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**RAFAEL
JARAMILLO:**

So today, we are going to talk about heat engines. So what's a heat engine? A heat engine is any machine that takes heat and turns it into work. So an engine is something that does mechanical work. A heat engine is an engine that runs on heat.

And as we'll see, no machine can turn heat into work with 100% efficiency. That would violate the second law of thermodynamics. But we try to engineer our systems to do it as well as we can. So here's the most obvious case of a heat engine in our day-to-day lives, and especially if you live in Texas right now, is that of power plants.

So power plants take some fuel resource, coal, oil, gas, or nuclear. And you burn that to create heat and then use that heat to make mechanical work. So this picture in the upper left here is the Mystic Generating Plant, just a couple of miles from where I am. That is a combined cycle natural gas power plant that provides much of the electricity that runs MIT.

And over here is some pictures of, I think, this is maybe a coal or maybe a nuclear plant. It doesn't matter. The point is I want to show you the cooling towers, which we'll come back to a little bit later. So those cooling towers have to do with cooling. And heat engines run on heat. So heating and cooling are somehow important here.

Another example of a heat engine. This is where the field all began, actually, is with steam engines. So a steam locomotive takes a heat resource, you burn coal. And you turn that heat into steam. And then the steam does some work for you on a piston. So that's a little bit more of an old-fashioned example.

Here is a more contemporary example, the jet engine. You burn an amount of fuel, it creates heat. You use that heat-- and how you use that efficiently is the engineering of the thing. So that's another example of a heat engine. Here's one which is familiar to all of us, an internal combustion engine. We burn something that creates heat. And that heat energy is partially transformed into work as the pistons work on a crankshaft.

And here's one which is maybe a little bit less obvious, but it's definitely a heat engine. It's the hurricane which harnesses a temperature difference between sea surface and the upper atmosphere and creates mechanical work. So these are all examples of heat engines.

So those are real-world examples. Now, what we're going to do is do this in a little more abstract. So let's see here, back to the board here. You guys can see the board, right? Yes? Yeah, OK. Thanks . All right, so we've seen some real heat engines. Now, we're going to abstract this a little bit. Heat engines abstracted.

So this is the way that heat engines are represented in a textbook or in a class like this. You have some cyclic machine normally with a circle with an arrow. Here's what it means to be a cyclic machine. It returns to the same state after each cycle.

All right, so now already we have some thermo words here that we know. State, so there's state functions involved. And this machine returns to the same state after each cycle. So here's another abstraction. We're going to have thermal reservoirs. We're going to have some high temperature reservoir. And we're going to have some low temperature reservoir, T_{hot} and T_{cold} .

So these are called thermal reservoirs. And they are maintained at T_{hot} and T_{cold} throughout. So this is an abstraction. In reality, nothing is maintained the same arbitrary precision for all time. But you can achieve this through engineering.

So for instance, you could have a boiling pot of water. That's a thermal reservoir. You know what temperature it is. It's at 100 C, because it's boiling. And as long as you keep enough water in it and keep the heat there, you know it's going to be regulated temperature. Another good thermal reservoir is an inlet of an ocean. And that describes a lot of why power plants are sited where they are.

So we have thermal reservoirs. And then there are some terms. We have total work, total heat, work in, heat in, work out, heat out, and η . So total work, total in, out, work, and heat. So tracking flows of energy across the system's boundaries throughout the cycle. And then there's an important term here, efficiency. η is defined as the total work that the system does on the surroundings.

And the reason I put this in green and circled it is because for today's lecture and today's lecture only, we're going to have some confusion over this sign of work and heat because we have defined for this class work as work done on a system. So positive work increases the energy of a system. But if you're engineering an engine, you probably want to flip the sign because you want to keep track of work with the system does on something, on the electric power grid, say, or on the road and your cars that push you down the road, or what have you.

So there's going to be a little bit of confusion there. It can be managed. So that's abstracted heat engines. So here is a typical representation of a heat engine. You have a high temperature reservoir. You have this engine, which operates with efficiency η . The engine receives heat q_{in} from the high temperature reservoir. And it dumps heat q_{out} to the low temperature reservoir.

