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MICHAEL SHORT: I got an interesting question in the anonymous comment box, and I want to see how many other people agree with it the comment when something like, is everything in this field of nuclear science engineering computational in nature? Because so far, we've pretty much just thrown theory and simulations at you, we haven't done any experiments.

So who shares this concern or wonder? So 1, 2, 3, 4-- OK, 5, yeah. So getting towards roughly half of you. So I can confidently answer, no, this is not a purely computational field. We just had to get you just enough science and physics so that you'd be able to understand some of the lab activities that we've got in store for you, one of whom, Mike Ames here from the Nuclear Reactor Lab is going to be helping you with-- actually, two of them he'll be helping with.

MICHAEL AMES: Two?

MICHAEL SHORT: Yeah. Well--

MICHAEL AMES: --your bananas.

MICHAEL SHORT: The bananas and the thing that we dreamed up today, so--

MICHAEL AMES: This morning? OK. So you want an intro those--

MICHAEL SHORT: Sure.

MICHAEL AMES: --and I'll, I don't know--

MICHAEL SHORT: Fill in what I get wrong. Also, Mike's going to talk about what we're going to do together, which is called NAA, or Nuclear Activation Analysis. There are many, many ways of measuring what sort of impurities may exist in materials, and this is among the most sensitive. We happen to have a nuclear reactor.

So what we will be doing, what I'm going to ask each of you guys to do for a special

assignment that not graded, except you'll need it for the problem set, so it kind of is, is I want each of you to bring something into me that weighs about 50 milligrams in one piece, fits in here, is not fissionable-- so if you have uranium at home, I shouldn't know about it. And we also ask that you don't bring anything in that is too salty, because sodium activates like crazy.

And each of you, using your knowledge of radioactive decay that we learned Tuesday and today, and the Bateman equations in serious radioactive decay next Tuesday and Thursday are going to calculate what impurities exist in your sample. What is your sample? It's--

MICHAEL AMES: You're not going to be able to do the calculations.

MICHAEL SHORT: We're going to make some estimates of--

MICHAEL AMES: Oh, OK.

MICHAEL SHORT: The isotopes that you'll let us see. The shorts, yeah. I know we're not going to get every impurity, right?

MICHAEL AMES: Well, we're not going to be able to run it next week.

MICHAEL SHORT: Yes, but I will want your materials next week. OK. So by Tuesday, I'd like each of you to come bring something in for what you'd like to know the elemental composition consisting of the following elements. Let's see. Problem sets, I think I've got it right up here. Let me just clone the screen.

So you can see what we can look for. So this is provided to me by Mike. We're going to do what's called a short nuclear activation analysis run looking for any of the elements up on this list. Shorts 1 with extremely short half-lives, and shorts 2 with elements in the half-lives of hours. And Mike, I had a question for you now that we're live. Can we count arsenic in that list? Because it's 24-point-something hours.

MICHAEL AMES: Yeah, we could do arsenic.

MICHAEL SHORT: OK.

MICHAEL AMES: It's not a great shorts element. You'd probably have to have something with a bunch of arsenic in it.

MICHAEL SHORT: So if any of you guys have some food that you bought online and you don't know what sort of

contaminants there are, or if you've got a piece of your fingernail and want to see if you are what you eat, or--

MICHAEL AMES: After telling you why fingernails would be a great sample, we might run afoul of the human subjects in research issue with fingernails.

MICHAEL SHORT: What about dog fingernails?

MICHAEL AMES: --believe it or don't. Ah, yes, your dogs are probably not--

MICHAEL SHORT: OK. So clip your pets' claws if you want to see if they are what they eat, or don't tell me what nail the thing came from, or get a-- slice up a piece of a peanut or whatever your favorite food is. Or if you want to see if there are any metal dyes used in your clothing, cut out a little 50 milligram square, it'll be like a fashion statement and an experiment at the same time, right?

