Incremental Analysis

Review

Nonlinear Analysis

- Analytical method
- Graphical method

Today

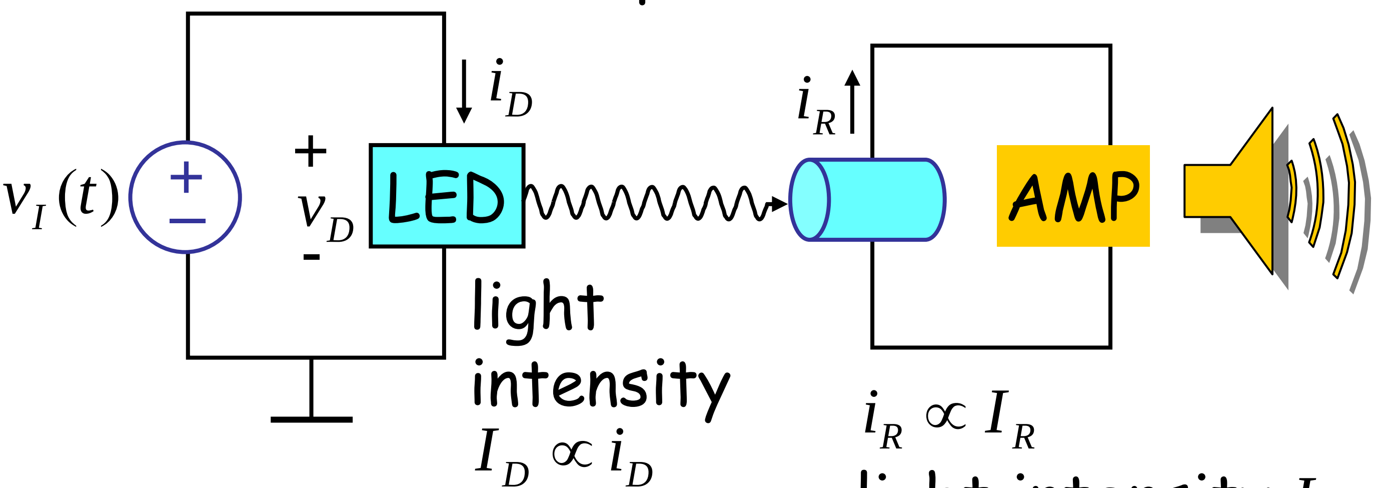
- Incremental analysis

Reading: Section 4.5

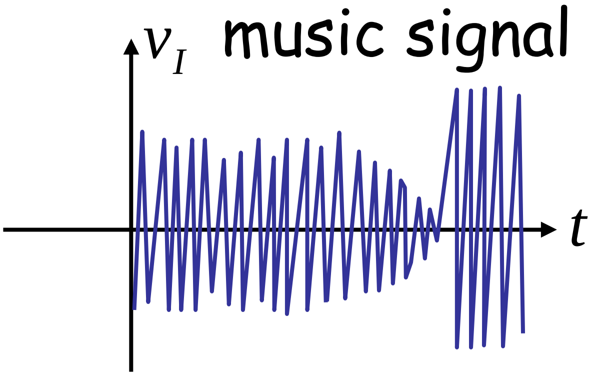
Method 3: Incremental Analysis

Motivation: music over a light beam

Can we pull this off?



