3.032 Problem Set 5
Fall 2006
Due: Start of lecture, 10/27/06

1. The potential energy between ions can be expressed as an equation from which the equilibrium spacing between ions r_o and the binding energy between ions U_b can be determined. Here, you will compare these quantities as determined from the functional form $U(r) = U(r)_{attractive} + U(r)_{repulsive}$ and from your graphical representation of $U(r)$ for the following Lennard-Jones potential approximation of an fcc metal crystal:

$$U(r)_{attractive} = -1.45/r^6 \quad (1)$$

$$U(r)_{repulsive} = 6.5x10^{-6}/r^{12} \quad (2)$$

   (a) From manipulation of the above equations, determine r_o as the inter-ion distance at which $U(r)$ is a minimum value.

   (b) From manipulation of the above equations, determine U_b as the binding energy at the equilibrium spacing r_o.

   (c) Graph $U(r)$ and compare the values you estimate from this graph to those calculated through evaluation of the above equations.

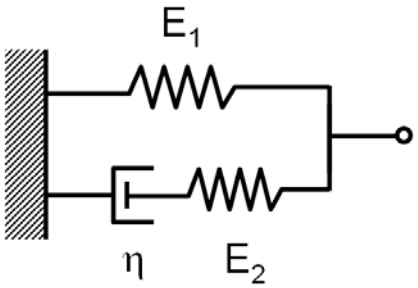
   Note that use of Mathematica or a similar program is encouraged for this analysis of a simple $U(r)$, and this will be helpful for your consideration of $U(r)$ in Lab 2. Note also that the units of the coefficients and of r are not given here, and often are omitted because they are normalized by the magnitude of U_b and r_o. In other words, it is often implied but not stated that you are really considering $U(r/r_o)/U_b$.

2. As noted in lecture, pseudo/superelasticity results from an austenite/martensite phase transformation under stress. The magnitude of the reversible strain is on the order of the strain induced when the austenite phase transforms to the martensite phase. For pseudoelastic AuZn, the martensite phase has been reported to be an ordered fcc of lattice parameter $a = 0.42$ nm (with Zn in the (0, 1/2, 1/2) lattice positions), and the austenite phase is body centered tetragonal of lattice parameters $a = 0.33$ nm and $c = 0.70$ nm (with Zn in the body centered lattice position).

   Explain the mechanism of pseudoelasticity, and predict the maximum reversible strain you would expect to observe in this alloy under a full cycle of uniaxial tension (loading such that all austenite $\rightarrow$ martensite, then unloading to $\sigma_{11} = 0$).

3. Consider a viscoelastic model that consists of a spring (E_1) in parallel with a Maxwell element (E_2, η for a spring and dashpot in series). The springs are purely elastic and the dashpot is purely viscous.

   (a) Draw a schematic of this model. What is its effective stiffness at high strain rates? At low strain rates?



   (b) Derive the constitutive relationship for this model, expressing the equation in a form that includes strain and its time derivative on one side of the equation and stress and its time derivative on the other.

   (c) Sketch the response of the model for a constant stress applied at t_1 and released a long time later at t_2. Assume (arbitrarily) that $E_1 = 2E_2$.

   (d) What characteristics of creep does this model simulate that are not available with the Maxwell model? With the Kelvin-Voight model? Are there any disadvantages of this three-element model?

4. There is very little information available on wikipedia.org about the viscoelasticity that characterizes the mechanical behavior of polymers. A cursory search indicates the following stubs:

   - http://en.wikipedia.org/wiki/Viscoelasticity
   - http://en.wikipedia.org/wiki/Kelvin-Voigt material
   - http://en.wikipedia.org/wiki/Maxwell material
   - http://en.wikipedia.org/wiki/Polymer

Your task, individually and as a class, is to modify existing articles and create new articles that describe the concepts covered in lecture (and beyond) as clearly, accurately, and interestingly as possible. This is supposed to be interactive and the article(s) will get better as everyone contributes. The credit for this problem will be as follows:

- 66% from the material you personally edit/create, turned in as a hard copy of the history comparison of the previous version and your new version, *every time you edit the document*. To do this, you will have to create one new wiki account on any computer (please use your MIT user name as your login "name"), and sign on using this account *every time you edit any wiki pages* from any computer. That way, we can also compare versions as the articles evolve.
- 33% for the total quality of the contributions to wikipedia that you as the 2006 3.032 class have made through your additions of material related to viscoelasticity.

It does not matter if you modify these pages or decide to create a new article that you all work on. You can add graphs, images, text, and equations. If you have not contributed much to wikis yet, there is a helpful FAQ at *http://en.wikipedia.org/wiki/Wikipedia:Editing_FAQ*. If you contribute early on, you'll have the most chance to shape the site but the least to work with; if you wait too long to get started, you will have to work harder to think of creative and accurate ways to add to the site. **Remember that citations in your own academic work should NOT come from wikipedia, because it is not rigorously peer-reviewed and is only as accurate as the last person who edited the page.** Have fun!