Engineering Economics: Session 3

Example Problem: Geometric Series

- Facility has aging cooling system which currently runs 70% of the time the plant is open
  – Pump will only last 5 more years. As it deteriorates, the pump run time is expected to increase 7% per year
- New cooling system would only run 50% of the time
- Assumptions
  – Either pump uses 250 kWh, Electricity cost $0.05/KWh
  – Plant runs 250 days per year, 24 hours per day
  – Firm's discount rate is 12%
- What is the value of replacing the pump?

Example Problem: Geometric Series

- **Current pump power cost =** 70% x 250kWh x \$0.05/kWh x 250 days x 24 hrs/day = \$52,500
- New pump power Cost = \$37,500

$$P_{Old} = \$52,500\left(\frac{1-(1.07)^{5}(1.12)^{-5}}{0.12-0.07}\right)$$

$$= \$214,400$$

$$P_{New} = \$37,500(P/A,12\%,5)$$

$$P_{New} = \$37,500(3.605)$$

$$= \$135,200$$

$$Value = P_{Old} - P_{New} = \$79,160$$

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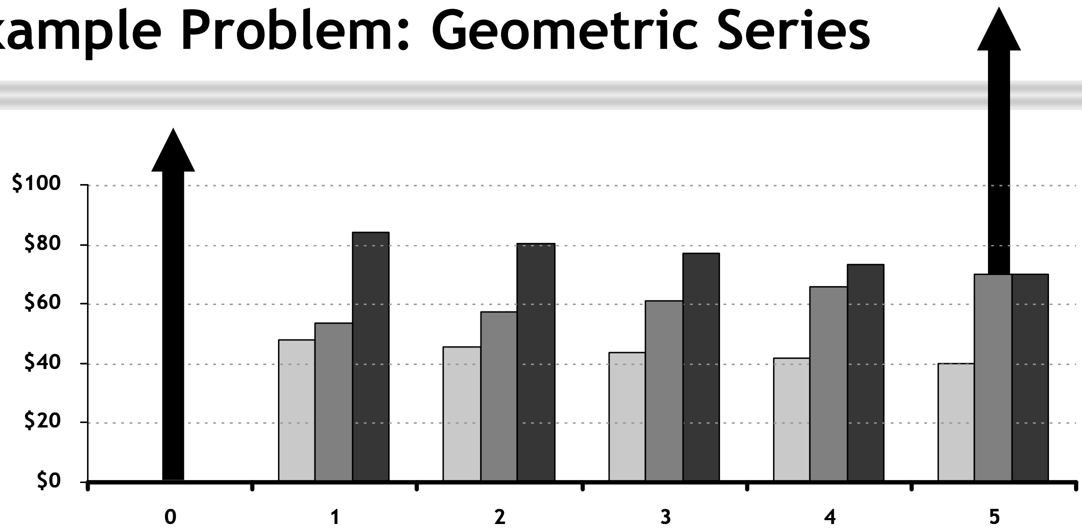
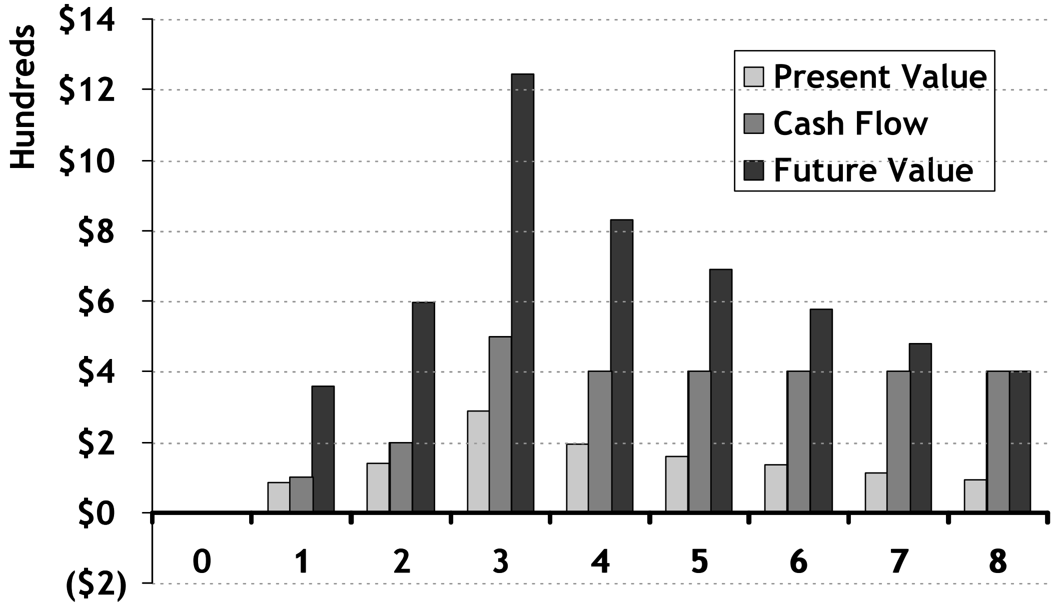
Example Problem: Geometric Series