light intensity I_R in photoreceiver



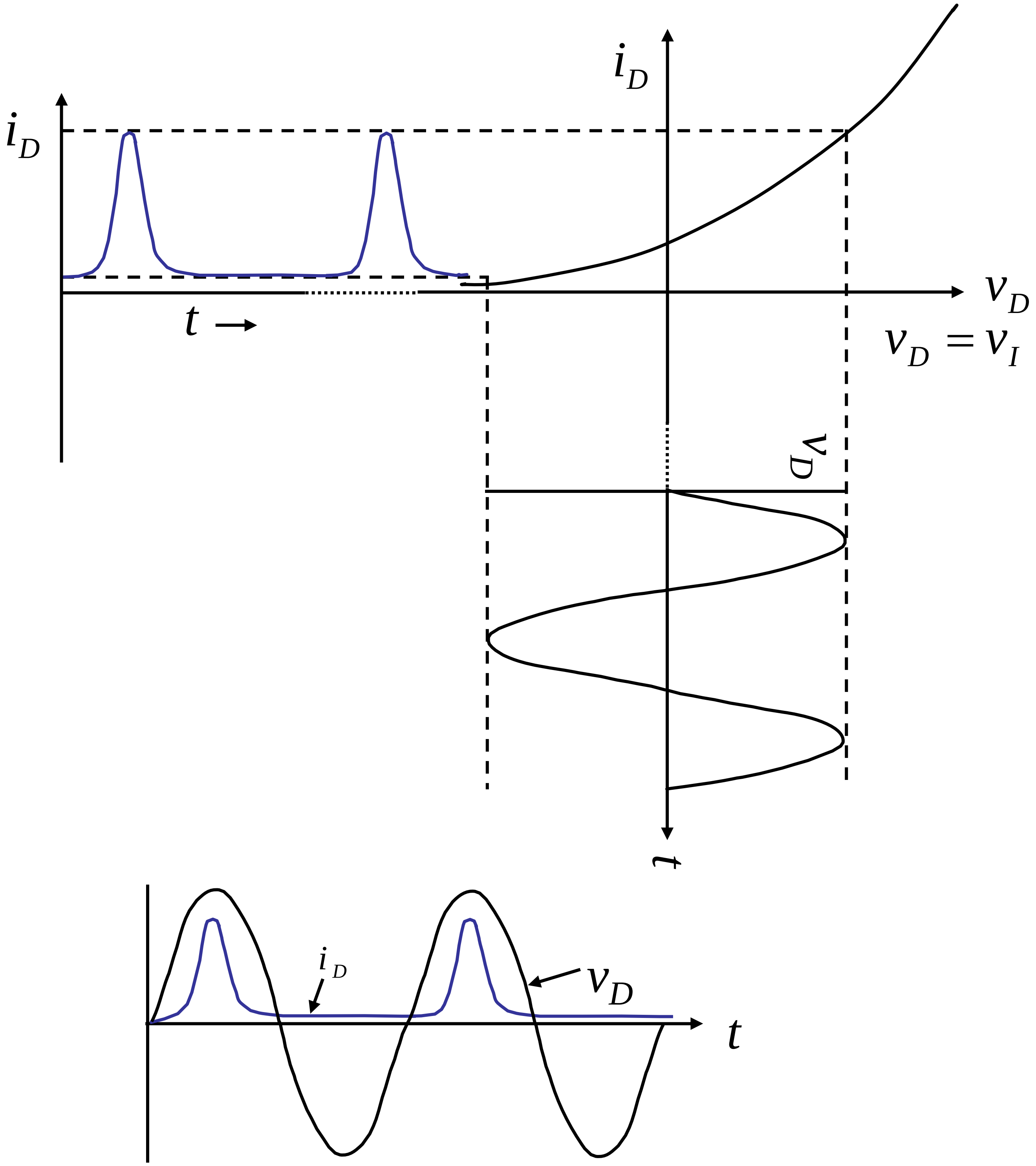
LED: Light Emitting expoDweep ☺

$v_I(t) \longrightarrow i_D(t)$ ⟿ light ⟿ $i_R(t) \longrightarrow$ sound

