6.002 Circuits and Electronics, Spring 2007

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Transcript – Lecture 10

Good morning, all. Good morning. I hope you guys did not spend all of last night celebrating the Red Sox victory, but there is one more tonight. OK. Let's see. I trust the quiz went OK. What I will do today is take off from where we left off on Tuesday.

And continue our discussion of the large signal and small signal analysis of our amplifier. Today the focus will be on "Small Signal Analysis". So let me start by reviewing some of the material. And, as you know, our MOSFET amplifier looks like this.

One of the things you will notice in circuits, as I have been mentioning all along in this course, is that certain kinds of patterns keep repeating time and time again. And this is one such pattern.

A three terminal device like the MOSFET with an input and the drain to source port connected to RL and VS in series in the following manner, this is a very common pattern. There are several other common patterns.

The voltage divider is a common pattern. We keep running into that again and again and again. The Thevenin form, a voltage source in series with the resistor is another very common form. The Norton equivalent form, which is a current source in parallel with a resistor is also very common.

And it behooves all of us to be very familiar with the analyses of these things. Voltage dividers in particular are just so common that you need to be able to look at it and boom, be able to write down the expression for voltage dividers.

I would also encourage you to go and look at current dividers. When you have two resistors in parallel and you have some current flowing into the resistors to find out the current in one branch versus the other very quickly.

The expression is very analogous to the voltage divider expression. And some of these very common patterns are highlighted in the summary pages in the course notes, so it is good to keep track of those and be extremely familiar with those patterns to the point where if you see it you should be able to jump up and shout out the answer just by looking at it without having to do any math.

So here was an amplifier. And then we noticed that when the MOSFET was in saturation it behaved like a current source. And this circuit would give us amplification while the MOSFET was in saturation.

So we agreed to adhere to the saturation discipline which simply said that I was going to use my circuit in a way that the MOSFET would always remain in saturation in building things like amplifiers and so on.

And by doing that throughout the analysis I could make the assumption that the MOSFET was in saturation. I didn't have to go through -- Analysis became easier. I didn't have to figure out now, what region is the MOSFET in? Well, because of my discipline it is always going to be in saturation.

But in turn what we had to do was conduct a large signal analysis. Again, in follow on courses you will be given circuits like this. In fact, this very circuit with a very high likelihood. And you will be looking at more complicated models of the MOSFET.

Or you will be given the MOSFET like this and, let's say in that course the designers do not adhere to the saturation discipline, in which case you have to first figure out is my MOSFET in its triode region or in the saturation region? And depending on the region it is in you have to apply different equations.

So it is one step more complicated than in 002. In 002 we simplified our lives by following a discipline. And let me tell you that following a discipline is quite OK. When it simplifies our lives and we can do good things with it, it is quite OK to do that.

We are not wimps or anything like that. It is quite OK to have a discipline and agree that we are going to play in this region of the playground and build circuits in that manner. By doing so, we could assume the MOSFET was in saturation all the time.

And analysis simply used a current source model. By the same token, what becomes important is to figure out what are the boundaries of valid operation of the MOSFET in saturation? To do that we conducted a large signal analysis.

And it had two components to it. One of course was to figure out the output versus input response. And what this usually does is that it does a nonlinear analysis of this circuit. If it is a linear circuit it is a linear analysis.

And figures out what the values of the various voltages and currents are in the circuit as a function of the applied inputs and chosen parameters. And the second step we said was to figure out valid operating ranges -- -- for input and corresponding ranges for the other dependent parameters such as VO.

You could also find out the corresponding operating range for the current IDS and so on. So by doing this you could first analyze the circuit, find out the "bias" parameters, find out the values of VI and VO and so on.

And then you could say all right, provided, as long as VI stays within these bounds my assumption that this is in saturation will hold and everything will be fine. The reading for this is Chapter 8.

And today we will take the next step and revisit small signal analysis. In the demo that I showed you at the end of last lecture, I showed you an input triangular wave. And the input triangular wave gave rise to an output.