So we ask that it's about 50 milligrams, it's gotta fit in here, it's gotta be not that salty and not fissionable, and we're going to pack a couple of these in one of these rabbits, these polyethylene rabbits. We call it those, one, because everything in nuclear is named after animals and farm implements for some reason. Did I go over that with you guys the first day of class? Barns, shakes, pigs, rabbits?

OK. So a rabbit is a little capsule. Do you pop open or you screw it open? Oh yeah, there's like a square nut at the top. Just a little capsule that goes through a pneumatic tube, kind of like the old bank machines, and it'll go firing into the reactor, sit there for a while, and get pneumatically sucked back out so that we can calculate the activation and decay of the isotopes within.

And a pig is just a big heavy thing of lead where you keep pieces of things that you irradiated for shielding. So if you notice the sort of methodology theme here, farms, pigs, rabbits, barns, shakes. Anyone else you know any other farmy nuclear units? Yeah?

AUDIENCE: [INAUDIBLE]

MICHAEL SHORT: Or farmy anythings?

AUDIENCE: They follow the [INAUDIBLE] detectors toads and bullfrogs.

MICHAEL SHORT: So the detectors in [? nif ?] are called toads and bullfrogs. Why is that?

AUDIENCE: Because the people who made the acronym [? like making their ?] acronyms, and [INAUDIBLE] bullfrog was not [INAUDIBLE].

MICHAEL SHORT: So actually bullfrog stands for something.

AUDIENCE: Yeah.

MICHAEL SHORT: That's pretty cool. OK.

AUDIENCE: [INAUDIBLE] acronyms are [INAUDIBLE]

MICHAEL SHORT: Yeah.

AUDIENCE: [INAUDIBLE] acronym for acronym.

MICHAEL SHORT: Oh yes, the acronym for acronym. As well as what I study, which is CRUD or Chock River Unidentified Deposits. It's the gunk that builds up on fuel rods. Well, I talked to a fellow from Chock River who took extreme offense to this detriment to nuclear power plants being attributed to his fine laboratory.

MICHAEL AMES: Oh, I thought it was chromium-rich.

MICHAEL SHORT: Oh, see, that would work, but they're not actually chromium-rich. Yeah. Cool. So Mike, do you want to say anything else?

MICHAEL AMES: Yeah. I think I want to say-- so yeah. So--

MICHAEL SHORT: You want to introduce yourself, too?

MICHAEL AMES: Oh yes, sorry. I'm Mike Ames. I work over at the Nuclear Reactor Lab mostly doing nuclear experiments, but I also run the NAA lab there. And I've been doing it for a while. So the idea-- the reason we've got these guys, it'll irradiate the samples in this. The easiest thing for me to do without losing any of your samples is [INAUDIBLE] [? there, ?] something gets radiated in this, I slip the top off, and you probably can't see that, but it's for a little poly bag. I'll dump your sample into the poly bag. So it's something that's like one piece that works the best.

What I usually tell people, it's something that you could pick up reasonably easily with a pair of tweezers. That way if they drop it, I can actually pick it up with tweezers. But that kind of gives you a good size. No-- nothing to powdery, because the powder is going to spread around and then get contaminations. Yeah, and that list there, I guess you said you posted it.

MICHAEL SHORT: Yep. The list has posted on the Stellar site.

MICHAEL AMES: Yeah, yeah. So yeah, we'll see mostly these light elements, the guys with the asterics. I don't see that well. Yeah, gallium I'm probably not going to see that. So--

MICHAEL SHORT: Interesting, we found gallium in your dog claws or something.

MICHAEL AMES: Yeah, magnesium, aluminum, titanium, vanadium, those are easy. Sodium, chloride, and potassium are easy. The manganese will come out nicely. Some of the elements of-- generally more interest down-- further down--

MICHAEL SHORT: Even further--

MICHAEL AMES: --chromium, those have longer half-lives, so I'm going to see those.

MICHAEL SHORT: Yeah.

MICHAEL AMES: Probably do like a five or 10-minute irradiation, let it decay for a little while, and then we'll do a couple of counts. Have you ever get over to the reactor?

