1 Energy, Frequency, and Wavelength

Last time, we discussed electromagnetic waves and how light is quantized as photons. We related the energy of a photon to its frequency and wavelength using the Planck-Einstein relation:

$$E = h\nu = \frac{hc}{\lambda}$$

In 3.091, the most common units we will use for energy are joules (J) or electron volts (eV). A joule is equivalent to a kg$\times\frac{\text{m}^2}{\text{s}^2}$, which is like a force integrated over a distance. Electron volts are also a unit of energy, but they are much smaller than a joule. An eV is literally the charge of one electron multiplied by one volt! We can convert between joules and electron volts using

$$1 \text{ eV} = 1.6022 \times 10^{-19} \text{ J}$$

Depending on the problem, it may be easier to work in J or eV. Either one works - double checking how your units work out is a great way to check your answer.

The units of frequency (ν) are $1/s$, or equivalently, Hertz (Hz), and the wavelength (λ) has units of m. Now, we can consider the speed of light, c. Last time, we said $c = \lambda\nu$: using the units above, we see that the speed has units m/s as it should!

Frequently, we will use units like nanometers (nm) or microns (μm) for small length scales: these can be converted to meters using the following conversion factors:

$$1 \text{ nm} = 10^{-9} \text{ m}$$

$$1\ \mu\text{m} = 10^{-6} \text{ m}$$

The last piece of the Planck-Einstein relation is the h: Planck's constant! Planck's constant is

$$h = 6.626 \times 10^{-34} Js = 4.136 \times 10^{-15} eVs$$

It is important that you use the version of h with the units of energy in a problem!

Example: What is the energy and wavelength of light that has a frequency of 440 THz?

$$E = h\nu = (4.136 \times 10^{-15} \text{ eVs})(440 \text{ THz})\left(\frac{10^{12} \text{ Hz}}{1 \text{ THz}}\right) = 1.82 \text{ eV}$$

$$\lambda = \frac{c}{\nu} = \frac{3 \times 10^8 \text{ m/s}}{440 \times 10^9 \text{ 1/s}} = 6.81 \times 10^{-7} \text{ m} = 681 \text{ nm}$$

2 Ionization

If a photon with sufficient energy is absorbed by an atom, the atom can become *ionized*: it loses an electron! In fact, when we discussed Bohr's model, the factor -13.6 eV is actually the *first ionization energy* of the hydrogen atom! That means it takes $13.6eV$ of energy to ionize an electron in the ground state- that is, to excite an atom in the n=1 state so much it leaves the atom. The final energy state of a Bohr model electron that is ionized can be thought of as the limit as $n_f \to \infty$:

$$\Delta E_{ionize} = \lim_{n_f \to \infty} (-13.6\ [\text{eV}])\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) = \frac{13.6\ \text{eV}}{n_i^2}$$

Example: A photon with a wavelength of $4.5\mu m$ strikes a hydrogen atom with an electron in an unknown energy level. The electron is then ejected from the atom, and flies through space. Determine a) the minimum energy state the electron could have been in, and how much energy would have been leftover, b) the velocity of the electron right when it leaves. Then, c) assume a beam of $4.5\mu m$ light shines on many hydrogen atoms in the state you determined in part a). If the beam power is 25mW, how many photons are ejected each second?

a)

$$E_{photon} = \frac{hc}{\lambda} = \frac{(4.14 \times 10^{-15}\text{eV s})(3 \times 10^8\ \text{m/s})}{4.5 \times 10^{-6}\ \text{m}} = 0.2757\ \text{eV}$$

$$E_{photon} = -13.6\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) = 13.6\ \text{eV}\left(\frac{1}{n_i^2}\right) = 0.2757\ \text{eV}$$

Solving this, we get $n_i = 7.02$ eV. The electron would have to be just above the 7th energy level – since it must be in an integer energy level, the lowest initial state is $n_i = 8$. The energy to ionize from the 8th energy level is

$$\Delta E_{8,\infty} = -13.6\left(-\frac{1}{8^2}\right) = 0.2125eV$$

It takes 0.2125 eV to ionize from $n_i = 8$. The remaining energy is leftover:

$$E_{left} = 0.2757\ \text{eV} - 0.2125\ \text{eV} = 0.063\ \text{eV}$$

The energy leftover is not absorbed by the atom, but it must go somewhere! One place it could go is to the kinetic energy of the electron that was ionized.

b) Assuming all the leftover energy becomes kinetic energy,

$$E = \frac{1}{2}mv^2$$

We need to convert to Joules to get a velocity in reasonable units:

$$0.063\ \text{eV}\frac{1.602 \times 10^{-10}\ \text{J}}{1\ \text{eV}} = 1.009 \times 10^{-20}\ \text{J} = \frac{1}{2}(9.11 \times 10^{-31}\text{kg})v^2$$

$$v = 1.5 \times 10^5\ \text{m/s}$$

c) First, let's break down the beam power:

$$25\ \text{mW}\left(\frac{1\ \text{W}}{1000\ \text{mW}}\right)\left(\frac{1\ \text{J/s}}{1\ \text{W}}\right) = 0.025\ \text{J/s}$$

We can then solve for the energy of each photon in joules:

$$E = \frac{hc}{\lambda} = \frac{(6.626 \times 10^{-34}\ \text{Js})(3 \times 10^8\ \text{m/s})}{4.5 \times 10^{-6}\ \text{m}} = 4.4 \times 10^{-20}\ \text{J/photon}$$

$$\frac{\text{photons}}{\text{s}} = \frac{\frac{\text{J}}{\text{s}}}{\frac{\text{J}}{\text{photon}}} = \frac{0.025\ \text{J/s}}{4.4 \times 10^{-20}\ \text{J/photon}} = 5.68 \times 10^{17}\ \text{photons/s}$$