And we noticed that we did have amplification, I had a small input and a much bigger output. I did have amplification when the MOSFET was in saturation but it was highly nonlinear. The input was a triangular wave and the output was some funny, it kind of looked like a sinusoid whose extremities had been whacked down and kind of flattened.

And its upward going peak had been shrunk. So it was a kind of weird nonlinear behavior. I will show that to you again later on. And so it amplified but it was nonlinear. And remember our goal of two weeks ago? We set out to build a linear amplifier.

So today we will walk down that path and talk about building a linear amplifier. So to very quickly revisit the input versus output characteristic, VI versus VO, this is VT and this is VS, this is what things looked like.

Also to quickly review the valid ranges, until some point here the amplifier was in saturation, the MOSFET was in saturation and somewhere here I had VO being equal to VI minus a threshold drop. At that point the MOSFET went into its triode region and I no longer was following the saturation discipline.

So therefore this is my valid region of operation. We also know that the output was given by VS minus K (VI-VT) all squared RL over 2. Again assuming the MOSFET is in saturation. It is very important to keep stating this because this is true only when the MOSFET is in saturation, when I am following the discipline.

Notice that this is a nonlinear relationship. So VO depends on some funny square law dependence on VI. The key here is how do we go about building our amplifier? Take a look at this point here. At this point here let's say I have a VI input.

Corresponding output is VO. Focus is this point. And left to itself this was a nonlinear curve. Remember the trick that we used in our nonlinear Expo Dweeb example? We used the Zen Method. Remember the Zen Method? We said look, this is nonlinear, but if you can focus your mind on this little piece of the curve here this looks more or less linear.

If I look at a small itty-bitty portion of the curve and I do the Zen thing, and kind of zoom in on here. This looked more or less linear. This means that if I could work with very small signals and apply the signal in a way that I also had a DC offset of some sort.

Then I would be in a region of the curve, I would be delineating a small region of the curve which would be more or less linear. This was a small signal trick. And what we will do here is simply revisit the small signal model.

Most of what I am going to do from here on will be more or less a repeat of what you saw for the light emitting expo dweeb. Just that here I have a three terminal device, with a little bit more complication.

The equation is different. I don't have to resort to a Taylor series expansion. I will just do a complete expansion of this expression and develop the small signal values for you. Recall the small signal model.

It had the following steps. The first step will operate at some bias point, VI, VO, and of course some corresponding point IDS. This is Page 3. And then superimpose a small signal VI on top of the big fat bias.

Remember the "boost"? So VI is the boost. Boom. And above VI, I have small signal VI that I apply. And our claim is that response of the amplifier to VI is approximately linear. The key trick with this is that for my small signal model here, this is Page 3 here, and Page 2.

The key trick here is that with the small signal model, I operate my amplifier at some operating point, VO, VI. I superimpose a small signal VI on top of small VI on top of big VI. And then I claim that the response to VI is approximately linear.

And let me just embellish that curve a little bit more. Notice that in this situation this was my VI, which is my bias voltage, this is VO, which is the output bias, and of course not shown on this graph is the output operating current which is IDS.

One nice way of thinking about this is to redraw this and think that your coordinate axes have kind of shifted in the following manner. This is VI. This is also on your Page 3. This is VT. Remember this was the operating point, VO and VI.

And notice that we were operating in this small regime of our transfer curve here. And in effect what we are saying is that I am going to apply small variations about VI and call those variations delta VI or small VI.

And the resulting variations are going to look like delta VO. Also referred to as small V, small O. So I will have small variations here. And they give rise to corresponding small variations there.

One way to view this is as if we are working with a new coordinate system. Another way to view this is that so the capital VI and capital VO correspond to my VI and VO as the total voltages in my circuit, but at this bias point I can think of another coordinate system here with small VI and VO out there.

And for small changes to VI, I can figure out the corresponding small changes to VO. Just that all the analysis I perform here is going to be linear. And I will prove it to you in a couple of different ways in the next few seconds.

When I am doing small signal analysis I am operating here in this regime at some bias point. You have also seen this before. How do I get a bias? This is my amplifier RL and VS. This is Page 4. VO.

