week. But let's say we somehow amplify the signal and then played out through a set of speakers. All right, so if I had some sort of a music signal here, then I could then transmit the music signal over to the side on top of this light beam.

But the problem, as I said the last time, was that our device, the Light Emitting Expo Dweeb had an exponential characteristic, so that I had some trouble in getting undistorted music. So, the characteristic of the VI characteristics of my device looked like so.

The ID versus VD curve looked as follows. OK, it was decidedly nonlinear. And because of that, I was getting a lot of distortions in my signal, and I showed you a little trick to plot, given an input waveform at a transfer function such as here to plot the output function.

OK, let me show you another little animation that I have created here for you that should give you even more intuition in terms of how it happens. So, this is a characteristic I showed you up here. It's on both sides, but I guess it points to only one unless I shuttle back and forth really fast.

So on average, I'll be in both places. But anyway, so here's my ID versus VD characteristic. And as I said, there's an exponential ID versus VD curve. And I want to see what the output looks like, for example, a sinusoidal input.

So I said, let's place the input along a little graph, rotate it so, and take a sinusoid, and apply a sinusoid to the input, VI, which would also appear across the Light Emitting Expo Dweeb. And then, what I wanted to see was how the output looked.

OK, so let me tell you that the output is going to look like this. OK, the output is going to look like so. And, a little artifice to discover curves like this is to think about a point here corresponding to the point on the transfer curve here, because this is VD, looking at the Y intercept.

That's a value of ID, and that's a value of ID here. And, time moves along here, and time moves along here. So, I did this little animation. You'd better be impressed. It took me six hours to do it.

So, here it goes. So, let's say I start by focusing on this little point that corresponds to this point on the transfer function, which then, in turn, points to a time, zero, this point on my ID curve.

OK, I hope this works. So, as my point moves down [LAUGHTER], this was fun to do, I promise you. So notice that as this point has the following excursion, this had the following excursions here. OK, all right.

So let me pause that little animation there. At the end of the lecture, I'll put that up again if you like, and you all can come and play with it. So, you can actually do this in PowerPoint. It took me quite a bit of time to figure out how to do it, though, but it's fun.

OK, so let me show you a little demo, and show you a sinusoid, and show you what the output looks like if I apply a sinusoid for VI. So, I'll show you ID as a function of VI when VI is a sinusoid. There you go.

So, I applied my sinusoid VI, and this is the current that I get. And notice, this is the transfer function that I talked about, the ID versus VD curve of my Light Emitting Expo Dweeb. And I get this highly nonlinear transformation of the input as I get to the output.

OK, so that is a problem. And then, I also played some music for you. Let's do that, too. I played some music for you. I applied the music as an input to the circuit, and that's the output. OK, that's the output that I'm observing at the amplifier.

It's highly distorted. OK, we can stop that. There you go. OK, so that was my problem. OK, so we had covered, we had gone this far last Tuesday. I set the problem up for you, motivated what we had to do, and showed you that I was able to transmit music over my garage door opener, but I did not think I could listen to that music for very long.

So, I challenged all of us to think about how a trick that I could use to be able to transmit music and have a linear response. So, did you people get time to think about it? So how many people here think they know the answer? It's OK, don't be modest.

Go ahead. Could you speak louder? Yeah, you find another something, kind of element, that's got the opposite graph so that when you add them together. Oh, this guy wants to cheat. No. He wants a new element.

So, no, no new elements. Pardon? Build an MP3 encoder. Ah-ha, so that will happen much later. Yes? Digitize the signal before you send it to the LED? Digitize the signal before you send it to the LED.

But in some sense, each of these solutions is a huge sledgehammer approach to look at solving it. There's a much simpler technique I can apply here. Yeah? Add a voltage offset. Ah, ah-ha, that might work.

What else? So let's say, here's my signal, right? If I add a voltage offset, that will just bump the signal up here. Then the curve is still nonlinear. But you're getting there. Well, I'll tell you what.