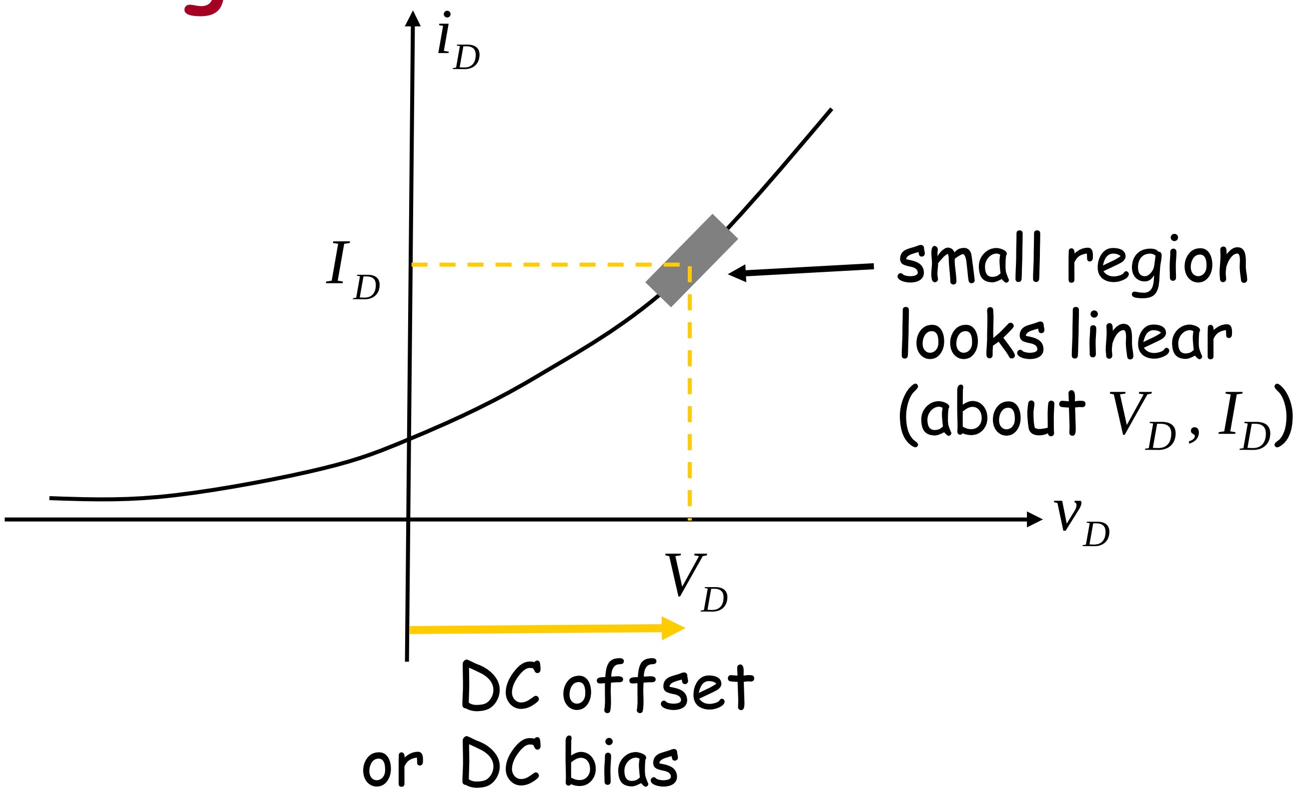
nonlinear (at $v_I(t) \rightarrow i_D(t)$) → problem! will result in distortion

linear (at the remaining steps)

