

- Reversible Adiabatic Expansion (or compression) of an Ideal Gas

$$1 \text{ mole gas } (V_1, T_1) = 1 \text{ mole gas } (V_2, T_2)$$

$$\begin{array}{ll} \text{adiabatic} \Rightarrow \delta q = 0 & \text{Reversible} \Rightarrow \delta w = -pdV \\ \text{Ideal gas} \Rightarrow & dU = C_V dT \end{array}$$

$$\therefore \text{ From 1}^{\text{st}} \text{ Law } dU = -pdV \Rightarrow C_V dT = -pdV \text{ along path}$$

$$C_V dT = -pdV \xrightarrow{p = \frac{RT}{V}} C_V \frac{dT}{T} = -R \frac{dV}{V}$$

$$C_V \int_{T_1}^{T_2} \frac{dT}{T} = -R \int_{V_1}^{V_2} \frac{dV}{V} \Rightarrow \left(\frac{T_2}{T_1} \right) = \left(\frac{V_1}{V_2} \right)^{R/C_V} \xrightarrow{C_p - C_V = R \text{ for i.g.}} \left(\frac{T_2}{T_1} \right) = \left(\frac{V_1}{V_2} \right)^{\frac{C_p}{C_V} - 1}$$

$$\text{Define } \gamma \equiv \frac{C_p}{C_V} \Rightarrow \boxed{\left(\frac{T_2}{T_1} \right) = \left(\frac{V_1}{V_2} \right)^{\gamma - 1}}$$

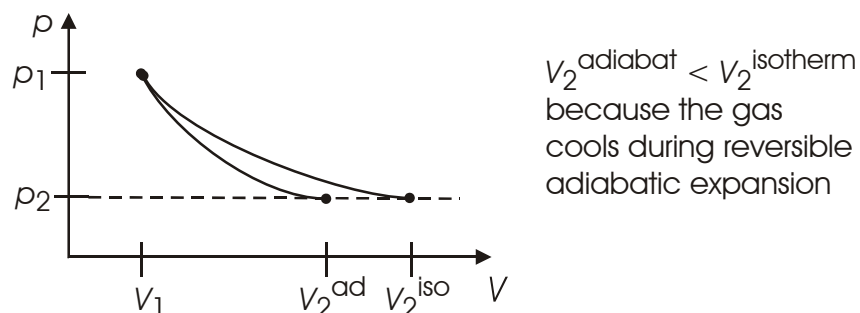
$$\text{For monatomic ideal gas: } \left. \begin{array}{l} C_V = \frac{3}{2}R \\ C_p = \frac{5}{2}R \end{array} \right\} \gamma = \frac{5}{3} \text{ (> 1 generally)}$$

In an adiabatic expansion ($V_2 > V_1$), the gas cools ($T_2 < T_1$).
And in an adiabatic compression ($V_2 < V_1$), the gas heats up.

$$\text{For an ideal gas (one mole)} \quad T = \frac{pV}{R} \Rightarrow \left(\frac{p_2}{p_1} \right) = \left(\frac{V_1}{V_2} \right)^\gamma \Rightarrow \boxed{p_1 V_1^\gamma = p_2 V_2^\gamma}$$

$$\boxed{pV^\gamma \text{ is constant along a reversible adiabat}}$$

$$\text{For an isothermal process} \quad T = \text{constant} \Rightarrow pV = \text{constant}$$



- Irreversible Adiabatic Expansion of an ideal gas against a constant external pressure

$$1 \text{ mol gas } (p_1, T_1) = 1 \text{ mol gas } (p_2, T_2) \quad (p_{\text{ext}}=p_2)$$

$$\begin{aligned} \text{adiabatic} &\Rightarrow dq = 0 \\ \text{Constant } p_{\text{ext}} = p_2 &\Rightarrow dw = -p_2 dV \\ \text{Ideal gas} &\Rightarrow dU = C_V dT \\ \text{1}^{\text{st}} \text{ Law} &\Rightarrow dU = -p_2 dV \end{aligned}$$

$$\therefore C_V dT = -p_2 dV$$

$$\text{Integrating: } C_V (T_2 - T_1) = -p_2 (V_2 - V_1)$$

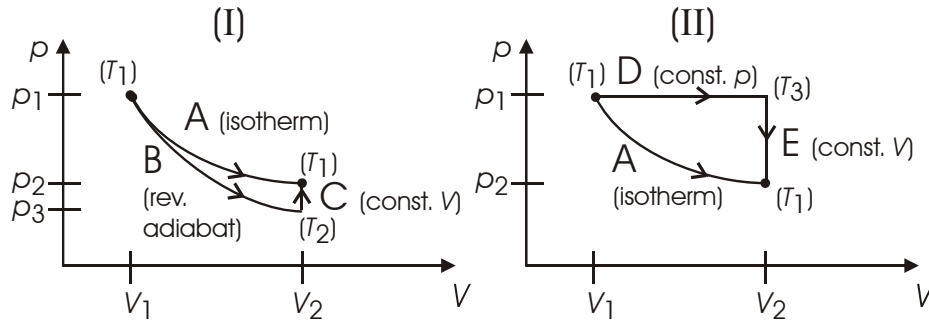
$$\text{Using } pV = RT \quad T_2 (C_V + R) = T_1 \left(C_V + \frac{p_2 R}{p_1} \right)$$

$$\text{Note } p_2 < p_1 \Rightarrow T_2 < T_1 \quad \text{Again, expansion cools}$$