MICHAEL SHORT: We will be there a fair bit.

MICHAEL AMES: OK, you guys are going to be there next week.

MICHAEL SHORT: Yeah. So what do you usually use NAA for?

MICHAEL AMES: NAA, well, the thing we've been using it for lately a lot is anything that's going to go in the core of the reactor, we want to analyze to see if there's any surprise elements-- to see how much cobalt is in a piece of steel that we're going to put in because people don't usually measure the cobalt, but it activates really well. And so that causes problems later on when we have to take some compartments for cobalt-60 or half the M that shows up and things that you don't expect it.

MICHAEL SHORT: Mm-hmm.

MICHAEL AMES: My past in doing NAA was all environmental samples. So we did a lot of trace element, heavy metal chemistry on atmospheric particulates, rain water, ice cores, lake sediments, crude oil, coal, fly ash. And so we would measure that whole stack of elements and those guys for environmental studies.

MICHAEL SHORT: Cool.

MICHAEL AMES: That was my time doing NAA. I don't know, I think that's enough. You want me back here next Tuesday or Thursday--

MICHAEL SHORT: Yeah, to explain a little bit more of the specifics of the process.

MICHAEL AMES: --give you the five, 10 minutes. And if you guys could-- I mean, are you guys can be able to come to the lab when we do the shorts?

MICHAEL SHORT: Depends on when you do them. If it's early November, then yes.

MICHAEL AMES: Yeah, OK. So when I do shorts, I put two samples in one of these guys, they'll shlink into the reactor and out.

MICHAEL SHORT: Which you guys should see. It's pretty cool.

MICHAEL AMES: Yeah, you can watch that part. And then I run it down the hallway and throw each sample on a detector. And while those samples are counting, I run another rabbit. So it's kind of an all-day thing running up and down the hall every half hour. So you could almost come anytime during the day while I'm running these and get one full round in half an hour.

MICHAEL SHORT: Cool.

MICHAEL AMES: I think that's the whole story. You can hang on to that if you want.

MICHAEL SHORT: Yeah. Thanks, Mike. So you heard the charge. Bring me your dog claws, your eyebrows, your skin flakes, your scabs, your--

MICHAEL AMES: Oh yeah, can we--

MICHAEL SHORT: --food pieces--

MICHAEL AMES: --from hair?

MICHAEL SHORT: Oh Yeah, so no hair.

MICHAEL AMES: We used to do a bunch of hair analysis. Hair is a pain in the neck because it--

MICHAEL SHORT: Staticky?

MICHAEL AMES: --clings to everything and it gets stuck to parts. Yeah, we did some hair analysis for the superfund site in Woburn, and it was a big success, but it was not pleasant to do the work.

MICHAEL SHORT: So don't bring us your hair, but bring us your skin flakes, your scabs, your dog claws, your food scraps, your--

MICHAEL AMES: No skin flakes.

MICHAEL SHORT: Just don't tell him.

MICHAEL AMES: Stuff that you can-- like I said, something that's like one good piece that you can pick it up--

MICHAEL SHORT: Yeah, get creative.

MICHAEL AMES: --be great.

MICHAEL SHORT: It doesn't have to be something that I said. As long as it's not fissionable or salty.

MICHAEL AMES: And we might veto samples-- I do need to know what they are roughly before we throw it in the reactor, and we might end up vetoing some samples.

MICHAEL SHORT: Yeah. So let's figure it out by Tuesday. That way if we have to veto, we have a while for you guys to find another sample.

MICHAEL AMES: Or-- yeah. I don't know if I'll be able to run a sample from everybody.

MICHAEL SHORT: We'll see.

MICHAEL AMES: We'll see.

MICHAEL SHORT: We'll see what we can do.

MICHAEL AMES: Anyway, I'll see you all next week.