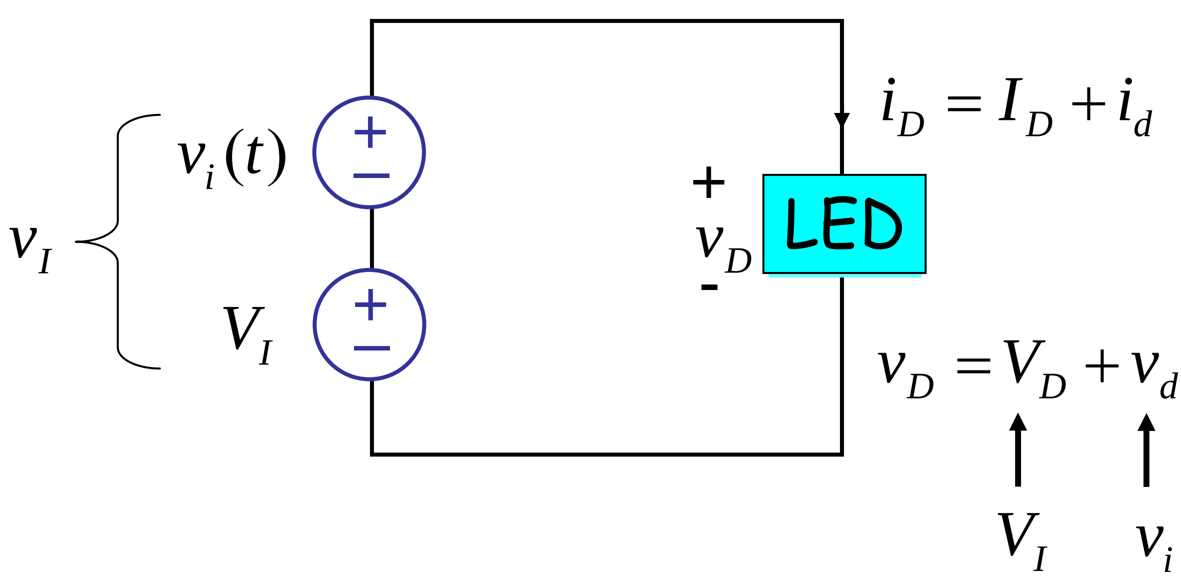
Problem:
The LED is nonlinear → distortion



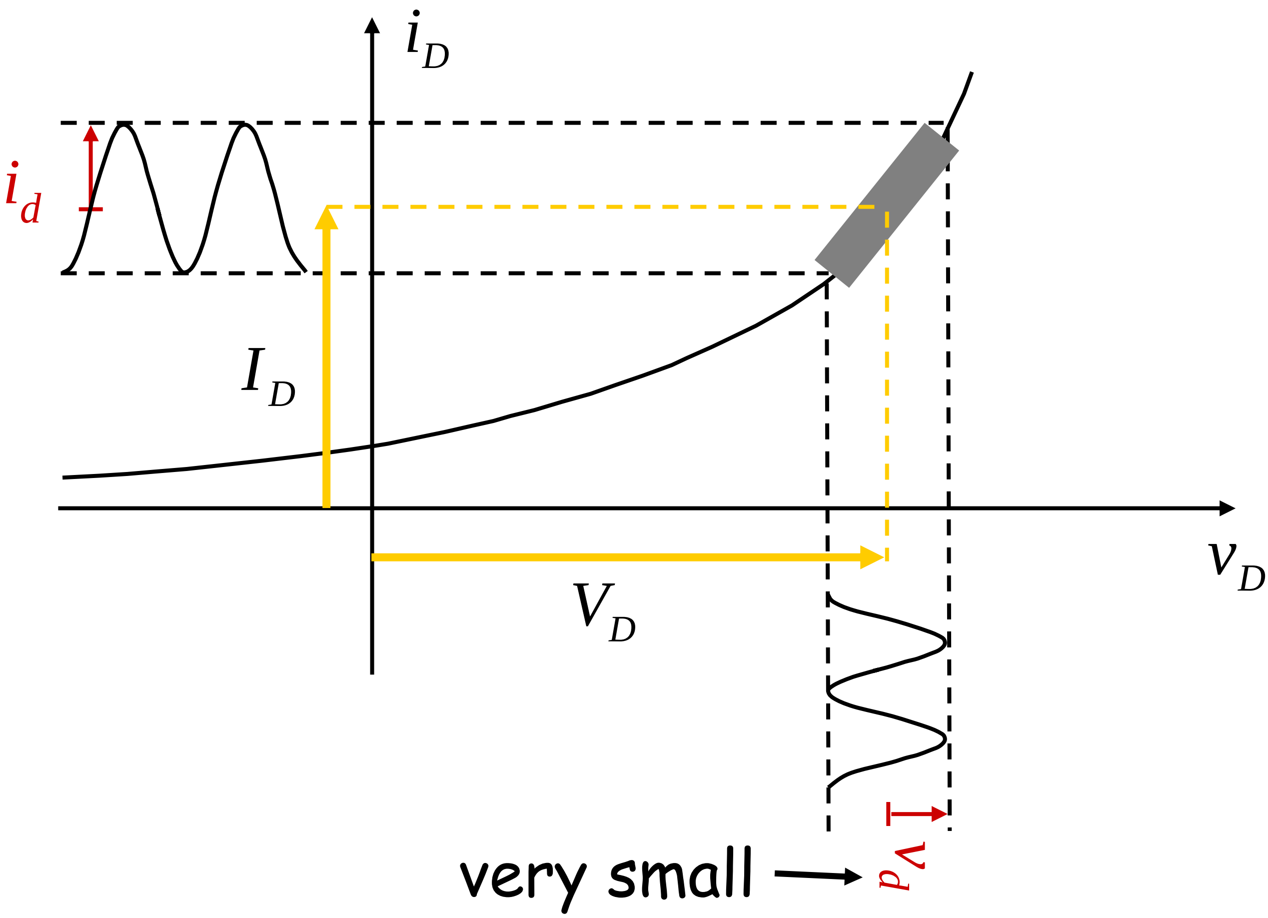
Insight:



