ROBERT FIELD: All right. So last time, I said that atomic sizes are interesting or useful to keep in mind, because you want numbers for them, which are somewhere between 0.1 and 100, or something like that. Because then you have a sense for how big everything is, and you're in the right ballpark. If you have to remember a number and an exponent, it's a little trickier.

And so I'm going to say something about atomic sizes in just a few minutes, but the main things from the previous lecture were this relationship between the wavelength and the momentum, which is true for both waves and particles, or things that we think are wave-like, like light, and things that we think are particle-like, like electrons-- and so this unifying principle.

Then I have a little question. And that is suppose the wavelength for a particle is known, and suppose we have n particles. And so if we say each particle requires a volume of lambda cubed, and there are n particles, and we stick them into a volume smaller than n times lambda cubed, what's going to happen? Yes?

AUDIENCE: They interfere with the structure?

ROBERT FIELD: Their identities get corrupted, and the person who does that for the first time gets a Nobel Prize, and that was Wolfgang Ketterle-- and others. So when you have quantum mechanical particles that are too close together, they lose their individual identity.

And so this is a very simple thing that anybody who was beginning to understand quantum mechanics in the 1920s would say, this is a puzzling thing, maybe we should think about that. And it's really hard, so it took a long time.

Then a whole bunch of experiments led to the idea that we need to have a way for atoms to fill space and satisfy all of the other stuff, and that was Rutherford's planetary atom picture. The problem with that is it does fill space, but there's no way for the electron to continue orbiting around a nucleus, because it will radiate its energy, and fall into the nucleus, and game over.

And so Bohr and De Broglie both had ways of fixing this. And Bohr's way was simply to say the angular momentum is not just conserved, but it has certain values-- an integer times h bar, h bar is h over 2 pi. And De Broglie said in order to keep the electron from annihilating itself, since it's going around in a circular orbit of known circumference, there must be an integer number of wavelengths around that orbit.

Now, that's a much more physical and reasonable ad hoc explanation, but it's still ad hoc. It assumes that the particles are moving around circular orbits. I hinted that the way out of this is going to be that the particles aren't moving.

Then we could still have angular momentum. We can still have all sorts of useful stuff, but they're not moving, and we don't radiate the energy of the particles. But that requires a completely new way of looking at particles in quantum mechanics.

But the thing about both the Bohr hypothesis and the De Broglie hypothesis is that any sophomore who made such a proposal would be laughed at in the 1910-1920 period, because it's just ridiculous. But the reason these hypotheses were taken seriously is that this planetary model with those corrections explain the spectra of all one-electron systems-- hydrogen, helium plus, lithium two plus, uranium 91 plus, all of them, to better than measurement accuracy at the time, and better than measurement accuracy for a long time after this was proposed.

So getting a whole bunch-- now, a whole bunch is an infinite number, actually-- of 10 digit numbers makes you think there's something here, but nobody knows where here is except the numbers. And the idea of the planetary model with the fixes doesn't explain anything other than the spectral lines, which is a lot, but it tells you there's something really good there.

I guess one thing I forgot to mention in the summary is that the energy levels that you get from the Bohr model are going to explain spectra if you say a spectrum are transitions between these energy levels. And that was also a brilliant suggestion, and it was suggested by numerology.

But let us go back to work now. We're going to talk about the two-slit experiment. And I have a personal thing to report about that.

The first time I gave a talk, it was going to be a 15-minute talk at a spectroscopy conference, and so I did a practice talk, and it had to do with the two-slit experiment in relationship to spectroscopy. But I did a practice talk, and it took two hours.

So I have a thing about the two-slit experiment. And I think this lecture is going to be not two hours. It's going to be on time.

So we're going to talk about the two-slit experiment, and the important thing about the two-slit experiment is that it's mostly ordinary wave interference. There is no quantum mechanics, and so most of the hard stuff to analyze is classical physics. And I'm going to do the best I can with that.

But after we get to understanding this problem, there will be a surprise at the end, which is a quantum surprise. And it's something that absolutely requires a postulate, the first postulate of quantum mechanics, some idea of what we are talking about here. What are we allowed to know about a system? And it tells you there's something there.

And then I'm going to give you a little description of something which I think I have to call a semi-classical optics uncertainty principle. Semi-classical or semi-anything is usually quantum mechanics, just a little bit, mixed into something that was well understood before, or that is very convenient to use, and you only bring in quantum mechanics when you have to. And it's the easiest thing to understand. And that introduces the uncertainty principle, and we get a taste of the first of several quantum mechanical postulates.