Let's pause here. Let me quit while I'm ahead. OK, so the answer here, folks, is Zen. OK, what I want you to do is, so, in Zen, what you have to do is you have to sit down in a courtyard, and look at a rock, like a small rock on the ground.

And you got a focus on it till the rest of Earth kind of vanishes. Just focus on the rock. OK, now make like you're in a courtyard, and you're looking at this little area here. Just look at this. OK, and I'll give you ten seconds.

Sit down quietly, and no sounds. Just stare at the spot here. OK, make believe this is your little rock, and just stand there and think about it. OK, I'll give you five seconds to do that. Just stare at it.

And very soon, the answer should pop into your heads. OK, what do you see? This guy, if I focus on this really small region of the graph, this small little piece looks more or less linear. OK, hmm, so that should give me some insight.

This whole thing, the macrograph is nonlinear. But I focus on a little rinky dinky piece of that graph like so, that appears more or less linear. If it's small enough, that appears linear. So, I'm staring at this, and that appears linear.

The question is, how do I exploit this little small, little, linear region to get a linear response from my device. OK, so here's the trick that I'm going to use. The little trick that I'm going to use is the following.

Notice that, let me call this voltage at the center of this region capital VD. What I can do, if I take my input signal, and I just pointed out earlier, I bump it up. I boost it. OK, so I apply a DC offset to my input signal, like so.

So I apply some input signal, VI, which is also equal to the VD if I look at a variable across the nonlinear element. If I apply a DC offset, VI, and I superimpose the music on top of that, let me call my music, just to distinguish between the two, capital VI, and the small vi.

OK, that's my music. So here's my capital VD, my DC offset. And I want to superimpose my music on top of that. OK, so I've gotten halfway there. By superimposing my music here instead of having excursions out here, I now have excursions out here.

OK, and so I'm using some portion of the graph here. But that's still way beyond the small little element there. So a second think that I do in addition to boosting up the signal is shrink it. Think of boost and shrink, BS.

So what I want to do is boost up the signal using a DC offset, and shrink the sucker. OK, so I'm going to go with a small signal and bump it up. OK, so now what happens is that small signal in its excursions, only uses that little portion of the graph.

OK, again, remember: bump and shrink, bump and shrink, two things, boost and shrink. So what do you think of that trick? So, by doing that, what happens is that signal that has excursions here will produce a corresponding response in this region, OK? And I argue that since this is more or less like a straight line, I invoke Zen here, and argue that this little signal now gets transformed, and I get a linear response.

OK: boost and shrink. So in terms of my circuit, let me draw it out for you. My Light Emitting Expo Dweeb, and this whole signal was what I used to call V capital I, and that's made up of two components now, a bump offset, and a shrunk voltage VI.

It shrunk, so therefore I've used the small v and small i, like, really, really small. In the same manner, I get a VD ID across the LED, and the corresponding values here will also have a DC offset and a small response.

Let me call that ID plus I small d. I'll do all this mathematically in a second as well, but first let me do it completely intuitively so you get some insight into what's going on. And, VD is simply capital VD plus small vd.

OK, and this is the same as VI, I, and VI. OK, so what have I done? I've done two things. I have said, as an engineer, OK, I care about getting music across my garage door opener. And I'll do what it takes to do that.

OK, so as an engineer, I'll do two things. I'm going to bump my signal up and shrink it. And the bumping and shrinking, and I do it like this. I shrink my signal, the music signal here, and add a DC offset.

OK, and I claim that the music I listened on the other side now, provided I have enough amplification there, is going to be undistorted. OK, so far I've showing this to you completely intuitively using little sketches, no math.

I promise you, I'll give you a bunch of math in a few seconds, but just get the basic idea, and get the intuition behind it. So let's go back to our demo and take a look. So remember, BS, right, bump and shrink.

So what I'm going to do is first of all, let me bump up the signal. So, what I'll do is I want to add an offset to my input, and let me bump it up. Let me shrink it first. It'll make the point a little clearer.