The way I get a bias is I apply some DC voltage VI and superimpose on top of that my small signal small VI. This is my DC bias that has boosted up the signal to an interesting value. And because of that what I can get is by varying VI as a small signal with a very small amplitude, I am going to get a linear response here.

And I can draw that for you as well. This is my bias point here. And if I vary my signal like so then my output should look like this. This is point VI, this is point VO, and this is my small signal VI and this is my small signal VO and this is capital VO.

So this small thing here is VI. I would like to show you a little demo. I will start with the same demo I showed you the last time. I showed you the amplifier. In the demo I am going to apply a triangular wave.

And initially I start with a large signal. And you will see that the output looks really corny, is going to look something like this. That's large signal response. And then I will begin playing with the input making it smaller, and you can see how it looks yourselves.

There you go. So this is where I stopped the last time. The last lecture I applied this input, time is going to the right, and the purple curve in the background is the output. It looks much more like a sinusoid with some flattening of its tips.

Nothing like an interesting triangular wave. What I will do next is that let me make sure I have enough of a boost here, enough of a DC voltage so that I am operating at some point here. I believe I already have that.

Notice that I can shift up the triangular wave input, or I can shift it down. So let me bias it here. I have chosen a VI that's about, I forget how many volts per division it is, but I have chosen some VI here.

And I biased it such that this is the input. You get a nonlinear response. It is amplified. It is much bigger. What I will do next is make VI that I apply smaller and smaller. I have already done the boosting.

Boom, that's a boost. So I have boosted up your VI already. Next is I am going to shrink it, and hopefully you will see that if all that I am saying is truthful here you will see a triangular response.

Let's go try it out. Watch the yellow. I am going to shrink the yellow and make it smaller and smaller. There you go. It is great when nature works like you expect it to. I have never seen a triangular wave looks so pretty in my life.

It is awesome. Look at this. Here is a tiny triangular wave. And the output is also a triangular wave but it is much more linear. Yes. Question? What's that? The question is that the output here is only as big as the input used to be before.

That's a good question. What I have done here is I am showing you a laboratory experiment. And let's assume that this input is the input I am getting from some sensor in the field. Assume that this is my input, not what I had before.

Assume that this is my input to begin with and this is the amplified output. What I can also do is I can also change the bias. And we will see this at the end of the lecture, in the last ten minutes of lecture.

How do you select a bias point? By changing your bias point you can change the properties of an amplifier to give you a preview of upcoming attractions. Let me ask you, what do you think should happen if I change the bias point? I have not shown you the math yet, so intuitively what do you think should happen? If I increase the bias what do you think is going to happen? Yes.

Good insight. Higher bias will be more amplification. Let's see if our friend is correct. Let me set a higher bias. Not necessarily, I guess. You're actually right, by the way. I am playing a trick on everybody here.

As I change my input bias. Notice that under certain conditions my output becomes smaller and gets more distorted. Under other conditions what is going to happen to my output is that it is becoming smaller and is going to get distorted again.

So there are a bunch of funny effects happening that reflect on the bias point, but for an appropriate choice of bias point as I increase the bias the amplification should increase. And I will show you that in a few minutes.

But it is a complicated relationship. Yes. This is finally getting fun. Here is the question. Professor Agarwal, we love your song and dance, but if you really want to get a high signal at the output and you want to amplify your big input signal how do you do it? So the question is let's say I have an input that is this big here, if it is this big, I have shown you how I can get things that are this big, but what if my input was this big? How do I get an output that is this big? Well, I will use one of those learned by questioning methods and have you tell me the answer.

Someone tell me the answer. How do I do that? Yes. Use another amplifier. So the answer is I will use one amplifier to go from here to here. And the suggestion is use another amplifier to go from here to here.

And, in fact, I believe that you may have a problem in your problem set where you will do that. And so you have only yourselves to blame. So how do you make this work? What you have to do is this VI has to be much smaller than the bias point VI on this one.

I have to build a different amplifier, choose a different set of parameters such that VI prime, which is the VI for this guy, is much less than V capital I prime for this guy. It's a design question.

You need to design it in a way that the signals of interest need to be much smaller than the bias voltage of this amplifier. So you may have to use much higher supply voltages. My amplifier, I believe, has a 4 volt supply or 5 volt supply.