So let's start with the sizes. I should have given you these sizes in the previous lecture, but I was being mostly number-free. So the radius of a Bohr orbit is given by n over z squared, where z is the integer charge on the nucleus, times 0.5292 angstroms.

So half an angstrom is the radius of, basically, any atom. And this charge on the nucleus, it gets smaller. And this is a very useful thing. The wavelength is given by n over z times 3.32 angstroms. And that's the situation where n times the wavelength is equal to 2 pi rn.

So we have n wavelengths around an orbit, and this is really the De Broglie hypothesis. But again, something which is on the order of something that you can remember.

And the energy levels-- these energy levels are z squared times 13.6 electron volts over n squared. If we were talking about energies in joules, or in any units you want, it's likely to have a big exponent. But this one is in electron volts, and that's actually what's happening. An electron is being attracted to a positive thing, and there's, basically, a voltage difference. And so that's another useful thing.

Now this z-- I left out the Rydberg. This Rydberg constant-- there's a bunch of fundamental constants, and since I'm a spectroscopist, I think in terms of wave numbers, reciprocal centimeters. And for me, that's energy, it's frequency, it's everything, but anyway, for hydrogen, it's 1097677.581.

For something with an infinite mass nucleus, it's 109737.3153 reciprocal centimeters. And this is the number that I have in my head, and I use it in all sorts of places, and you can imagine where. And to get to any particular nucleus, this is just r infinity mu nucleus, or mu atom, over the mass of the electron.

And it turns out that almost everything except hydrogen is very close to this number. And so this hardly matters, but it does give you a little bit of dependence on the nuclear mass.

So I said before that this Rydberg equation, or this equation, tells you nothing. It tells you where all the energy levels are, and anyone could tell you where the rest are. So it's a pattern, which is nice, but a pattern which says if things are well-behaved, like hydrogen atom, this is what the energy levels will be.

But life is difficult. Life is not with everything well-behaved, and so this is a pattern which says I'm interested in how the real life is different from that pattern. It's a way of thinking about structure and how we learn about structure. Information about the details of a molecule or an atom is encoded in the spectrum, and this is the magic decoder, or one of the magic decoders, we use to begin to assemble the new insights.

This doesn't appear in textbooks. In textbooks, you get the equations, you get the truth, and you don't get any strange interpretations. That's what you're getting from me. You're getting strange interpretations, and you'll have them throughout the course.

So now what we want to do is talk about the two-split experiment. Let's just begin. So here's a diagram. And here we have a source of light. It's a light bulb in your notes. It's a candle here.

And then we have two slits, and the slits are separated by distance d. And so this is the first slit, s1, and the second slit, s2. And the distance between them is much larger than the width of each slit.

And now we go down to the screen. And the distance from the slits to the screen is l, and l is much, much larger than d. This means, of course, we're going to be using small-angle approximation and simple solutions, because everything is much larger than something else.

And that's very convenient.

Now I want to just put on axes. So this is the x-axis, and this is 0, and this is l. And now the screen-- we're going to see something on the screen that looks like this. I'm giving it away. And the distance here is 0, and the distance here is the z-axis.

So the distance to this slit is on the x-axis, and the pattern, the diffraction pattern, is on the z-axis. And this 0 is right in the middle of the pattern. It would correspond to this point, the midpoint here.

So what we want to do is calculate what's going to appear on the screen, and I've already given it away. What you see is a bunch of equally-spaced intensity maxima where we have constructive interference. And in between, we have less constructive interference-- or destructive interference, and we want to understand that.

Now this is optics. This is no quantum mechanics at all. Let's look at this in more detail.

So we have the z-axis, horizontal, and we have a path to the screen, and another path to the screen. So what we're interested in is here's one slit, here's the other slit, and we have two parallel lines that meet at infinity, which is where the screen is-- and what we're going to be interested in is what is the path difference between this one and this one.

So we have an angle which is given by the perpendicular to this right. So this distance is d. This distance is L. And this angle is theta, as is this angle.

And what we're interested in is this-- the extra path traveled by the lower slit. So we use trigonometry, and we can figure that out. And so delta is the path difference. Delta is equal to d sine theta.

And in order for it to be constructive interference, we have to have this path difference to be an integer number of wavelengths. Now this is optics, and so that's something that we don't need quantum mechanics for. We know we're going to get interference.