Energy Cost	-	A_1	$A_1(1+g)$	$A_1(1+g)^2$	$A_1(1+g)^3$	$A_1(1+g)^4$
Energy Cost	$0.00	$52.50	$56.18	$60.11	$64.31	$68.82
Present Value	$0.00	$46.88	$44.78	$42.78	$40.87	$39.05
Future Value	$0.00	$82.61	$78.92	$75.40	$72.03	$68.82

Combining Concepts



Effective Interest Rate

- **So far -- assumed compounding at end of period** (& mostly 1year periods)
- **What if compounding happens more frequently?**

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Screenshot of bank website page describing its savings products, highlighting the statement "Daily compounding interest with tiered interest rate structure."

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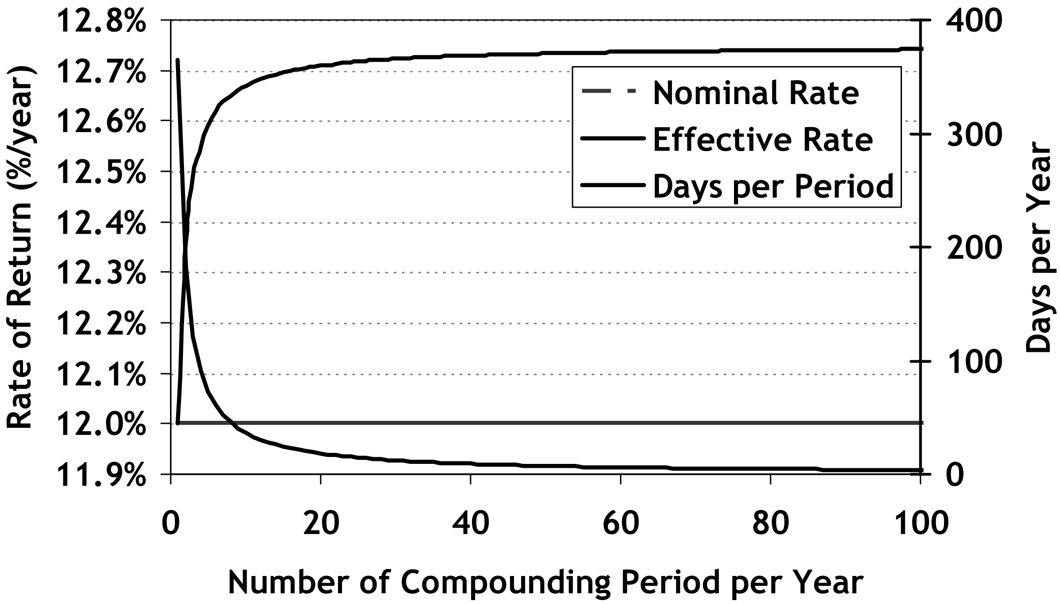
Screenshot of Bankrate.com mortgage comparison table; highlighting the difference between published Rate and APR.