MICHAEL SHORT: Thanks, Mike. So yeah, so Mike's going to be helping us do some nuclear activation analysis, and in addition, next week we're going to be counting our big bag of burnt bananas, because now that you're learning about radioactivity, and this-- when we go over activity and series decay, you'll have enough of the science to understand to calculate how to what is the radioactivity of one banana. And to do so, we need like 500 bananas to get enough statistics. So we'll be going to the reactor for that. We've also set it up so that next week and the week

afterwards you guys are going to be manipulating the power levels of the reactor. So you'll actually get to sit in the control seat, raise and lower the control rods, and watch the power of the reactor change in ways you probably won't expect unless you're getting operator training. And all of that stuff is going to be used in the lab components of the problem sets.

So you guys might have noticed there was some spinthariscopescope thing on problem set 3. Radiation protection did not want me taking smoke detectors apart and giving them all to you because that's distributing open radiation sources and I probably shouldn't do that, but we've got plenty of lab stuff for you to do to see what is actually hands-on in this field. And if you do want to know what else is hands-on, I have an experimental group and you're always welcome to come see what we do at the lab. There usually isn't an explosion happening, but there's usually something that 1,000 to 1500 Celsius temperature, blue uranium fluoride salts, nanonewton forces, extreme pressures, or what have you, we do a lot of it.

So I wanted to give a quick review of where we were in radioactive decay last time. I think we left off somewhere around "Particle Physics Telescope Explodes" was my favorite BBC headline ever when we were talking about-- we've already gone over alpha decay, we've gone over beta decay, we started talking about positron decay and the neutrinos that come out of that, and this Kamiokande detector that is set up with lots of expensive phototubes to detect the cones of light left as neutrinos pass faster than the speed of light in water through water. And then there is my favorite headline.

I believe we made it up to the end of positron annihilation spectroscopy or ways that you can actually use positron emission to look at the number and types of atomic defects in crystalline materials. And yep, that's where we left off, interested in PAS? Lots of papers to check out. In the meantime, let's look at one of the competing mechanisms for positron emission, which is electron capture. In this case-- so I will warn you, it's sometimes a little easy to get mixed up between electron capture, internal conversion, isometric transition, so I've left these slides on here, and I also took pictures of the board from last time and posted them on the Stellar site. So all the blackboards where we filled the boards, there's pictures of those.

And I'm going to keep doing that. So if you've learned better just by looking and listening rather than writing everything, feel free. If you want to write stuff down, also feel free. So an electron capture, another way of, well, destroying positive charge would be for the nucleus to capture an electron. So either it can emit a positron, giving away some positive charge, or it can capture an electron, destroying one of the positive charges. And in each case here, we've

got a proton that becomes a neutron and something. I'm won't be specific as to which one because positron and electron capture, well, two different but similar kind of decaying mechanisms.

And then what you get is this hole where the electron used to be. And that's not a very stable configuration for an atom to have, let's say, one fewer electron than protons and especially to have a hole in the inner shell. So you get this cascade from straight-up from high school chemistry of electrons falling from one shell to the next and giving off characteristic X-rays-- that's me that cross that out there-- because you will find misinformation all over the place online, and someone might make a great figure and mislabel an electron-emitted photon as a gamma ray, and remember, we said gammas come from the nucleus, otherwise they're indistinguishable photons.

And so in electron capture, you don't need much of an energy difference between the parents and the daughters, unlike positron decay where for positron decay to happen, you have to have Q at least equal to 1.022 MeV, which is the same as 2 times the rest mass of the electron. For electron capture, you don't. This can happen at just about any energy. As long as you can overcome just the binding energy of the electron, which is negligible compared to these sort of nuclear energy levels.

And so this is the Q equation. Keep in mind, these deltas here are excess masses. So I'll put this up again, the excess mass is the real mass minus the terrible approximation of a nucleide's mass. And this way, excess mass and real mass are directly related, so you could plug in masses here, you could plug in binding energies by making everything with a minus sign, and so on. I think I've repeated myself enough for the Q equation stuff, would you guys agree? Yeah, OK.

