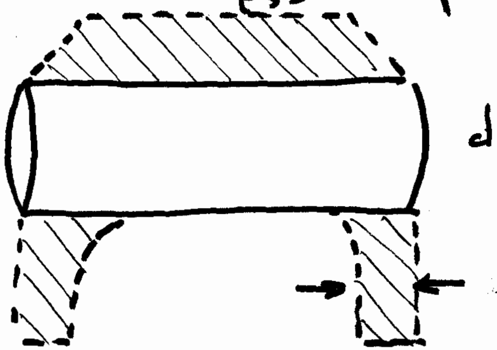
Short-fiber composites

- Easier fabrication, especially complex curvatures
- Critical fiber length

$\sigma_{f,b}$ - fiber failure stress

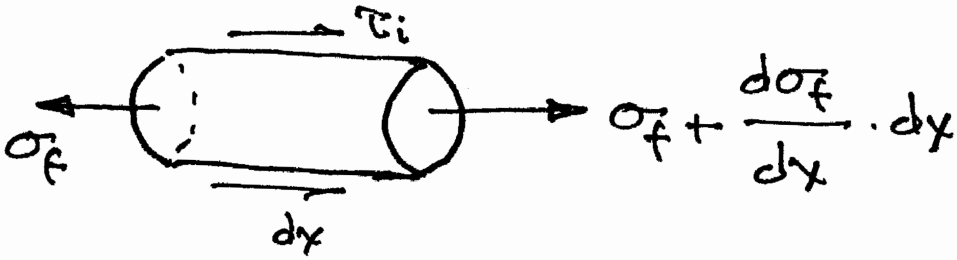


δ^* = debond length

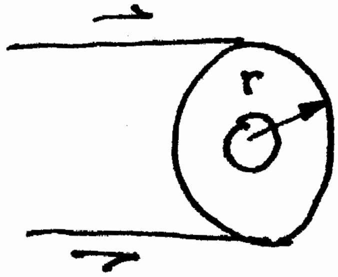
τ_i = interfacial shear strength

$$l_c = 2\delta^* = \frac{\sigma_{f,b}}{2\tau_i}$$

Shear Lag Theory (Cox)



$$\left(\frac{d\sigma_f}{dx} \cdot dx\right)\frac{\pi d^2}{4} = -\tau_i (\pi d)\, dx \rightarrow \frac{d\sigma_f}{dx} = -\frac{4}{d}\tau_i$$



shear force per unit length at r

$$2\pi r \cdot \tau = \pi d \cdot \tau_i \rightarrow \tau = \frac{d}{2r} \cdot \tau_i$$

shear strain in matrix:

$$\gamma = \frac{\partial u}{\partial r} = \frac{\tau}{G} = \frac{d}{2r} \cdot \frac{\tau_i}{G}$$

$$u = \int \frac{du}{dr} \cdot dr \rightarrow u_1 - u_f = \frac{d}{2}\frac{\tau_i}{G} \ln\frac{2R}{d}$$

$$\rightarrow \left(\frac{d}{2n}\right)\frac{d^2\sigma_f}{dx^2} - \sigma_f = -E_f \epsilon_1, \quad n = \sqrt{\frac{2G_m}{E_f \ln\frac{2R}{d}}}$$

$$\sigma_f = E_f \epsilon_1 + C \sinh\frac{2nx}{d} + D \cosh\frac{2nx}{d}$$

b.c.: $\sigma_f = 0$ @ $x = \pm \ell/2$

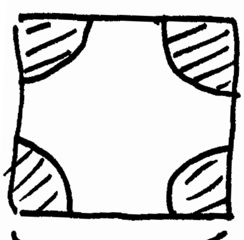
$$\sigma_f = E_f \epsilon_1 \left[1 - \frac{\cosh\left(na\frac{2x}{\ell}\right)}{\cosh(na)}\right] \qquad a = \ell/d$$

Shear Lag Theory

$$E_1 = \eta v_f E_f + v_m E_m \quad \begin{cases} \eta = 1 - \dfrac{\tanh(na)}{na} \\ n = \sqrt{\dfrac{2G_m}{E_f \ln\left(\frac{2R}{d}\right)}} \\ a = l/d \end{cases}$$

eg 30% E-glass in N66

$\tau_i = 20$ MPa, $d = 12\,\mu$, $l = 1$ mm



$d/2$

$2R$

$$v_f = \frac{\frac{\pi}{4}d^2}{(2R)^2} \rightarrow \frac{2R}{d} = \sqrt{\frac{\pi}{4v_f}} = 1.618$$

$$a = \frac{l}{d} = \frac{10^{-3}}{12\times10^{-6}} = 83.3$$

$$n = \sqrt{\frac{2(1.015\times10^{9})}{76e9 \ln(1.618)}} = 0.2356$$

$$\eta = 1 - \frac{\tanh[(.2356)(83.3)]}{(.2356)(83.3)} = 0.949$$

$$E_1 = (.949)(.3)(76e9) + (.7)(2.7e9)$$

$$= 23.5 \text{ GPa}$$