So, the big input, green, is a big input. Let me shrink it. OK, so I've made my input small, and in the middle of that picture out there, you see the region of the transfer curve that's being articulated.

OK, this region of the curve is being articulated by the small signal. It's a much smaller signal. And the output is still distorted because I have to do two things: bump and shrink. I've only shrunk.

OK, let me bump it up now. What's the yellow curve? It's going to get linear. It's going to get proportional to the input. Then I'm bumping it up now. I can make it smaller, make it even smaller, there you go.

Isn't that fantastic? So, I'm making nature do my bidding here, OK? So, this is one of those, when I learned electronics and so on many, many years ago, this was one of those really big ah-ha moments for me, saying, wow, that stuff is cool.

It's something that I couldn't think about myself, and it's not obvious, and by being disciplined and creative in how I use circuits, I can do really, really cool things. OK, remember this as a big ah-ha moment for you.

So, here's my little signal that I've shrunk and bumped up, and my output is a sinusoid, and not this funny, distorted waveform. And notice that this is the region of the curve that is being articulated.

So, I can make the signal even smaller if I like. OK, and what I'd like to do next is play music for you, and if you don't believe your eyes, you can at least believe your ears. Let me go to the distorted signal again, switch to music, and raise it up.

OK, now what we'll do is shrink the music signal and then bump it up. Can I turn the volume down a little bit? That's good. OK, so if I shrunk the volume a little bit, and let me bump it up, now. [MUSIC PLAYS] Just remember this as a big ah-ha moment.

OK, the signal is really, really small. I like that. I like the enthusiasm. OK, so the signal's very small, and I get a more or less linear response. OK. All right, so that's intuition, and the approach that I've taken is called, it's variously called small signal analysis, incremental analysis, small signal method, small signal discipline, whatever you want.

OK, this simply says that by boosting and shrinking my signal, I get a response that's more or less linear even when I have a nonlinear device. And this technique is called the small signal approach.

So, just to focus on that a little bit longer, switch to page five of your notes and let me draw something out for you. OK, so what I have here, this is my offset VD, and from the VD offset I have my little signal V small d, and the total signal is called V capital D.

Offset, small signal, and that's my total signal. OK, notice the offset is all capital. The total signal is small v capital D, and the music or the small signal is small v small d. Similarly, the output is going to look like this, and here I get an offset in the output ID.

I get a corresponding signal, I small d, and I get a total signal, I capital D, OK? The cool thing to notice is that the signal here, the output signal here corresponding to the input signal, the music signal, VD, is small I small D, and that is more or less linear.

OK, and I can even plot the signal like so. This is my input, v capital D. That's T. This is VD, V small d. That is my total input. And similarly, I have an output. And this is my output ID. And, that looks like this, I capital D, small i small d, total signal I capital D.

OK, so that's the small signal method. So, let me summarize that for you. There are three steps to the method. So, first of all, operate at some DC offset. This is also called DC bias, and in that example it's VDID.

OK, so I choose an operating point that bumps up the operation in some region of interest. The second step is to superimpose small signal on top of VD, capital V capital D, to superimpose a small signal, and the third step is observe the response -- -- and the response, small i small d, that's the music part of the response, ID, is approximately linear.

OK, three steps to the method here, and just remember this notation. And, my notation in the small signal model is as follows. My total signal ID is the sum of two signals, I capital D plus small i small d.

This is called the total signal. That's called the DC offset. And this is the superimposed small signal. OK, total signal, DC offset, plus the small signal. And sometimes, especially when doing math, and so on, we may oftentimes represent ID as a delta, I capital D, OK, to show that ID is incremental change in the value of I capital D.

And because of that, this method is also often called the incremental method, incremental analysis. OK, so far what I've done is given you some intuition. I've developed a small, simple method, given you some insight into why we use this method, and also shown you some demonstrations that show that when I bump and shrink, and observe the response, I do get a more or less linear response.

