3.020 Lecture 12

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1 Integrating the C-C equation for vaporization of a condensed phase

$$\frac{dP}{dt} = \frac{\Delta H}{T\Delta V} \quad \longleftarrow \textbf{in general, functions of T and p}$$

Approximations

$$1.\ \Delta H \approx \textbf{const.} \quad 2.\ V^g \gg V^s, V^l \quad 3.\ V^g = RT/P$$

$$\frac{dP}{dT} = \frac{\Delta H}{TRT/P} \quad \textbf{separable !}$$

$$\frac{dP}{P} = \frac{\Delta H}{T}\frac{dT}{T^2}$$

$$ln(P/P_0) = \frac{\Delta H}{R}(\frac{1}{T} - \frac{1}{T_0})$$

$$P = P_0 e^{\Delta H/RT_o} e^{-\Delta h/RT}$$

$$P_{SAT} = Ce^{-\Delta H/RT}, \quad C = P_0 e^{\Delta H/RT_0}, \quad P_0 = P_{SAT}(T_0)$$

slides: data for water

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2 Dew point and relative humidity

- Air always has some water in it
- Relative humidity (RH) = Water vapor pressure

  normalized to its saturation vapor pressure

$$RH = P_{H_2O}/P_{SAT}$$

- Dew point = temperature at which water at its current vapor pressure will condense

$$P_{SAT}(T_{DP}) = P_{H_2O}$$

- When raining, water is in 2 phases, and $P_{H_2O} = P_{SAT}$

$$\Longrightarrow \quad 100\%\ R.H., \quad T_{DP} = T$$

- Otherwise, $P_{H_2O} < P_{SAT}$, $\quad R.H. < 100\%$, $\quad T_{DP} < T$

slides: Boston Weather

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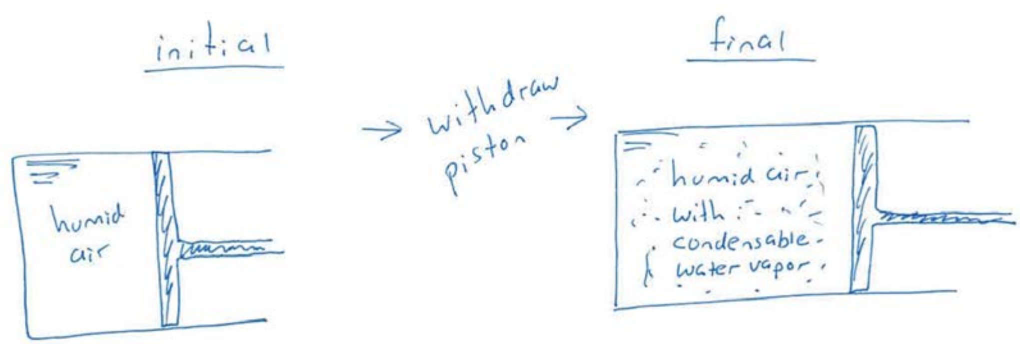
3 The Cloud Chamber Problem

slides: Cloud chambers

- Cloud chamber initially at 298K (77^oF) and 1 atm
  filled with air at dew point 288K (59^oF)
- Expands quickly (read: adiabatically) by $\Delta V/V_i$

control parameter

$$\Longrightarrow \quad \Delta V/V_i \quad \textbf{needed to achieve saturation}$$



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(1) Write expression for water vapor pressure as f'n of $\Delta V/V_i$

- initial v.p. $P_{H_2o,i} = P_{SAT}(dewpoint) = P_{SAT}(288K) = 1863\ Pa$
- system initially at $P_{TOT,i} = 1\ atm = 101,325\ Pa$
- $P_{H_2O} = P_{TOT} X_{H_2O}$ (mole fraction), by Dalton's law of partial pressures (next lecture)
- for adiabatic process $P_f = P_i(\frac{V_f}{V_i})^{-\gamma} = P_i(1+\frac{\Delta V}{V_i})^{-\gamma}$
- applying Dalton's law

$$P_{H_2O} = X_{H_2O}\ P_{TOT,i}\ (1+\frac{\Delta V}{V_i})^{-\gamma}$$

$$X_{H_2O} = 0.0184$$

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(2) Write expression for temperature as f'n of $\Delta V/V_i$

- for adiabatic process

$$T_i V_i^{\gamma-1} = T_f V_f^{\gamma-1}$$

$$T_f = T_i(\frac{V_f}{V_i})^{-\gamma+1} = T_i(1+\frac{\Delta V}{V_i})^{-\gamma+1}$$

(3) Solve for saturation condition

$$P_{H_2O} = P_{SAT}$$

$$P_{H_2O} = X_{H_2O}\, P_{TOT,i}\, (1+\frac{\Delta V}{V_i})^{-\gamma}, \quad P_{SAT} = Ce^{-B/T}, \quad T = T_i(1+\frac{\Delta V}{V_i})^{-\gamma+1}$$

- Parametric in $\frac{\Delta V}{V_i}$
- Solve numerically or graphically
- For γ, use value for ideal diatomic gas
  - applies to N_2 and O_2, main components of air
  - $C_P/C_V \approx 1.4 - 1.7$ for air

slides: solution for $\frac{\Delta V}{V_i}$