So we can now solve for where the constructive interference occurs. And so we have theta n is equal to the inverse sine of n lambda over d. But this is a small angle, even though I drew it not as a small angle.

And so we can replace sine x by x or inverse of sine y by y. And anyway, we can say that the angles for constructive interference are given by n lambda over d. So we've derived the diffraction equation. We've solved.

And now what we want to know is where do the spots occur. They occur at z equals 0, z equals plus or minus l sine theta, which is approximately equal to l over d n lambda.

So we have l times lambda over d times an integer. So what we're going to see on the screen is a series of bright lines for constructive interference-- and they're not lines. It's a curve. But we can measure the maximum of the intensity, and we can say they're like this, and they're equally spaced.

And they tell us things we knew. We knew l. We knew d. We knew lambda. So there's nothing surprising. This is just optics. So now suppose we go in and we cover one slit. What happens? Yes?

AUDIENCE: The interference stops?

ROBERT FIELD: The interference goes away. And you could imagine that there would be a little sign. If you covered the top slit, the pattern would be skewed a little bit in the direction of the bottom slit.

And so there'll be a little bit of asymmetry, but you could actually know which slit your colleague covered. So if both slits are open, you have interference. If one slit is covered, you have no interference.

We're getting into the realm of quantum mechanics. In quantum mechanics, one of the things we do is we say, suppose we did a perfect experiment. Maybe it's an experiment that's beyond what you're capable of doing with the present technology, but you can say, I could measure positions in time as accurately as I want, and one could also say that I could do this an infinite number of times. I could do the same experiment an infinite number of times.

Without quantum mechanics, if you did the same experiment an infinite number of times, you'd get the same answer an infinite number of times. But with quantum mechanics, you're going to discover you don't.

It's probabilistic, not deterministic. And under certain conditions, the range over which you have a finite probability is very small, and it looks deterministic, but it isn't. The perfect experiment business is an interesting hypothesis, but you can imagine defining what is perfect in terms of what is intrinsically possible to achieve, even if it's not currently possible.

So what we want to do is decrease the intensity of the light that's going into the apparatus, so that there is never more than one photon in the apparatus. Never is a strong word, and we could never do that, and so that's not legal.

But we can say, suppose we decrease the intensity so that for the time it takes for the photon to go from the slit to the screen, which we know because we know the speed of light, and for the intensity of the light, which we can measure with an energy meter, we can say the probability of there being more than one photon at a time in the apparatus is small, as small as we want, but not zero.

And so then we do the experiment. And what we discover when we do the experiment is instead of having a uniform intensity or some kind of continuously varying intensity on the detector screen, we get a series of dots-- events. The photon went in, and the photon was a wave when it went in. There was interference, maybe, and the photon died on this detector screen.

This is an example of destructive detection, which is something that is very important in quantum mechanics, because in quantum mechanics, most measurements destroy the system, or destroy the state that the system was in during the experiment. So this business of what is the state of the system is a really important quantum mechanical concept, which you don't normally encounter in classic mechanics.

We send photons one at a time through the apparatus, and we get something like this. And we get something like this whether both slits are opened or one slit is covered. So we do this, and we do this for a long time, and what we see is we see a lot of events, and they're starting to arrange themselves where the interference fringes were supposed to be.

So this pattern only emerges after you'd allow a large number of photons to go into the apparatus. There is no way classical optics gives you that. And so if one slit is covered, you get a uniform distribution of dots. If both slits are open, you get this kind of a distribution.

I'm going to ask you to vote on this. So we do the experiment-- and I've sort of given away the answer, but I still want you to vote on this. What are the possible things?

So we know there's only one photon in the apparatus at a time. Our concept of interference is light interference with itself. And this is a possibility that says, I think I know.

And here is another, weak interference on top of constant background. This would reflect. Even though we decided that we would have one photon in the apparatus, occasionally there are two. And when there's two, there could be interference, and so we'd have some weak interference superimposed on the constant background. Now we get this 100% modulated interference structure.

And the last thing is something else. You have to transport yourself back in time to around 1910. You haven't heard this lecture, but you do know what the experiment is.

And so what would you expect? First, no interference, raise your hand. I've got one-- two-- I've got a few votes for no interference. In 1910, that's what you would have said.

Weak interference on top of constant background-- that would be when you're hedging your bets and saying, the experiment, it isn't perfect, and this is really the right answer, but if someone was a little sloppy, I'd get this.