So let me now do this mathematically and show you that mathematically, you can also derive your response to be a linear response. This is page seven. So, I know that ID is some function of the diode voltage.

F was my nonlinear function. OK, so my function F was a nonlinear function. So therefore, ID was nonlinearly related to VD. So, let's do the math. So as a first step, what we did was replace VD by a DC offset, the small signal method, a DC offset, plus a small incremental change.

OK, by doing the math, let me simply use the delta VD notation to show you that I'm dealing with small increments, and also because in the mathematics community, when you learn about some of these techniques, they will use the incremental change notation, which is the delta VD notation.

In electrical engineering, we use a small v, small d notation. So, this is a large DC offset, and this is a small change about that offset. So, you folks have taken math courses before, and been looking at finding out the value of a function, which is a small change for an input value, which is a small change about a big input value or a big DC point is Taylor's expansion.

OK, so let's use Taylor's series expansion, OK, and substitute VD plus delta VD into this, and see what ID looks like. Again, let me tell you where I'm going with this. ID equals F of VD. This is a nonlinear function, OK? I claim that by replacing VD, the input, with the DC offset plus a small value, the resulting response to the small value will be linear, OK? So what I'm going to do next is replace VD with this sum here, and then do the math, and show you that the response corresponding, or the change in ID corresponding to the change in VD is going to be linear.

All right, so let's expand this function using Taylor's series near the DC offset point, capital V capital D. OK, so ID is simply, by Taylor's series, I want to find out a value of the function close to V capital D.

OK, so I take the value of the function at that point, and then I add a few terms in my Taylor's series expansion. The first term is simply the good old Taylor's series stuff. OK, the first term is the first derivative of the function times the change.

And then, the second one is second derivative. OK, and then I get higher order terms. So this is nothing new here. This is good old Taylor series expansion, and again, let me tell you where I'm going.

I want to look at the response for an input that looks like this, and I want to show you at the end of the day that the response in ID, the effect on ID of using an input like this is as if that effect, the incremental change is linearly related to the small input, delta VD.

So here's my Taylor's series expansion for delta V. Now remember, I told you that delta VD is much, much smaller than V capital D. OK, it's a very, very small quantity. But that quantity is really very small.

Then what I'm going to get is that my output is, I can begin to ignore my second order terms. OK, delta VD is very, very, very small. Then, what I'm going to do is that ignore higher order terms. So I'll go and ignore higher order terms.

They'll all go to zero. Remember, I can do this because by design I've chosen delta VD to be very, very, very small. OK, remember, we are engineers. I've chosen it in a way that this is very small.

OK, so I'm telling you that's the case, and under those conditions, I can ignore second higher order terms, in which case I am left with this expression here. So let me rewrite this. Let me rewrite this down here.

OK, I've just copied this turnout, I've ignored all these terms here, and so I have a more or less equal to sign that remains. So what I'm going to do is when I apply a small input of this form to a large DC offset, my output is also going to look like some output offset with a change in the output offset.

And let me call the output offset I capital D, and some small change in the output delta ID. OK, we'll make sure we can convince ourselves that this is indeed the case. Notice that this guy here, F of capital V capital D is a constant.

That's a constant with respect to the incremental change, delta VD. Similarly, this part here is a constant. Notice that this term here is the first derivative of the function evaluated at the DC bias point, capital V capital D.

OK, so this term is also a constant with respect to delta VD. So notice, then, I have a constant term plus a constant term multiplying a small change, delta VD. So what I can do next is, in this case, given that I have a constant term on both sides, and on this side it's a time varying term, what I can do is equate the two constant terms.

I can go ahead and equate these two terms. Remember, I have a constant plus a time varying term, OK, if I'm assuming here that delta VD, my little music signal is a time varying term. So, this constant will equal this, so ID must equal F of VD.

And I know that's the case because the function evaluated at the DC offset gives me the DC current ID. And similarly, ID is equal to that component. Delta ID is equal to D, F of -- OK, so my incremental change in the output is the first derivative multiplied by the small change in the current.