And so these are actually two competing mechanisms. So shown here is the decay of sodium-22 which we don't want to happen in our nuclear activation analysis because it gets pretty toasty. It can either proceed-- there's a kind of hidden part of the diagram that I drew in to make a little more sense. You start off with the nucleus at 2.8 MeV above the neon nucleus' energy level. You need 1.022 MeV to create the positron-electron pair, at which point you can emit the positron with a certain energy. You're left in an excited state, and the next thing we'll go over is gamma decay or Isometric Transitions or IT. That's the next method of decay we'll talk about.

Or the nucleus can just capture an electron, getting to that same energy level and emitting the same gamma ray. So these are two competing mechanisms of decay. And then you might ask, well, when is one going to happen and not the other? Well chances are, the lower energy that transition is, the more likely electron capture is going to happen. So when you look at these energy diagrams, you can see that as the transitions get bigger, the probability of positron decay goes up and up and up. So you need 1.022 MeV to make the positron an electron, but the probability of positron decay very close to this is fairly low. Possible, but unlikely.

Is everyone clear on these two competing mechanisms? So one way of reducing the number of protons is emit a positron, another is gobble up an electron. In the end, they make the same daughter products, but they go by different mechanisms. And they give off different bits of radiation which we can actually sense and measure. Cool. So on to gamma decay or isometric transition. These range from the dead simple, like technetium-99 metastable giving off a characteristic 140 keV gamma ray for technetium, that's the medical imaging procedure we've talked a lot about. To the ridiculously complex, like americium-241, which has a lot, a lot, a lot of different nuclear energy states, all of which release anywhere between 1 and a lot of gamma rays.

And this is what's referred to as isometric transition. So we'll say gamma or isometric transition is like the same thing, they're just different words for it. These are called isomers. They've got the same number of protons and neutrons, so it's the same nuclide, but at an excited state. And we call it gamma decay because we emit gamma rays or photons.

I think this one is the easiest one to understand, because the reaction goes something like-- let's say we have a parent nucleus with Z protons and A neutrons. Nothing happens. It's about the easiest nuclear reaction there is. Except you do give off a gamma ray. And we'll usually put a star or something to denote an excited state. So when you see a star in the reading over there on a nuclide where the charge would be, that's an excited energy state that will likely decay by IT or gamma decay.

There is also a competing mechanism for isometric transition or gamma decay, and that's what's called internal conversion. In this case you can kind of think of it-- this isn't the correct physical explanation, but it's a perfectly good mental model, that the gamma ray would either just be emitted from the nucleus, at which point you would see it, and the energy of the gamma is the same as that Q , or it kind of hits an electron on its way out, ejecting that

electron. So instead of finding a gamma ray, you may just get an electron emitted at an energy very, very close to that gamma ray.

The difference between the gamma ray energy and the electron energy is its binding energy. Because if a gamma hits an electron on the way out, it has to overcome the binding energy of that electron, at which point the rest of the energy is just its kinetic energy. So again, I don't think that's the precise physical mechanism, but it's a perfectly good mental model to remember what this is. A gamma can either just get out on its own or it can hit an electron on its way out. If you hit the electron on your way out, just like in electron capture, then you get a larger shell electron falling down to the inner shell emitting an X-ray just like before.

And then there's one other process I want you guys to be aware of. Has anyone here ever heard of Auger electron emission? Er, yeah. So in this case, instead of sending out an X-ray, you can think of it like the X-ray kind of hits the-- another electron on its way out. That's not the actual process that happens, but let's just think of it like that. And then that electron is ejected usually from a much outer shell.

And we can actually use these Auger electrons because they have specific but very low binding energies to do imaging and elemental analysis of materials. So this is another one of those things where the stuff you're learning today is used in an Auger electron microscope up and Building 13 to do combined imaging and elemental analysis of materials.

I want to skip back a sec, because let's say we have this decay diagram right here, a pretty simple one. Cesium-137, that isotope that everyone was worried about from the release from Fukushima. Can either proceed by just beta decay, or beta decay followed by an isometric transition. And shown here is a spectrum of all the different electron energies that you'll get out.

