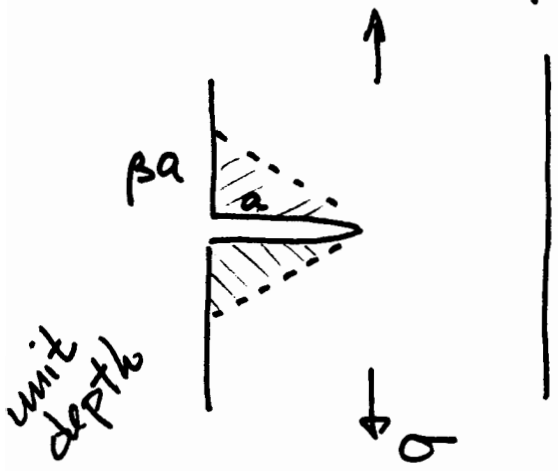
Fracture - Griffith Energy Balance

- Strain energy released by crack extension



energy per unit volume

$$= \int \sigma \, d\epsilon = \frac{1}{2}\sigma\epsilon = \frac{\sigma^2}{2E}$$

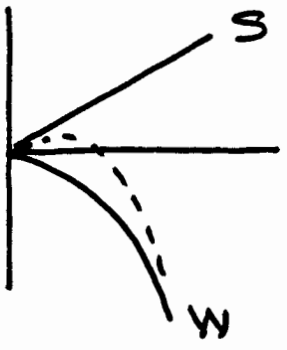
volume

$$= 2 \times \frac{1}{2}(a)(\beta a) = \beta a^2$$

Inglis: $\beta = \pi$

$$\rightarrow W = -\frac{\sigma^2}{2E} \cdot \pi a^2$$

- Equate to surface energy $S = 2a \cdot \gamma$



$$E = S + W$$

catastrophe when

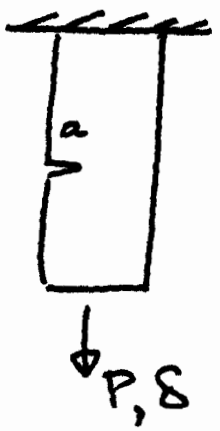
$$0 = \frac{dE}{da} = -\frac{\sigma_f^2}{2E} \cdot 2\pi a + 2\gamma$$

$$\rightarrow \sigma_f = \sqrt{\frac{2\gamma E}{\pi a}}$$

- Irwin / Orowan: use $\gamma = 2\gamma + W_p$

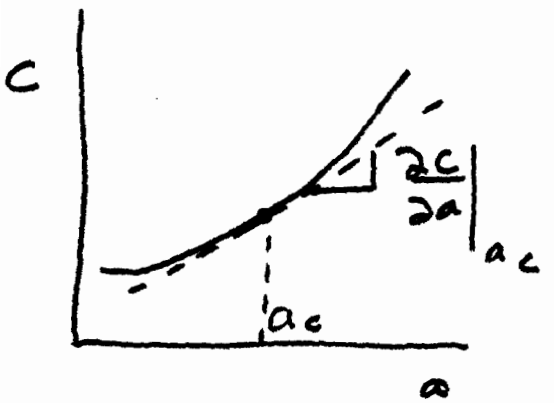
$$\sigma_f = \sqrt{\frac{\gamma_c E}{\pi a}}$$

Compliance Calibration



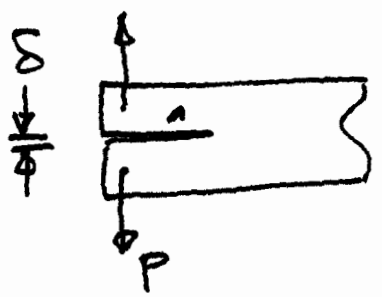
$$C = \frac{\delta}{P}, \quad U = \frac{1}{2} P\delta = \frac{1}{2} CP^2$$

$$\mathcal{G} = \frac{\partial U}{\partial a} = \frac{1}{2} P^2 \frac{\partial C}{\partial a}$$



$$\mathcal{G}_c = \frac{1}{2} P^2 \frac{\partial C}{\partial a}\Big|_{a=a_c}$$

eg DCB:



from beam theory

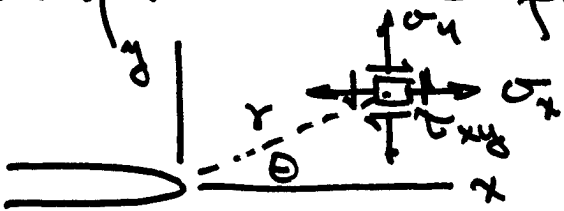
$$\frac{\delta}{2} = \frac{Pa^3}{3EI}, \quad I = \frac{bh^3}{12}$$

$$C = \frac{\delta}{P} = \frac{2a^3}{3EI}$$

$$\mathcal{G}_c = \frac{1}{2} P_c^2 \cdot \frac{2a^2}{EI} = \frac{12 P_c^2 a^2}{b^2 h^3 E}$$

Fracture - Stress Intensity

- Singular stress field



$$\sigma_y = \frac{K_I}{\sqrt{2\pi r}} f_y(\theta)$$

$$K_I = \begin{cases} \sigma\sqrt{\pi a} & \text{, central crack} \\ 1.1\,\sigma\sqrt{\pi a} & \text{, edge crack} \end{cases}$$

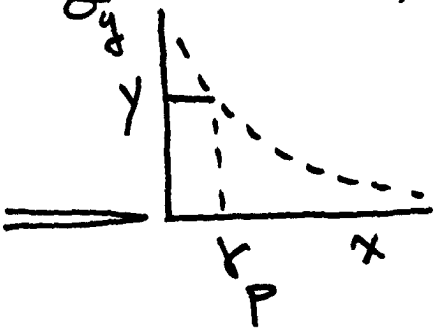
fracture when $K_I \geq K_{IC}$

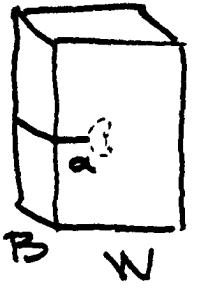
- Relation to energy approach

$$\sigma_f = \frac{K_{Ic}}{\sqrt{\pi a}} = \sqrt{\frac{\mathcal{G}_c E}{\pi a}} \rightarrow K_{Ic}^2 = \mathcal{G}_c E$$

- Plane stress vs plane strain
  - $K_{Ic}^2 = \frac{\mathcal{G}_c E}{1-\nu^2}$ for plane strain
  - elastic constraint: $\sigma_z > 0$
    - → τ smaller → less plastic flow
    - → smaller $\mathcal{G}_c$ → less tough

- Limits of applicability





$$Y = \frac{K_I}{\sqrt{2\pi r_p}} \longrightarrow r_p = \frac{K_I^2}{2\pi Y^2}$$

$$\longrightarrow \text{keep } B,\ (W-a) > 2.5 \left(\frac{K_{Ic}}{Y}\right)^2$$