Note also $(-w_{\text{rev}}) > (-w_{\text{irrev}})$ Less work is recovered through an irreversible process

Some Thermodynamic Cycles

- Reversible Ideal Gas processes: Find ΔU , ΔH , q , w , $\int \frac{dq}{T}$



$$[A] \quad 1 \text{ mol gas } (p_1, V_1, T_1) \stackrel{\text{const. } T}{=} 1 \text{ mol gas } (p_2, V_2, T_1)$$

Ideal gas isotherm:

$$\Delta U_A = 0$$

$$\Delta H_A = 0$$

$$w_A = -RT_1 \ln \frac{V_2}{V_1} \quad q_A = RT_1 \ln \frac{V_2}{V_1} \quad \int \frac{dq}{T} = R \ln \frac{V_2}{V_1}$$

$$[B] \quad 1 \text{ mol gas } (p_1, V_1, T_1) \stackrel{\text{rev. adiabat}}{=} 1 \text{ mol gas } (p_3, V_3, T_2)$$

Adiabat:

$$q_B = 0$$

Ideal gas:

$$\Delta U_B = C_V (T_2 - T_1)$$

$$\Delta H_B = C_P (T_2 - T_1)$$

1st Law:

$$w_B = C_V (T_2 - T_1)$$

$$\int \frac{dq_B}{T} = 0$$

[C] 1 mol gas (p_3, V_2, T_2) $\stackrel{\text{reversible}}{=}_{\text{const. } V}$ 1 mol gas (p_2, V_2, T_1)

Constant V :

$$w_C = 0$$

Ideal gas:

$$\begin{aligned} \Delta U_C &= C_V (T_1 - T_2) \\ \Delta H_C &= C_p (T_1 - T_2) \end{aligned}$$

1st Law:

$$q_C = C_V (T_1 - T_2)$$

$$\int \frac{\dot{d}q_C}{T} = C_V \ln\left(\frac{T_1}{T_2}\right)$$

[A]

vs.

[B] + [C]

$$\Delta U_A = 0$$

$$\Delta H_A = 0$$

$$\Delta U_B + \Delta U_C = 0 = \Delta U_A$$

$$\Delta H_B + \Delta H_C = 0 = \Delta H_A$$

$$q_A = RT_1 \ln \frac{V_2}{V_1}$$

$$w_A = -RT_1 \ln \frac{V_2}{V_1}$$

$$\int \frac{\dot{d}q_A}{T} = R \ln \frac{V_2}{V_1}$$

$$q_B + q_C = C_V (T_1 - T_2) \neq q_A$$

$$w_B + w_C = C_V (T_2 - T_1) \neq w_A$$

$$\int \frac{\dot{d}q_B}{T} + \int \frac{\dot{d}q_C}{T} = R \ln\left(\frac{V_2}{V_1}\right) = \int \frac{\dot{d}q_A}{T}$$

This result suggests that $\left(\int \frac{\dot{d}q_{\text{rev}}}{T}\right)$ is a state function!

[D]

$$\Delta U_D = C_V (T_3 - T_1)$$

$$\Delta H_D = C_p (T_3 - T_1)$$

$$q_D = C_p (T_3 - T_1)$$

$$w_D = -R(T_3 - T_1)$$

$$\int \frac{\dot{d}q_D}{T} = C_p \ln\left(\frac{T_3}{T_1}\right)$$

[E]

$$\begin{aligned}\Delta U_E &= C_V (T_1 - T_3) \\ \Delta H_E &= C_p (T_1 - T_3)\end{aligned}$$

$$w_E = 0$$

$$q_E = C_V (T_1 - T_3)$$

$$\int \frac{\mathrm{d}q_E}{T} = C_V \ln\left(\frac{T_1}{T_3}\right)$$

[A]

vs.

[D] + [E]

$$\Delta U_A = 0$$

$$\Delta U_D + \Delta U_E = \Delta U_A$$

$$\Delta H_A = 0$$

$$\Delta H_D + \Delta H_E = \Delta H_A$$

$$q_A = RT_1 \ln \frac{V_2}{V_1}$$

$$q_D + q_E = R(T_3 - T_1) \neq q_A$$

$$w_A = -RT_1 \ln \frac{V_2}{V_1}$$

$$w_D + w_E = -R(T_3 - T_1) \neq w_A$$

$$\int \frac{\mathrm{d}q_A}{T} = R \ln \frac{V_2}{V_1}$$

$$\int \frac{\mathrm{d}q_D}{T} + \int \frac{\mathrm{d}q_E}{T} = R \ln\left(\frac{V_2}{V_1}\right) = \int \frac{\mathrm{d}q_A}{T}$$

Here again $\left(\int \frac{\mathrm{d}q_{\text{rev}}}{T}\right)$ looks like a state function!