So heat received, heat rejected. And each cycle, it performs some amount of work on the surroundings. And I ordered new pens. And they haven't come yet. So I'm sort of fading here. But let me switch to purple. So a typical representation of this would be like this.

The heat engine, we're just learning terminology here, with efficiency η operating between T_{hot} and T_{cold} . So that's what the engineers will say to you. We have this heat engine. It's got efficiency and it operates between these two temperatures.

In each cycle, q_{in} is absorbed at T_{hot} . q_{out} is rejected at T_{cold} . And work out is performed. OK, quiz, what is q_{in} minus work out minus q_{out} ? Somebody, what is that?

AUDIENCE: Zero.

RAFAEL JARAMILLO: Zero. It's zero because this is a cyclic engine. It returns to the same state every cycle. That means every state function has to return to its starting point. Energy is a state function. So the energy of the system cannot change around each cycle. So again, coming back to our bookkeeping, this here, q_{in} minus work out minus q_{out} , this is the energy exchange.

This equals Δu , which has to be zero because all the state functions return to their starting point. So let's go back to our slides here just for fun. So here's some real engines. For the thermal power plant, say, somebody tell me where would I find T_{hot} ? Where would I find the high temperature? Does anybody know?

AUDIENCE: Burning temperature of the fuel?

RAFAEL JARAMILLO: The burning temperature of the fuel. And actually, these combined cycle natural gas plants, their first cycle is a jet, actually. It's very similar to a jet. So the burning region is going to look something like this. You have a region where the fuel is burned. And that's going to be the highest temperature inside of these units. And where's the cold temperature? Where's $T_{\text{sub cold}}$?

AUDIENCE: Water.

RAFAEL JARAMILLO: Yeah, that's a good guess. In this case, it's actually wrong. And I put this up here as a trick here because it's kind of cool. The water here is cold. We're in the harbor. So that'd be a good resource. But the water is pumped up into these evaporative cooling units. This thing here, as far as you can tell, it's just a building on stilts. You don't know what that is. But I've been there. It's very cool. These have the same function as these cooling towers do. And they evaporative-- they cool the water down below the temperature of the inlet.

So they're able to lower $T_{\text{sub c}}$ a little bit that way. And that turns out to be important to do that for the efficiency of the plant. But yeah, you're basically right. This is-- the reason these plants are on the water, the reason for it. And it's because it's a nice big reservoir of cold. So for a steam locomotive, they don't carry around cold reservoirs with them. So their cold would be just the ambient.

In hurricane, $T_{\text{sub hot}}$ is the warm surface currents that fuel hurricanes. And $T_{\text{sub cold}}$ is the upper atmosphere, the top of this engine. And internal combustion engine, $T_{\text{sub cold}}$ is your tailpipe. So it's pretty much-- I think that's kind of neat. And as we'll see, the actual numerical value of $T_{\text{sub hot}}$ and $T_{\text{sub cold}}$ is really important to determining the efficiency of the engine. So let's calculate a cyclic process.

Work and heat are process variables. Thermodynamics doesn't describe-- thermo doesn't describe real world processes. So what do we do? We describe a hypothetical process for which the system remains in equilibrium at all times.

And this is weird. And it is just weird. It's a weird thing to do. But if we do this weird thing, we can use state variables. We can use equations of state if they're available, if we know them. But here's something to keep in mind.

In practice, such a cycle would take infinite time. So when you are venture capitalists and you have somebody coming, pitching you too good to be true energy technologies, normally, you're going to think back to 3020 and try to poke holes in their argument. And here's one hole you might find, which is power is work divided by cycle period.

As you approach this ideal of being in equilibrium at all times, you have to slow your cycle down. As you slow your cycle down, your cycle period goes up. And the actual power you get out of your unit goes to zero. So even if you could design and build an ideal heat engine, no one would buy it because it might be maximally efficient, but you get no power out.