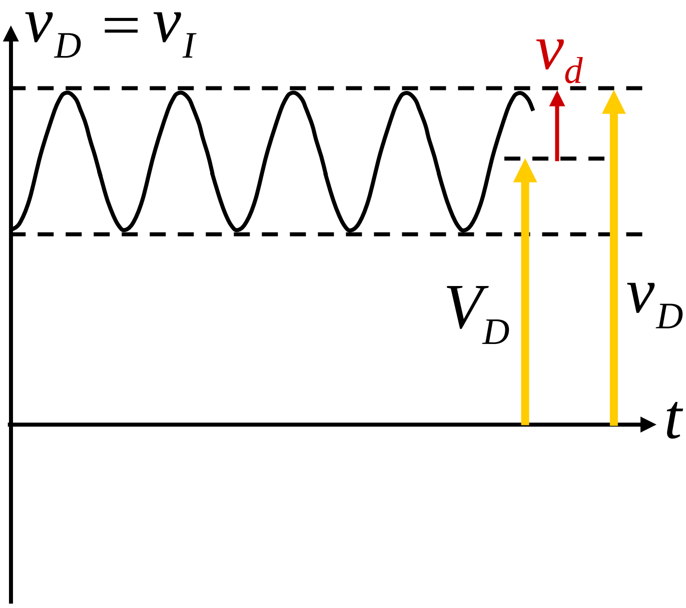
Trick:

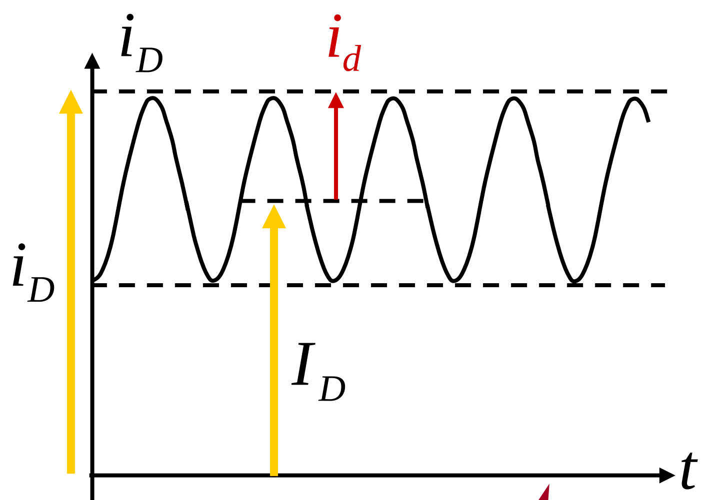


Result



Result





~linear!