Raise your hands for this one. What would you have said in 1910? I got nobody with the courage to say this. You would have gotten a Nobel Prize if you could have defended it. Something else-- maybe something else-- maybe the photons come in pairs or something ridiculous.

So the correct answer is 100% modulated. What people would have said in 1910 is this. Some curmudgeons, like the climate change deniers, would have said there is a little bit of this or maybe that, but I can't possibly accept that, which is the truth. We won't belabor that anymore.

This means that one photon can interfere with itself. It's a very disturbing idea, but it leads to a critical idea in quantum mechanics. In quantum mechanics, the state of the system is described by some state function, which is a function of position in time.

And so what happens is you prepare the system in some state. You do something to it, like force it to go through two slits. Then we get some new state function. And then we detect it, and we get something else.

So the actual experiment is a click, the preparation, and the click, detection. And somehow, what is the nature of the experiment is expressed on this initial state of the system. This is all very abstract, but it's about interference.

So this guy had better have phase. So we have a wave that can constructively or destructively interfere with itself. And so we start talking about things like amplitude, but the crucial word is amplitude.

And mostly, when we detect things, we're detecting probability. This is always positive. These guys can be positive and negative. This is essential for quantum mechanics. It's essential for understanding the two-slit experiment, but we have to do an awful lot more to make all this concrete.

Why don't we look at the classical wave equation? Actually, we'll look at the classical wave equation next time, but I will say that the solution to the wave equation is some function of x and t, which has the form a sine kx minus omega t.

So I'm doing this to introduce you to the crucial actors in this game, which is amplitude, wave number, and frequency. And this is a probability amplitude. It can have either sine, because you can see the sine function.

So this is a wave of frequency omega propagating in the plus x direction. Now let's just identify the crucial quantity. Wavelength is the repeat distance. So that if we said we have u of x and t, it has to be equal to u of x plus lambda and t.

So that's how we define the wavelength. And we discover that the wavelength has to be related to k times lambda is equal to 2 pi. Because this part has to change by 2 pi in order for there to be an exact replica of what we had before.

So we know that the wavelength is the repeat distance, and k is 2 pi over lambda. It's called wave number, and it's the number of waves, complete waves, that occur in 2 pi times the unit length. Now, in 3D, we have a vector as opposed to a number. And that points in the direction of propagation of the wave.

We know from quantum mechanics-- or from the experiments-- that the wavelength is related to the momentum. There's several reasons for this for waves. This could still be optics, but it could have been relativistic optics, because Einstein proposed that the momentum is e over c.

So we put that together, and we get the relationship between k and momentum. Now h bar is h over 2 pi. And so the wave number is large if the momentum is large, and the wavelength is small if the momentum is large.

Now what about the velocity? So we have a wave. Let's sit on this wave here and say, we're now sitting on some point where the phase is constant. I like to call this the stationary phase point, but that has other meanings, so you just have to be careful with this.

So we want to know how fast this moves in space. And so we say, the phase is the phase of this function here, and so it's k x-- I'll put a little phi on this-- minus omega t. And we want to find how this moves in time, this stationary phase.

We could choose this to be zero. We could choose a phase. This is zero, this is some maximum, but we can choose anything we want. So let's make it zero. And so then we can solve for x phi is a function of t, and that's omega t over k.

We want the phase velocity, the velocity of this wave. So we take the derivative with respect to t. And so the velocity, which we'll call c, is equal to omega over k. That's true for light traveling in a vacuum.

This is called the dispersion relation. Whenever you do a calculation of waves in material, the goal is to get the dispersion relation. This is the simplest possible one, because it says everybody at the same frequency-- I'm sorry-- that there is a relationship between omega and k so that everybody travels with the same speed.

If that didn't happen, the waves would get out of phase with each other. That's what's dispersion is. So we have from simple optics, basically, everything we need.

And now, the last thing is that the intensity of this wave is given by kkix minus omega i t. So this is the wave function. It's something that has sines.

And this is the intensity, which is proportional to this quantity. We sum the amplitudes and then square. We do this in quantum mechanics all the time.

And this is like quantum mechanics in the sense that we have our fundamental building block, which is something with phase. And the relationship of the thing with phase to the thing which is probability is sum of the square. Yes?

AUDIENCE: What's the first character after the capital sigma?

ROBERT FIELD: I'm sorry?