If you remember from last time when we talked about, let's see, the energy of a beta particle emitted versus let's say the number of those particles emitted, if this is the Q value for that reaction, you don't always get a beta particle out at the Q value-- in fact, you never do. It looks something like this, where they'll be some, let's say, average or some most likely beta energy, which is about one-third Q. Depends on the reaction, but that's a good rule of thumb.

So if you've got a 1.174 MeV beta particle, you're going to see a spectrum of electron energies ranging from 0 to 1.174. And you've got this other beta transition possible at about half an MeV, so you're going to see that same spectrum right there. And then there's these two,

what's called the conversion electrons. That's evidence of the competing process with gamma decay. Which is to say that this gamma decay can either just get out, at which point you see a gamma ray of that energy, or that gamma hits an electron on its way out, knocking those electrons out.

Does anyone know here what it means by K-shell or L-shell? If you do, just shout it out. So that there depends on the-- oh, that's correct. It's the energy level. So let's say we'll draw kind of a Bohr model of a nucleus, we'll call it N. And let's give it three electron energy levels. And let's say there's a couple electrons in the first shell and some electrons in the outer shells. And let's say this electron was struck on its way out by a gamma ray. So it's gone.

At this point, you might have an electron fall from let's call this level 2 to level 1. And so this 2-to-1 transition is called the K-transition. Don't ask me why the letters are the way they are. I probably have read it and have forgotten it because it wasn't that intuitive, but this is referred to as a K-transition. So you may have what's called the K-alpha or a K-beta line, that depends on if you have an even higher energy shell, but whatever this letter is, it tells you what an energy level the electron is going to.

So the K lines would be here. The L-line-- yeah, I'm sorry. Let me back up and say that again. So the idea here is that if this the-- but let's see. Which one is this? 0.662. So if the gamma ray is at 0.662 MeV, which would be about there, notice that these K-shell and L-shell lines aren't quite 0.662. That's because they have to overcome the binding energy of the electron to get out.

So to jump back to this diagram right here, the gamma ray loses a little bit of energy in freeing the electron, the rest of which can become kinetic energy, which is why you can see that the electron, let's say, was ejected from the K-shell here. And-- yeah?

AUDIENCE: So internal conversion is the actual process of-- when we figure out the process of a gamma ray hitting the electron?

MICHAEL SHORT: I will say that internal conversion, you can imagine a mental model of the gamma ray hits an electron on its way out.

AUDIENCE: But it's not the actual-- that's not physically happening?

MICHAEL SHORT: Physically it's more complicated.

AUDIENCE: OK.

MICHAEL SHORT: Yes.

AUDIENCE: Looks like a [INAUDIBLE].

MICHAEL SHORT: Yeah. Yep. So if you want remember like what's what, I would say, just remember this diagram right here. Yeah, Kristen?

AUDIENCE: You get a gamma ray and an electron or just the one--

MICHAEL SHORT: Just the-- you get just the electron, good question. Is the gamma ray is effectively absorbed in freeing the electron from its bound shell and then imparting kinetic energy. Yep?

AUDIENCE: Is the Auger emission is when another electron hops down an energy level and that [INAUDIBLE] electron?

MICHAEL SHORT: That's right. So that's correct. So I'll spend the next couple of slides going over what Auger electron emission is, but not till we finish the easier stuff, because Auger is a little complicated. Did I see another question out here? Cool. So again, all the competing methods for gamma decay, one, the gamma can just get out; two, the gamma can knock out an electron from, let's say, the K-shell or the L-shell or the M-shell and so on and so on depending on what elements you have. So I'll just label these. Like this would be like the K-shell, this would be the L-shell, this would be the M-shell.

Now I want you to notice something, too. The K-shell electron ejected from the innermost electron is slightly lower than the L-shell electron. Why do you guys think that is? Let's look at the energetics for this process, right? The electron energy level is whatever the gamma ray level is minus the binding energy. Which of these two electrons, the K-shell or the L-shell, do you think is going to be more tightly-bound? The K-shell. The innermost electron is more bound, so it takes away more energy from that gamma ray. Let's say this is the gamma. It takes more energy to eject an electron from the K-shell than the L-shell. Or in other words, the gamma loses less energy ejecting a less-bound electron.