The incremental method:
(or small signal method)

1. Operate at some DC offset or bias point V_D, I_D.
2. Superimpose small signal v_d (music) on top of V_D.
3. Response i_d to small signal v_d is approximately linear.

Notation:

$$i_D = I_D + i_d$$

- i_D: total variable
- I_D: DC offset
- i_d: small superimposed signal

What does this mean mathematically?

Or, why is the small signal response linear?

$$i_D = f(v_D)$$ ← nonlinear

We replaced

$$v_D = V_D + \Delta v_D$$

large DC (V_D); v_d (Δv_D); increment about V_D

using Taylor's Expansion to expand $f(v_D)$ near $v_D = V_D$:

$$i_D = f(V_D) + \left.\frac{df(v_D)}{dv_D}\right|_{v_D=V_D} \cdot \Delta v_D$$

$$+ \frac{1}{2!}\left.\frac{d^2 f(v_D)}{dv_D^2}\right|_{v_D=V_D} \cdot \Delta v_D^{\ 2} + \cdots$$

neglect higher order terms because Δv_D is small

$$i_D \approx f(V_D) \quad + \quad \left.\frac{d\,f(v_D)}{dv_D}\right|_{v_D = V_D} \cdot \Delta v_D$$

constant w.r.t. Δv_D — constant w.r.t. Δv_D, slope at V_D, I_D

We can write

$$\otimes: \; I_D + \Delta i_D \approx f(V_D) \quad + \quad \left.\frac{d\,f(v_D)}{dv_D}\right|_{v_D = V_D} \cdot \Delta v_D$$

equating DC and time-varying parts,

$$I_D = f(V_D) \longrightarrow \text{operating point}$$

$$\Delta i_D = \left.\frac{d\,f(v_D)}{dv_D}\right|_{v_D = V_D} \cdot \Delta v_D$$

(constant w.r.t. Δv_D)

so, $\Delta i_D \propto \Delta v_D$

By notation,

$$\Delta i_D = i_d$$

$$\Delta v_D = v_d$$

In our example,

$$i_D = a\, e^{b v_D}$$

From ⓧ: $I_D + i_d \approx a\, e^{bV_D} + a\, e^{bV_D} \cdot b \cdot v_d$

Equate DC and incremental terms,

$$\boxed{I_D = a\, e^{bV_D}}$$ → operating point
- aka bias pt.
- aka DC offset

$$i_d = a\, e^{bV_D} \cdot b \cdot v_d$$

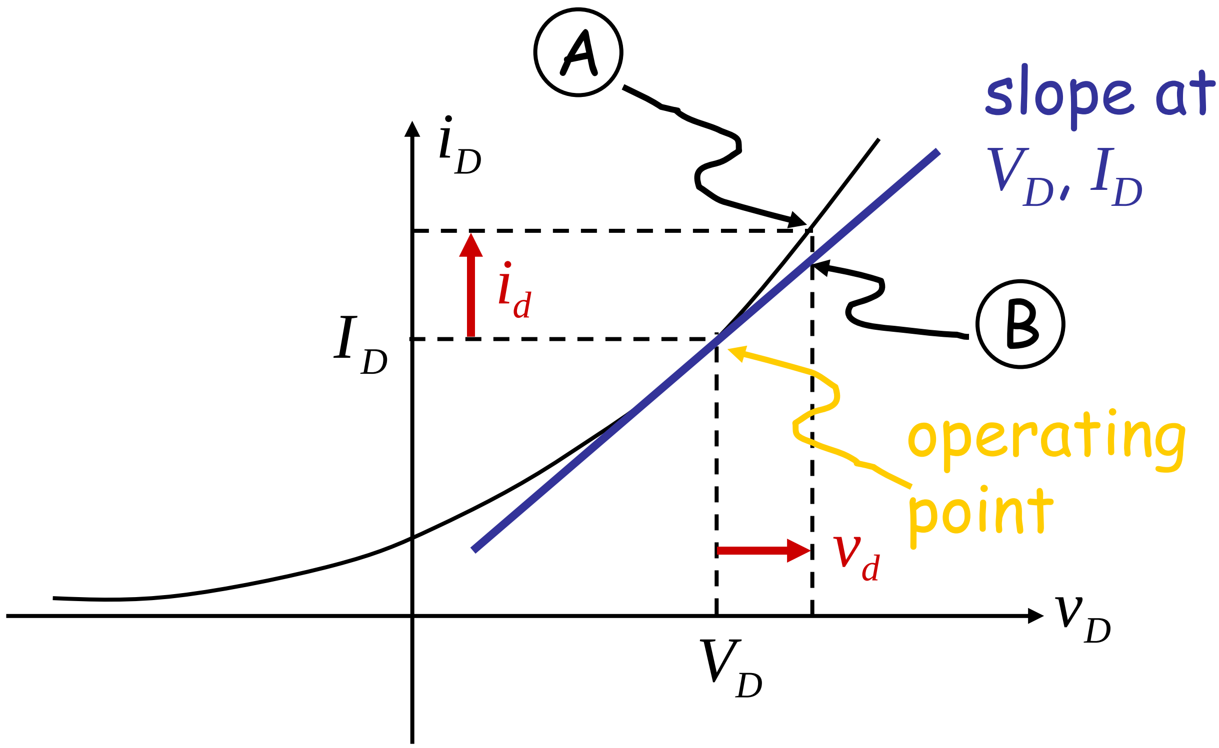
$$i_d = \underbrace{I_D \cdot b}_{\text{constant}} \cdot v_d$$ → small signal behavior