You might have to use an amplifier with a much bigger supply, different values of RL and so on. And I know that the course notes also have some exercises and problem sets that discuss that in more detail.

Yes. This is even more fun. The question is, good question. The question is why do you need this guy here? Just use this guy, right? Why do you need this guy? Big guys rule, right? Who needs the little guys? Well, let me use the Socratic method again.

Why don't you give me the answer? You guys are smart. Why do you need little guys? Why do you need the small guy here? Anybody with the answer? Yeah. The big guy may not be as sensitive. I like that.

You know what? He is almost correct. I will show you why in a second. Anything else? Any other reason? Yes. Bingo. That is another good answer. So let me address both the answers. The answer given was that look, this amplifier is amplifying the signal by a certain amount, by a factor of 7.

And I have designed this such that this amplifies a signal by a factor of maybe 10. So in all I am getting an amplification of 70. This would be a great design question for lab next year. I give you a bunch of components and ask you to design an amplifier given the constraints with the highest amount of amplification.

It turns out that when you design your amplifier, in order to meet the saturation discipline and so on, you have to choose values of RL and VS and stuff like that and be within power constraints so the amplifier doesn't blow up and stuff.

And by the end of it all you are going to get a measly 7X gain out of it. The same way here, to be able to deal with a very small signal here and get some amplification, another set of values and you get 10X.

So they multiply. It is much harder to build one amplifier with a much larger gain. You know what? I just realized that we will be looking at this in the last five or seven minutes of lecture. I am going to show you what the amplification depends upon.

It depends upon K. It depends upon RL. It depends upon VI. Now the question is I have had all this time to think about how to stitch in sensitive into this, and I believe I can. It turns out that when you have large voltages and so on and you have practical devices, it turns out that the more current you pump through devices they tend to produce noise of various kinds.

So very powerful amplifiers are not very good at dealing with really tiny signals because they have some inherent noise capabilities. And so I guess that is sensitive. It is sensitive to noise. Another question? Yes.

Ask me the question again. I didn't follow. Let me just explain it. It turns out that I will not be able to pass this through the big amplifier to begin with because it is just going to give me a gain of just a factor of 7.

However, if I have a signal that is this big to begin with then I may just need this amplifier. I don't need the smaller guy. If my signal was this big to begin with, if I had a strong sensor that produced a strong signal to begin with, yeah, I can deal with just a single stage.

I don't need to two stages. It is all a matter of design. And it is actually a fun design exercise. Given a budget, dollars, right? You go to your supply room and look at the parts that you have and you go to build what you have to build with the parts that you have.

And so sometimes you need to build two amplifiers to get the gain or build a signal amplifier. It's all a design thing. All right. Moving on to Page 7. That brings us to the small signal model. Page 5.

What I showed you up on the little demo was that provided the signal input in this example VI was much smaller than capital VI out there as I shrank my input, I was able to get a more or less linear response at the output.

And so to repeat my notation at the input, the total input is a sum of the operating point input plus a small signal input. This is called the total variable. This is called the DC bias. It is also called the operating point voltage.

And this is called my small signal input. It is also variously called incremental input. This is more a mathematical term relating to incremental analysis or perturbation analysis. So VI, call it small signal, call it small perturbation, call it increment, whatever you want.

Similarly, at the output I have my total variable at the output a sum of the output operating voltage and the small signal voltage. I do not like using Os in symbols because big O and small O is simply a function of how big you write them.

It is not super clear. And in terms of a graph, let me plot the input and output for you. Let's say this is the total input and that is the total output. I may have some bias VI. And corresponding to that I may have some bias VO.

Hold that thought for a second while I give you a preview of something that we will be covering in about three or four weeks. Notice that as I couple amplifiers together, the output operating point voltage of this amplifier in this connection becomes the input operating point voltage of this amplifier, right? So when they connect this output to this input, the output operating point voltage becomes coupled to the input here so it becomes the input operating point voltage here.

Now I have a nightmare on my hands. As I adjust the bias of this guy, the bias of this guy changes, too. The two are dependent. It is a pain in the neck. And we being engineers find ways to simplify our lives.