AUDIENCE: The first character after the sum there?

ROBERT FIELD: This? No, this. That's a constant. That's the amplitude of that particular frequency and wave vector. I'm sorry about the mess on the board. We have enough time. Oh, good.

So this is sort of a taste of what you're going to be doing, but now let's produce a form of the uncertainty principle. You've heard about the uncertainty principle, and it's a very important part of quantum mechanics, but it's also something that you can have in optics.

And so again, we resort to this simple idea of sending a particle through a slit and looking at what happens over here. Instead of having two slits, we just have one. But there are two important things.

The two edges of this slit are special, because they're edges. And so we can ask, what about the interference between particles that was diffracted by this edge versus that edge.

And the same thing goes. You get constructive interference when the difference in path length is an integer number of waves, and destructive interference when it's an odd integer number of half waves. We're interested in the destructive interference.

So we analyze this in exactly the same way we did the two-slit experiment. And now this is the z direction. And this is the x direction.

And what you end up finding is that the uncertainty in the z direction is going to be related 2 times lambda over delta s over l. Delta s is the width of the slit, so width of the image along the z-axis. So we have a maximum here, and we have minima here and here, and so this delta z is the distance between minima.

So we can say, between minima, we have the particle localized. Its position in space-- or the photon localized-- its position in space is uncertain by this quantity.

What about its momentum? And so now we just have to draw a little bit of conservation of momentum. So here we have the magnitude of the momentum starting from the middle of the slit, and this magnitude is constant along a circle-- the magnitude.

So if we do draw a line here, this has got p. And if we ask the length here, that's also p. But now what we want to know is what is the uncertainty of the momentum in the z direction-- delta pz.

This is the same sort of calculation we did before, and what you find is delta pz is approximately equal to magnitude of p lambda or delta s. And now we put in the Bohr relationship here, and we get that this is equal to h over lambda, lambda over the ds, which is equal to h over ps.

Or ds in the z direction-- no, ds is the slit width. I got something wrong here in my notes. The final result of this calculation is dz is dpz approximately equal to h. Sorry about the glitch here. I don't know how to fix it right now, but if this were done correctly, we would have gotten this result.

This is an uncertainty principle. If you tried to measure z and pz simultaneously with classical optics, you still would get something like this. Using the relationship for momentum that lambda is equal h over p, we've got to use that. But because of that relationship, we get this result.

If you do a perfect experiment, and you make the slit smaller and smaller, you make the uncertainty in the momentum larger and larger. The best you can do is this.

Now, this is actually a reasonable and rigorous derivation if I had done it a little better. But I've never liked this introduction to the uncertainty principle, because it says, for the kind of experiment we thought about, you can't do better than this.

Maybe there's a different kind of experiment. It's sort of an artifactual as opposed to a physical derivation. We care. We will do a physical derivation of the uncertainty principle. And it will have to do with the ability of two operates to commute with each other. That's a purely mathematical definition, but this is the first sign of the uncertainty principle.

At the end of your notes, there is a set of postulates from which, essentially, all quantum mechanics can be derived. Now, there are different sets of postulates proposed by different people, but these are things that can't be proven. They are things that you think is going to be true, and then you look at the consequences.

The first postulate says, the state of a quantum mechanical system is completely specified by a function, psi of r and t, that depends on the coordinates of the particle and on time. This function, called the wave function or the state function, has the important property that this quantity times its complex conjugate integrated over the volume element is the probability that the particle lies in the volume element centered at r at time t.

So we are saying there is a way, a complete way, of describing the state of the system. It's a function. It's not a bunch of discrete quantities, like velocity and position.

And things are smeared out. And what we want to do when we want to calculate anything, we're going to be using this function. And this function comes from the Schrodinger equation. And we're going to get to the Schrodinger equation-- not in the next lecture. I'm going to spend the next lecture really laboring the wave equation, because the Schrodinger equation is just a tiny step beyond the wave equation.

I've given you the five postulates. You have not a clue what any of them mean, except maybe a little bit about the first one. But what I don't want to do is give a lecture on the postulates. I want to bring them into action when you need them, because they'll mean much more.

And so you won't be asked to memorize the postulates. You'll know when they are applicable and how to apply them. So it's a little premature to say you understand the first postulate, but that's what's at play here in the two-slit experiment and in this hand waving derivation of the uncertainty principle.

So next time, we will look at the wave equation. Thank you. Hey, I finished on time this time. That's a bad sign.