constant → linear!

Graphical interpretation

$I_D = a\, e^{bV_D}$ ➡ operating point

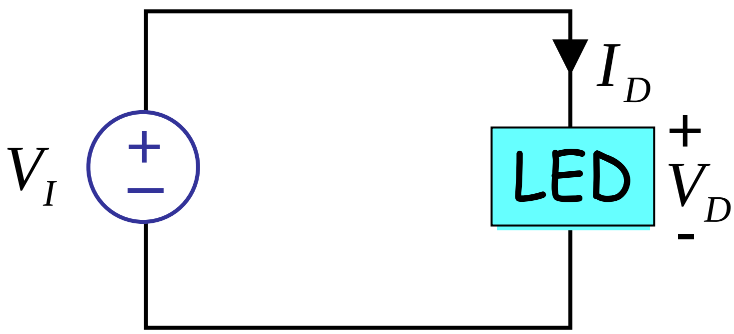
$i_d = I_D \cdot b \cdot v_d$



we are approximating (A) with (B)

We saw the small signal → graphically / mathematically / now, circuit

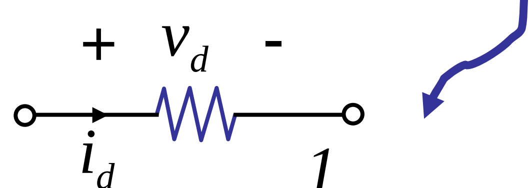
Large signal circuit:



$$I_D = a\,e^{bV_D}$$

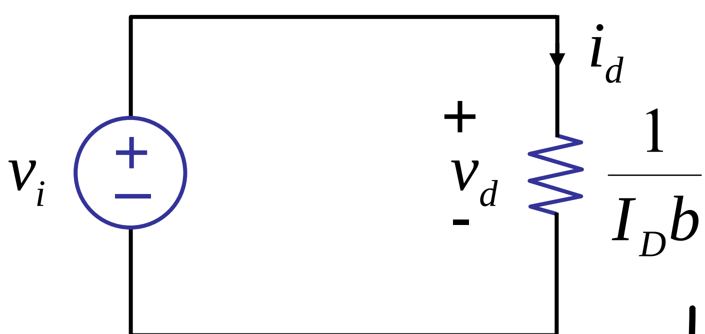
Small signal reponse: $i_d = I_D\, b\, v_d$

behaves like:



$$R = \frac{1}{I_D\, b}$$

small signal circuit:



Linear!