And you will learn another trick in about three or four weeks. And that trick will let you decouple these two stages in a way that you can design this stage in isolation, go have a cup of coffee and then come back to this stage and design this stage in isolation.

For those of you who want to run ahead and think about how to do it, think about it. What trick can you use to get them in isolation? Moving on. What I would like to do next is address this from a mathematical point of view.

And much as I did for the light emitting expo dweeb analyze this mathematically and show you that if VI is much smaller than capital VI, I indeed get a linear response. This time around I won't use Taylor series because it turns out that this expression can be expanded fully.

So you don't have to buy into Taylor series and so on. I am going to list everything down for you. We know, to begin with, that VO for the amplifier is VS-RLK/2 (VI-VT)^2. What I am going to do for this, much as I did for the LED, what I'm going to do is derive for you the output as a function of the input when the input VI is very small.

In other words, when I substitute for VI, V capital I squared plus small VI. Much as I did for the expo dweeb, I want to substitute for VI a big DC VI. So VI is much smaller than VI. And show you for yourselves that the output response, V small O is going to be linearly connected to VI.

Notice that, let me write another equation here. This is a total variable. This simply says that if the input is VI then the output is going to be VO, which means that the operating point input voltage should satisfy this equation, correct? In other words, the operating point output voltage V capital O should equal VS-RLK/2 (VI-VT)^2.

This is at VI equals capital VI. This is very simple but may seem confusing. All this is saying is that look, this equation gives me the relationship between VI and VO. Therefore, if I apply capital VI as the input, I'm given that my corresponding output is capital VO, so they must satisfy this equation, right? Those are bias point values and that must satisfy this equation.

Simple. I know that. So hold that thought. Stash it away in the back of your minds. Now let me go through a bunch of grubby math and substitute for VI in this expression here. Let me go ahead and do that.

$VS-RLK/2((VI+vi)-VT)^2$. When I do something that is other than math I will wake you up. I will just keep doing a bunch of steps that are pure math. No cheating. No nothing. Watch my fingers. When I do anything that is not obvious math I will wake you up.

Next I am going to simply move VT over and rewrite this as follows, $RLK/2((VI-VT)+vi)^2$. Again, I haven't done anything interesting so far. I have just substituted this. I am just juggling things around just to pass away some time, I guess.

All right. Next what I am going to do is simply expand this out and write it this way RLK/2, expand that out and treat this as one unit $VS-RLK/2((VI-VT)^2+2(VI-VT)vi+vi^2)$. Nothing fancy here. This is like the honest board.

Nothing fancy here. Standard stuff. Only math. I will move to this blackboard here where I do some fun EE stuff. Yes. Good. At least one person isn't asleep here. Thank you. So just math here.

Nothing fancy. Plain old simple math. I have not done any trickery. I still have all my ten fingers. Now what I am going to do, now watch me. I am not using Taylor series here because this expression lends itself to this analysis.

Notice VI squared here. I made the assumption that VI is much smaller than capital VI, so what I can do is assuming that VT is small enough that VI minus VT is still a big number compared to small VI, what I can do is ignore this in comparison to the capital VI terms.

So I have a capital VI term here. I am going to ignore VI squared. So, for example, if capital VI was 5 volts and small VI was 100 millivolts 0.1, so 0.1 squared is 0.01. So it is comparing 0.01 to 5.

So I am off by a factor of 500. So now watch me. Now I begin playing some fun and games here. I eliminate this, and because I eliminate that it now becomes approximately equal. What I do in addition is let me write down the output.

The total variable is the sum of the DC bias and some variation of the output. And let me simply expand that term and write it down again. $VS-RLK/2(VI-VT)^2-RLK/2$. I get a two here. And I get VI-VT.

I won't forget the VI this time. Again, from here to there nothing fancy. This is the one step where I have used a trick. I have said small

VI is much smaller than capital VI, and so I have simply expanded this out and written it here.

So do you see the obvious next trick here? From star look at this guy. I can cancel this out from star because I know that at the operating point these two expressions are equal, and so therefore I can cancel out the operating point voltage and this.