And this comes up again and again. So I'm glad that you've seen this now. All right, let's talk about one ideal cycle. Let's talk about the most famous one, the Carnot cycle with an ideal gas. There are many cycles you can calculate, reversible cycles you can calculate from ideal gas. Carnot's just one of them. But it's a famous one. So that's what we're going to do. So here's the Carnot cycle.

You start with isothermal expansion at T_{hot} . Let's try that. Now, while I'm drawing this, there was a point on Piazza about this that I think I replied to this morning. For an ideal gas, we're going to tell you-- you don't have to know this. We're just going to tell you that the energy is depending only on the temperature. The energy is depending only on the temperature.

So isothermal processes don't change the energy of an ideal gas. So if the gas is expanding, it's doing work on the surroundings. Does it have to be receiving heat from the surroundings or does it have to be heating the surroundings? It's a question for you.

AUDIENCE: Receiving heat.

RAFAEL JARAMILLO: Has to be receiving heat. Has to be receiving heat because it's losing energy in the form of work. So it must be gaining energy in the form of heat. So we're going to start at 1 here and we're going to go down to-- get a different color-- 1 here and we're going to go down to 2. So this is step one.

We're starting at a smaller volume. And we're expanding to a higher volume, isothermally. This dashed line is the T_{hot} isotherm. Step two, adiabatic expansion. to T_{c} . So now, we're going to expand this is step two. And this dotted line is a T_{c} isotherm.

All right, step three, Isothermal compression at T_{c} . That's this. And then step four, step four, which is adiabatic compression back to the starting point, back to point 1. So here's my arrow, going around this way, good.

All right, here's a note. The area enclosed by the cycle, the work total is area enclosed. That's the integral. The integral pdv with p is the area enclosed by the cycle. So if you have a cycle on a $p-v$ plane, you can already figure out the work done geometrically. This is true for any cycle, not just Carnot. So let's analyze the isotherms.

Isotherms, pV equals nRT $p dv$ plus $v dp$ equals what? Zero. It's isothermal. So dv equals minus v over p dp equals minus nRT over p squared dp . So $p db$ equals minus nRT over p to the power of 1 times dp . All right, so that was useful.

So the integral of work equals the integral dp nRT over p equals nRT natural log p_{final} over p_{initial} . Which by, again, using equations of state equals nRT natural log v_{initial} over v_{final} . That's useful. Let's do a sanity check.

Expansion, does work on surroundings. v_{final} larger than v_{initial} means v_{initial} over v_{final} is less than 1, which means log v_{initial} over v_{final} is less than zero. That means the system loses energy. OK, good, checks out.

So this is the expression for an isotherm for an ideal gas. Isotherms continued. For ideal gas, internal energy is a function of T only. du equals $n C_v dT$. We're not going to derive this in this class. That actually gets kind of beyond the scope of the class. But we'll use it from time to time. This is single variable calculus. Makes things easier.

I just want to make sure for ideal gas and only for an ideal gas, and only for an ideal gas. So what this means is du at fixed t equals zero. du for an isothermal process equals zero. And what that means is dq equals minus d work or q equals minus work. And this, again, gets to the point that was on Piazza this morning. And it's a useful expression when calculating properties of heat engines. Now, let's talk about the adiabats.

Adiabat is a process with no heating across the boundary. So we know that d work equals minus $p dv$, as usual. This also has to equal the change in energy, since there's no heating. How do we calculate this?

For ideal gas and only for the ideal gas, du equals d work plus dq . But that's zero. Equals minus $p dv$ equals the thing I gave you on the previous page, $ncv dt$. So single variable calculus, we can easily integrate that. Work equals $n cv t$ final minus t initial. So that becomes simple for an ideal gas.

For an ideal gas, adiabatic curves are described by $tv^{\gamma-1}$ equals constant. Or in other words, p final over p initial equals v initial over v final to the power of γ where γ equals the heat capacity ratio.

Now, we're not going to spend a lot of time on this in 020. So I'm just giving you equations. These are derived well in Wikipedia. You don't need to go to the textbook. But they're also described well in De Hoff chapter 4. And we'll come back to it at lectures 6 and 7. Again, because today's lecture is a little bit of a detour from the mainstream of 020, we're going to just give you some expressions. So that you're familiar with them. This is not a mechanical engineering class.