So that's why you see these. If there were an M-shell, there-- I don't know what element this was drawn for, but let's say-- oh, for cesium. So there probably is an M-shell. It's just that as you get down in energy levels, the probability of finding an electron from these outer and outer energy levels gets way and way less likely. So you'll usually just see a K or an L-shell electron.

And if I asked you to draw one of these on a problems set or an exam, just drawing the K and the L-shells is perfectly sufficient. Because that way at least you'll know that there's a couple of possibilities. Yeah?

AUDIENCE: What are the two curves on the right graph there?

MICHAEL SHORT: On this one?

AUDIENCE: Yeah.

MICHAEL SHORT: So these two curves represent the probability of finding an electron emitted at that energy. So this curve right here where you get a maximum beta energy of 512 keV comes from that beta decay. And the maximum for the 1.174 comes from this beta decay. So the total curve would be the sum of each of these four things. If I just said draw the total probability of detecting an electron at that temperature, you just add those up. I saw two other questions or were they the same thing? Yeah?

AUDIENCE: Yeah, I was going to ask what the [? lower ?] [INAUDIBLE], but his question made me think. So is it kind of like the area under both those curves sums to 1? Because the probability given that like 0.512 max is 95% of--

MICHAEL SHORT: Ah. So the question was is the area under each of these curves 1? Not with this scale. Here we're just showing a relative number of electrons. So if you want to find what's the total probability that cesium will emit an electron of each energy, if you integrate under all of these curves, that will sum to 1. If you're looking at just one of these decays and you're saying, if cesium undergoes this decay, what's the probability of each of these energy levels? Then you only integrate under the relevant curve. What's more practical is usually what's the probability of finding any electron at any energy from cesium? Then you take into account all the possible decays, draw all the curves independently, add them up, and you get the total probability function whose area will be 1? Yes?

AUDIENCE: So maybe you could say that all L-shell electrons would be ejected if [INAUDIBLE] in the K-shell due to the fact that it's less tightly-bound?

MICHAEL SHORT: Wait, can you say the last part again?

AUDIENCE: Shouldn't we say the L-shell electron will be ejected if [INAUDIBLE] energy in the K-shell due

to the fact that it's not as tightly-bound?

MICHAEL SHORT: That's correct. So the L-shell electron in the second shell is less tightly-bound than the first one, which is why it's ejected with more energy. It doesn't take as much of the gamma's energy to get it out. If you were to get, let's say, the outermost electron ejected, which happens and Auger, which we'll go over next, it can take up anywhere from 1 to 7 eV. Really, really low energy. That's what we call the work function or the energy required to make the outermost electron out.

So any other questions on this before I go Auger and show you that process? Cool. So then let's get into what is Auger electron emission? It's exactly-- Luke, is that what you said? Were you asking about it?

AUDIENCE: Yeah.

MICHAEL SHORT: Yeah So it's exactly like what Luke said. Normally if you have a hole in a lower-level energy shell, an electron from a higher shell will fall down to fill it, emitting an X-ray. A competing process for this is another electron from a very similar energy shell will get ejected instead. The mental model for this, which again, is not the true physical picture but it's perfectly fine to think of it like this, the X-ray hits another electron on its way out. And you can look at the energetics accordingly.

Where for Auger emission, let's say the kinetic energy the Auger electron is the difference in the final and initial electron energy states minus the binding energy of the Auger electron, which will usually be very low, because the Auger electron that's emitted is usually one of the outer shell electrons. So to help make this a little more concrete, I wanted to show-- I will do a little calculation example-- it's just addition, so it's not that hard.

So let's say we were measuring-- I don't even know what this is. And we started to see some characteristic Auger electrons for copper, platinum, carbon, and oxygen. And the question is, why do we see oxygen coming out right there at about 501 eV? Very, very low energy compared to what we've been talking about.