Effective Interest Rate

- By convention, most interest rates are quoted on an annual basis
  - e.g., 2% interest, with an interest period of 1 month
    → stated as 24% compounded monthly
- How does this differ from 24% compounded annually?
  - F@1month = P(1.02)
  - F@2 months = P(1.02)(1.02)
  - F@12months = $P(1.02)^{12}$ = 1.2682P
  - Effective interest rate = 26.82%
- Effective interest rate (i_e)
  r = nominal annual interest rate
  M = compounding periods per year

$$i_e = (1 + \frac{r}{M})^M - 1$$

The impact of compounding



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3.080 Econ & Enviro Issues In Materials Selection
Randolph Kirchain

Engineering Economic Analysis: Slide 54

Effective Interest Rates

When you want to express i_e annually:

$$i_e = (1 + \frac{r}{M})^M - 1$$

Or more generally, for an arbitrary period:

$$i_e = (1 + \frac{r}{CK})^C - 1$$

C = The number of interest periods per payment period
K = The number of payment periods per year
Note that r is still assumed to be expressed annually

Suppose you deposit into your retirement account $1,000 at the end of each year

The retirement account compounds quarterly,
Nominal rate = 12%

What is the value after 10 years?

Off Period Compounding

- **Payments more frequent than compounding**
  - **Either**
    - Use effective rate for payment period, or
    - Aggregate payments within a given period
      - Use this only if, interest doesn't accrue during period
- **Payments less frequent than compounding period**
  - **Compute**
    - Effective interest rate for period
    - Effective payment for interest rate

Example of off period compounding:
Cash Flows Less Frequent then Compounding

- Suppose you deposit into your retirement account $1,000 at the end of each year
- The retirement account compounds quarterly
  - Nominal rate = 12%
- What is the value after 10 years?

Example of off period compounding:
Cash Flows Less Frequent then Compounding (cont)

- **Equivalent rate**

- **Value of payments after 10 years**

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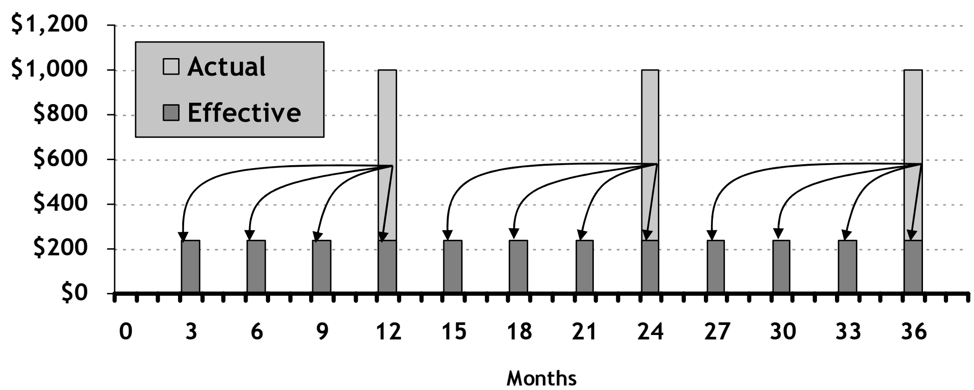
Example of off period compounding:
Cash Flows Less Frequent then Compounding (cont)

- **Equivalent payment**



Very Small Compounding Periods

$$i_e = (1 + \frac{r}{M})^M - 1$$

$$\text{let } x = \frac{M}{r}$$

$$i_e = \left(\left(1 + \frac{1}{x}\right)^x\right)^r - 1$$

$$\text{As } M \to \infty, x \to \infty$$

$$\lim_{x \to \infty} i_e = \lim_{x \to \infty}\left[\left(\left(1 + \frac{1}{x}\right)^x\right)^r - 1\right] = e^r - 1$$

$$F = P(1+i)^N$$

$$= P(1 + e^r - 1)^N$$

$$= Pe^{rN}$$

$$F = Pe^{rn}$$

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Using Spreadsheets: Two Key Strategies

- Parametrize
  - Try to always put values in their own cells
  - Try not to embed values in formulae
- Rely on Data Tables for sensitivity analysis