Question, adiabats are steeper than isotherms in p - v plane. Why? Find my plot. Here's my plot. So here, adiabats, which are sections 4 and 2, they're steeper than the isotherms, which are sections 1 and 3. Mathematically, it's because γ is greater-- γ is greater than 1.

So if this were 1, you'd have the same curvature as isotherms. But γ is greater than 1 for reasons we discussed last lecture, because c_p is bigger than c_v . So those are some properties of adiabats added all up.

We're going to have work. We're gonna have heat. And we're going to have the different segments. So for segment one, segment one was isothermal. We calculated the work. It was nR temperature $\log v_1$ over v_2 using the notation of the cycle that we set up before. So this is an isothermal process. There is the work. Somebody, what's the heat?

AUDIENCE: Be negative of the work.

RAFAEL JARAMILLO: Negative of the work, exactly. And $nR t_{\text{sub hot}} \log v_1$ over v_2 . Isothermal process, ideal gas, energy doesn't change. So work and heat have to be equal and opposite. OK, two, the work here was the change of energy, which was $ncv t_{\text{cold}}$ minus t_{hot} . What was-- what was heat for the adiabat?

AUDIENCE: Zero.

RAFAEL JARAMILLO: Zero, right, adiabatic. 3 and 4 are going to be the mirror images of 1 and 2. So 3, the work is $nR t_{\text{sub cold}}$ now $\log v_3$ over v_4 . And of course, the heat is going to be the opposite of that. Minus $nR t_{\text{sub cold}} \log v_3$ over v_4 . And the final leg, which is an adiabat is going to be $ncv t_{\text{hot}}$ minus t_{cold} .

So that's the table of the contributions to heat and work. And then using this table, we're going to calculate the Carnot efficiency. So I actually wish I had-- I'll bring it back. I threw that scrap of paper on the floor. Put it up here. Let's calculate the efficiency. Calculating the-- I'll put this in parentheses-- because we're really just calculating the efficiency, happens to be for a Carnot cycle.

Calculating the Carnot efficiency. Work total, the total amount of work the thing does is the negative of all of these added up. It's the negative of all of these added up. And you see that step two and step four are going to cancel exactly. So it's the difference between step one and step three. So here we go. It's going to be this minus $nR \ln v_1/v_2$ plus $nR \ln v_3/v_4$. So that's the total work, total work done by engine-- by the engine. What about the heat in? How much-- how much thermal research-- how much fuel did you burn to run this thing?

What's that going to be? That's the heat received at the high temperature isotherm. OK, that's the input. That's the thermal research that you used. So minus $nR \ln v_1/v_2$. Heat absorbed at T_h . So the efficiency equals the total work that the thing does over q_{in} equals $1 - T_c/T_h \ln v_3/v_4 / \ln v_1/v_2$.

Now, using the property of adiabats, $Tv^\gamma = \text{constant}$, you can show that v_3/v_4 equals v_1/v_2 inverse. And we'll come back to that. And it's just three or four lines of algebra. If we have time, we'll come back to that.

But if we use that, we get the Carnot efficiency equals $1 - T_c/T_h$. So after all that, some steps of which we skipped, we have a very simple expression. So you are running a power plant. Do you want-- what can you do to improve the theoretical limiting efficiency of your plant?

AUDIENCE: You can increase T_h and decrease T_c .

RAFAEL JARAMILLO: Yeah, increase T_h and decrease T_c . So a lot of engineering decisions, which you will see if you go studying thermal engines-- and we could talk about jet engines, we could talk about power plants. Let's talk about power plants. A lot of engineering decisions which you see come down to trying to increase T_h and decrease T_c .

Once you understand this is the theoretical limit, a lot of the designs make a lot of sense because you see what the engineers are trying to do. Increasing T_h , for example, you could do by using supercritical water in your working cycle, allowing you to burn your fuel at a higher temperature. There's a lot of material innovation that's needed in order to enable that because you're increasing the high temperature of-- high temperature part of the system.