OK, so I'm pretty much done. So, therefore, notice that delta ID is proportional to delta VD. OK, and that's what I had set out to show. Remember, I had set out to show that provided my input is a small excursion around a large DC offset, then my output could also be a large DC offset with a small excursion on top of it where the two excursions, the input excursion and the output excursion would be linearly related like so.

OK, and the method is very simple. I simply expanded the function about that point, that DC point, neglected higher order terms, and notice that my incremental term was simply the derivative plus the incremental change, a derivative times the incremental change in the input.

Move onto page nine, and I'd like to give you a quick graphical interpretation of this. So I gave an intuitive explanation earlier. This is a mathematical explanation that shows you that the input could be linearly related to the output, provided, the outputs would be linearly related to the input, provided the input has a DC offset, and small excursions about that DC offset.

So, let me give you some intuition in what you've really done here, using a little graph here. So, I'm going to plot ID versus VD, and notice that I have some point here, V capital D, I capital D. That's my DC bias.

So, I have some DC bias point here. OK, what is this? That is simply the slope of the curve at that point. OK, it's the slope of this curve evaluated at this point. So this guy here is simply the slope of this curve evaluated at ID VD.

OK, now, what I care about is this point here, and this point here. So let's say that this is delta VD, all right, and that corresponds to this point here. So what I've done is taken the slope and multiplied that by delta VD.

So I've taken the slope, and multiplied it by delta VD, OK, and that gives me this component here. OK, and so, this is the point that I'm going to get. So in other words, what I've done is approximated point A using the Taylor trick by the point B.

OK, so this is a point, A, which is what I really want, and I've approximated that by taking the slope of the function at V capital D, and multiplying that by the change in the input to get the corresponding Y offset, and that's the point that I get.

And notice that if I make this delta VD small enough, then the error between these two points becomes smaller and smaller. So back to our example, so ID was a e to the BVD. This was the relation for our Expo Dweeb, and let me just plug in the values.

So, ID plus small id. Notice, I'm just shuttling back and forth between the notation delta VD, and small v small d. OK, and so that is given by a e to the BVD, oops, plus, I'm just writing that equation up there.

Let me call this equation X. And so, I get the second term is the derivative, ab times e to the BVD times delta VD, small VD, and equating this term that the DC offset. Notice that this is the DC offset in the output, and the small signal, ID is, further notice that in this particular example, what's that? a e to the BVD.

That's simply ID again. It just happens to be that way in this example. So, I get ID times BVD. So, for my input, small id, my incremental change in the output is some ID times B times VD. And notice that this is a constant.

And because that is a constant, my small signal behavior ID is going to be linearly related to the signal, VD, the input signal VD. OK, in the last three minutes, I'd like to give you one additional insight.

So what we've shown so far is if I have an offset and a small change above it, then my output ID will be linearly related to my input. Now let's stare at this thing again. Let me rewrite it. It's some constant IDB times VD.

So, where have we seen such an expression before? OK, where ID was some constant times VD. OK, remember, I equals V divided by R: Ohm's law. What I want to show you now is how we constantly keep simplifying our lives.

The moment we hit some complication and things get too painful to analyze, as engineers, we come up with some clever tricks to make an analysis and use of circuits simple again. And so, notice that this is similar to some, one by RD VD, where RD is simply one over IDB.

I'm just defining this to be RD. And what that means is that I can take a nonlinear circuit that looks like this. OK, and what I can do is replace this by its incremental equivalent, and build what is called a small signal circuit.

And I'll just introduce it here. And we will revisit the circuit in much more gory detail a couple of weeks from now. So, what I can do is build a small signal circuit where I have all the small signal variables, and replace a nonlinear device by a simple little resistor whose value is given by IDB.

OK, so therefore, what I can do is take my nonlinear circuit, and for small, incremental changes, replace that circuit with this equivalent small signal circuit, and go back to doing simple stuff again.

Thank you.