Using Spreadsheets:
Excel Formulae for Equivalence

Type	Notation	Formula	Excel
Single	Compound Amount (F/P,i,N)	$F = P(1+i)^N$	FV(i,N,,P)
	Present Worth (P/F,i,N)	$P = F/(1+i)^N$	PV(i,N,,P)
Uniform Series	Compound Amount (F/A, i, N)	$F = A\left(\frac{(1+i)^N - 1}{i}\right)$	FV(i,N,A)
	Sinking Fund (A/F, i, N)	$A = F\left(\frac{i}{(1+i)^N - 1}\right)$	PMT(i,N,0,F)
	Present Worth (P/A, i, N)	$P = A\left(\frac{(1+i)^N - 1}{i(1+i)^N}\right)$	PV(i,N,A)
	Capital Recovery (A/P, i, N)	$A = P\left(\frac{i(1+i)^N}{(1+i)^N - 1}\right)$	PMT(i,N,P)
Linear Gradient	Present Worth (P/G, i, N)	$P = G\left(\frac{(1+i)^N - iN - 1}{i^2(1+i)^N}\right)$	*manual*
Geometric Gradient	Present Worth (P/A_1,g, i, N)	$P = \begin{cases} A_1\left(\frac{1-(1+g)^N(1+i)^{-N}}{i-g}\right) \\ \frac{NA_1}{(1+i)}, \text{if } i = g \end{cases}$	*manual*

Engineering Economic Analysis: Slide 63

Evaluating Alternatives

Is This Worth Doing?

Evaluating “Projects” or Alternatives

- Does a project provide benefits (revenues) that exceed
  - Equivalent expenditures
  - Return on capital

Over a 5 year period, your firm can make $22,000 by running copy service. The associated costs are:

$4,000 to buy copier (salvage value of $1,000)

$2,200 to maintain and operate copier

Does it make sense to invest in this business?

Evaluating “Projects” or Alternatives

- Does a project provide benefits (revenues) that exceed
  - Equivalent expenditures
  - Return on capital
- Minimum Attractive Rate of Return (MARR)
  - Cost of using funds (e.g., if borrowed)
  - Return from other available options
  - Risk of project under consideration

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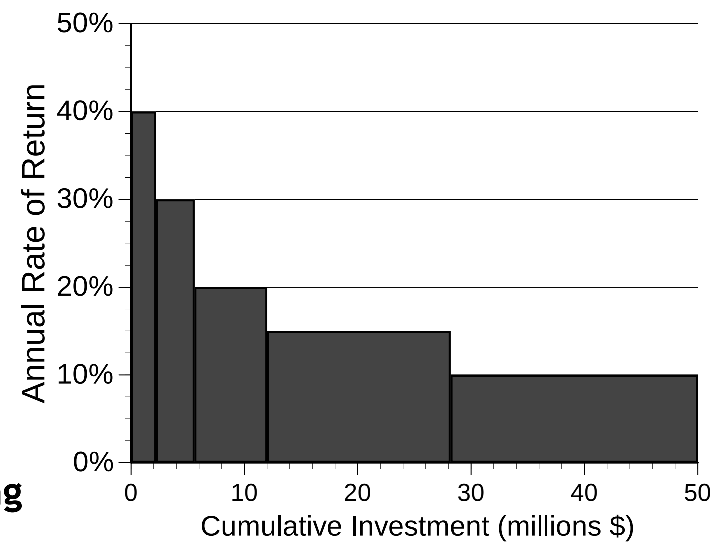
Determining MARR
Logical Method: Capital Rationing

- **Insufficient capital to fund all projects**
  - **Actually, capital available, but at escalating cost**
- **MARR comes from**
  - **return of best *rejected* project**
  - **with return *greater* than cost of acquiring capital**



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Determining Equivalence: Present Worth Method – Net Present Value

- Present Worth (PW) Method – comparing equivalent worth of all cash flows at present time
- Positive PW =
  Project has return greater than MARR
  - PW measures how much money one could afford to pay for the investment in excess of its cost
  - Positive PW for an project is amount of profit over the minimum amount required

Computing Present Worth

- To compute present worth
  - Calculate discounted cash flow
    (i.e., present worth of each cash flow)
  - Sum all discounted cash flows
- Project is worth considering if PW > 0