And we know that you need really advanced alloys, and ceramics, and thermal barrier layers and such to do that. And then a lot of other engineering decisions you see are around decreasing T_c . And so that's where, for example, the cooling towers come in. Especially when you have a power plant, which is in the middle of, let's say, the prairie, and there's no ocean nearby, you don't have a really convenient cold resource.

So you build evaporative cooling towers to try to get a lower T_c . So for example, here, the combined cycle power plant, the Mystic Generating Station, I looked it up. Very typical of GE, General Electric combined cycle units, the inlet temperature is 1,400 C. So most of your metals melt at this temperature. So you've got to have really special materials at the inlet.

And then the heat rejected is at 15 C. That's what the website said. So I don't know if that's representative of the water or what-- probably what you get in these cooling units. I'm not sure. So if you plug these numbers into Carnot efficiency, you find that the theoretical limit is 82%. And the actual efficiency of these is close to 60%. So that's really remarkable. It's really, really good. This tells you a lot about the state of technology for natural gas combined cycle plants.

All right, I want to come back to some math here to round us out. So that was kind of neat. But now, we're going to head back towards materials. Let's consider heat transfers. Carnot cycle, and then we're going to consider a less efficient cycle that burns the same quantity of fuel.

So ideal and realistic. Let's consider the heat absorbed and the heat released. OK, so heat absorbed is q_{in} . We just wrote this down. $nR \ln \frac{v_2}{v_1}$ just-- let lowercase v equals v_2 over v_1 , just to keep the writing simple. So that's the heat absorbed. And the heat released is-- we'll call this q_{out} Carnot equals $nR \ln \frac{v_1}{v_2}$.

Now, that was just copying results from the previous slides. Less efficient engine, if it burns the same quantity of fuel, it takes the same heat. That was how we set up the problem. It burns the same quantity of fuel. You burn fuel, you give off a certain amount of heat. But it's going to be less efficient, the heat it rejects, how is that going to compare to the Carnot case? Is it going to be greater than the Carnot case? Or is it going to be less than the Carnot case?

Keeping in mind that $\Delta u = q_{in} - w_{out} - q_{out}$. So this has to be zero. q_{in} is the same. It's a less efficient engine. So it does less work. So it has to reject more heat.

So a less efficient cycle, burning on the same quantity of fuel, rejects more heat. That's the way to talk about this. It rejects more heat with the low temperature. Makes sense, right? It's losing energy that it could-- a better cycle could use-- could exploit as work.

And here is where we get to something which puts us on the road to the second law of thermo. We're going to consider this quantity, $\frac{dq}{T}$. And for now, just bear with me because this is probably seeming random. The notation here, the integral sign with the circle in the middle means we're going to integrate that around the cycle.

So for Carnot, this thing is $\frac{dq}{T} = nR \ln \frac{v_2}{v_1} - nR \ln \frac{v_1}{v_2}$. So this funny thing just happens to be zero for Carnot. For less efficient-- less efficient cycle, the $\frac{dq}{T}$ equals $nR \ln \frac{v_2}{v_1} - \frac{q_{out}}{T_{cold}}$. So these are the same.

But this term is bigger than this term. So that means the overall line is negative. This is because q_{out} is greater than $q_{out, Carnot}$. So this is where I want to leave it today because we will soon see that this is related to entropy generation by a non ideal cycle.

And $\frac{dq}{T}$ is related the change of the state function ds . So what we're going to see in the next lecture or two is that a Carnot cycle, or more generally, any reversible process is one which leaves the entropy unchanged. And we'll see that a less efficient cycle, or any irreversible cycle, is one which-- well, the sign will make sense, increases the entropy of the surroundings. Or you could say increases the entropy of the universe.

All right, so now, that's how the-- our detour into heat engines ends. It's 10:51. I hope that this lecture gives you some recognition of the role of heat engines, some insight into how they're engineered, why they're engineered the way they are. Some equations which you can use in later classes, especially in things like course 2 and course 16.

But with respect to our class, this slide here is really the most important one because we've taken this data in order to justify the importance of this funny thing, dQ over t , which goes to the heart of the second law, which is what we're going to start on next time.