What I am left with is small VO is simply minus RLK(VI-VT) times vi. Only one place where I did something funny. Other than that it is purely math. So this is what I get. Notice that this whole thing is a constant, minus RLK(VI-VT).

This whole thing is a constant. And so VO is equal to some constant times VI. Let me just define some terms for you that you will use again and again. For reasons that will be obvious next lecture, I am going to call this term here GM.

I am going to call this term a constant, K(VI - VT). It is a constant for a given bias point voltage. So I am going to call that GM. And then I am going to call this whole thing A. And of course this is VI.

There you go. I have my linear amplifier. A is the gain times small VI. And the gain has these terms in it. I just call this GM. You will see why later. But notice that the gain relates to RL. The size of the load resistor RL, how big it is, 1K, 10K, whatever.

K, this is a MOSFET parameter, and VI minus VT. That is a constant for a given bias point voltage and small VI. So VO equals small VI. I won't give you a graphical interpretation, but I encourage you to go and look at Figure 8.9 in the course notes.

And it gives you a graphical interpretation of that expression. Move to Page 7. Another way of looking at this, another way of mathematically analyzing it, here I went through a full blown expansion and pretty much deriving the small signal response.

What I can also do is take a shortcut here. So let me just give you the shortcut. You might find this handy. VO=VS-KRL/2(VI-VT)^2. And my shortcut is as follows. My small signal response is simply this relationship.

I find the slope at the point capital VI and multiply by the increment. Slope times the increment gives me the incremental change in VO as follows. $d/dI\ (VS-KRL/2(VI-VT)^2)$ evaluated at $vI=VI$ times vi.

This is math again. I want to find out the change in VO for a small change in VI, and I do that by taking the first derivative of this with respect to VI substituting V capital I and multiplying by the small change delta VI or small VI.

So this is simply the slope of the VO versus VI curve at VI. And so therefore taking the derivative here of this. This is a constant so it vanishes. But twice 2 to cancel out, so I get KRL(VI-VT) times small vi evaluated at capital VI.

So I get twice KRL, VI evaluated at capital VI, so it is VI minus VT times small VI. Same thing. Oh, and I have a minus sign here. I get the same expression that I derived for you up there, and this is just taking the slope and going with it.

And this, as I mentioned before, this is A. The last few minutes let me kind of pull everything together and also hit upon something that many of your questions are touched upon. And that all relates to how to choose the bias point.

So here I have taken an analysis approach. When teaching we often teach you are given something, you analyze it, but as you begin to master it you can begin to design things where you can ask a lot of questions and so on.

And here what we have is an analysis given a value of RLK, VI and so on. How to choose the bias point becomes more of a design issue. If you are designing an amplifier, you asked me the question, how do I choose two small amplifiers versus one big amplifier, that sort of stuff? It boils down to how do you choose the bias point? How do you choose VI? How do you choose RL and so on? What I would like to do is touch upon some of these things.

First of all, gain or the amplification. One of the most important design perimeters for an amplifier is what is the gain? Let's say you get a job at Maxim Integrated Technologies, and they say we would like you to build a linear power amplifier for cell phones.

You can say I know how to do that. And then they say the next stage needs a 100 millivolt input. While this thing coming from the antenna is only a few tens or a few hundreds of a microvolt. So you sit down and say oh, my gosh, I need an amplification of so much, and you go design an amplifier.

So gain tends to be a key parameter. And notice that gain is proportional to RL. It relates to VI minus VT, so proportional to VI. It is also related to RL. The second point is the gain point determines where I bias something.

If I choose my bias too high I get distortion, or if I choose my bias too low I get distortion. So depending on how I choose my bias point, as a signal goes up it may begin clipping or begin distorting.

And I will show you a demo the next time on that particular example. So bias point will determine how big of a signal you can send without getting too much distortion. And the other thing is that, relates to how big of an input, what is a valid input range? So let's say you have a signal.

And you want that signal to have both positive and negative excursions of the same value. Then, depending on where you choose a bias point, your input range may become smaller or larger. And we will go through these in the context of and amplifier and look at some design issues in the next lecture.