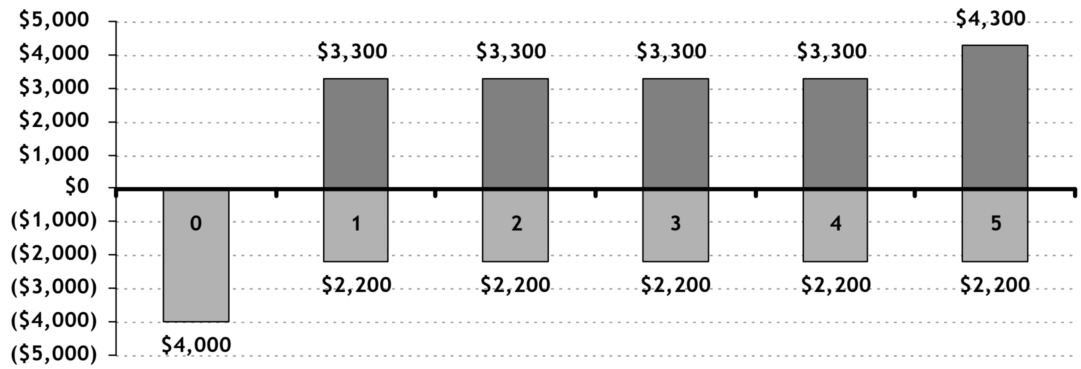
Over a 5 year period, your firm can make $22,000 by running copy service. MARR=12% The associated costs are:

$4,000 to buy copier (salvage value of $1,000)

$2,200 to maintain and operate copier

Present Worth Example: Invest in the Copier?

- How does this set of expected cash flows compare to simply investing $4,000 at 12%?



Determining Equivalence
Annual Worth Method

- **Annual Worth (AW) of a project is the series of equals payments with value equivalent to that project at the MARR**
  - **AW = Annual Equivalent Revenues**
    - Annual Equivalent Expenses
    - Annual Equivalent Capital Recovery
  - **Capital Recovery**
    = Equivalent uniform annual cost of capital invested
    - **Loss in value of the asset**
    - **Return on invested capital**

Annual Equivalent – Calculating Capital Recovery

- AW = R – E - Capital Recovery
- Capital recovery =
  Annual Equivalent of Investment
  - Annual Equivalent of Salvage Value

Annual Worth Example

- Producing cars is highly capital intensive with a new car factory costing nearly $1 billion dollars
- A new technology has been developed that speeds the assembly of vehicles. It has the following characteristics:
  - Current plant - 30 stations, output = 200k per year
  - Profit = $1000 per vehicle
  - Investment - $300k per station, 30 stations per plant
  - Salvage value $60k per station
  - Maintenance cost - $20k / station / year
  - Improved throughput - 1%
  - Investments replaced after 5 years
  - MARR = 15%

Should you install this tech?

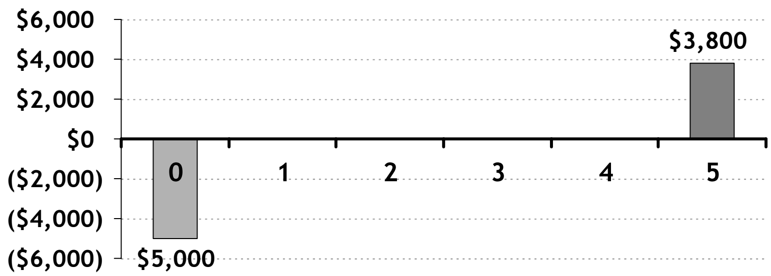
Annual Worth Example (cont)

Determining Equivalence: Internal Rate of Return (IRR)

- IRR method solves for the rate of return for which

  Equivalent worth for cash inflows =
  Equivalent worth of cash outflows
  - i.e., Present Worth = Annualized Worth = 0
- This rate is called the IRR
- If IRR < MARR, then project is NOT acceptable
- If IRR ≥ MARR, then project IS acceptable

IRR Example



- What interest rate will make worth of inflows = outflows?

Issues with the IRR method

- IRR assumes that cash flows can be reinvested at the IRR over course of study
  - For projects with IRR >> MARR this may be unrealistic
- Can have multiple IRR for cash flows, if net cash flows reverse signs more than once over study period

Payback Period

- Simple payback
  - Number of periods (θ) for cash inflows to equal / exceed cash outflows

$$\sum_{k}^{\theta}(R_k - E_k) \geq I$$

where: R_k is revenues in period k
E_k = expenses in period k
I = initial investment

- Payback period does not indicate profitability
  - Represents liquidity
  - Used to estimate risk → lower = less risk