We can actually look at the binding energies of some of the different electrons in oxygen. Luckily there aren't that many electrons in oxygen. The first-- the only-- well, the only K electrons, let's say, have a binding energy of 532 eV, the L1 is 24, and then the L3's something else, and one of the other p orbitals is 7 eV. And so the formula is pretty simple. It's

just 532 minus 24-- that's the difference between the final and initial energy levels-- minus the seven to free that outer electron-- comes to 501 eV, which is exactly where you see the Auger line for oxygen.

So when I ask you what are all the possible things that you could see during the decay of something, something, something, if I were to show you this curve and ask what's missing, what would you do? Where would you draw the Auger electrons on this curve? Yep?

AUDIENCE: Like almost on the vertical axis because it's [INAUDIBLE].

MICHAEL SHORT: Yep. 500 eV would be like, I don't know, one pixel away on this graph. But if you want a complete answer to this question, you've got to take into account all the possible beta energies for all the possible beta decay mechanisms; all of the possible conversion electrons for whatever gamma come out-- in this case, there's only one gamma; and Auger electrons which could compete with X-ray emission. So everyone clear on that? Yeah?

So the question is if you eject an electron from one of the inner shells, does that eventually create an Auger electron, right? It can. These are competing processes. So to the X-ray can just escape during that transition, or we'll assume that it hits another electron on its way out and emits an Auger electron. So these are also competing processes. So you'll see one or the other-- in reality, you'll see a lot of both with different probabilities. Because you're usually not looking at one atom, it's usually looking at a lot. Cool. Any other questions on IT as an isomeric transition or internal conversion or Auger, what this is all about?

Cool. Wanted to give you one note, too. That these Auger electrons are really, really low energy, which means the only ones that get out of the material are in the top, like, tens or so nanometers of the material. So it's a very surface-sensitive technique. So if you want to do a really detailed surface analysis or profiling, you can scan an electron beam across the sample and then collect the Auger electrons that come out-- skipping ahead to our calculation-- and get an elemental profile that'll tell you how much of each element is where depending on how many of its Auger electrons you can count.

And it's pretty-- it's a pretty cool technique. There are just machines that do this now. Any interest in seeing one of these at the Center for Materials Science and Engineering, because we could try to arrange that, too? Cool, OK. I'll see what I can do. That'll be fun. And the last decay that we haven't talked about and did not show up in our generalized decay diagram from last time. We did talk about neutron decay, there's one other one that probably wouldn't

fit on this diagram. Does anyone know what it is? Spontaneous fission.

So this can happen-- that's right. This happens with very heavy elements. Then usually the heavier it is, the less stable it is, the higher probability this is at which the nucleus can once in a while, it-- nuclei just explode sometimes. Giving off two fission products, any number of neutrons-- usually between 1 and 3, a couple of gamma rays, some anti-neutrinos, and a whole bunch of other crazies.

And so that, of course, doesn't fit on the diagram, but it is another type of decay that I'll ask you guys to analyze on the homework. And here's a hint-- you already analyzed part of it in problems set 1. I'll ask you to go a little deeper in problem set 3. So anyone have a question for myself? Cool. OK. Bear with me because I skipped back to like slide really far away. Oh cool. That thing actually works. Right to the summary.

So in summary, the radioactive decay processes are more-- I think the energetics are pretty easy. The formulas aren't that hard to remember because most of them are just parents minus daughters minus something. But what I do want you to remember is which mechanisms compete with which other ones and why. And if I were to tell you, draw me a spectrum of photons that you may see from the decay of caesium-137, or draw me a spectrum of electrons, you'd be able to draw what that is so that when you go do lab number 4 and we count our big bag of burnt bananas and you know that there's potassium-40 in there, you know what peaks to start looking for.

Because you're not just going to see-- let's do a little flash-forward to detectors now. And some other stuff that nuclear engineers actually do on a daily basis. Let's say you're counting the energy of photons as a function of, let's say, the number of photons that you count. You're never-- you're almost never just going to see a lone potassium-40 peak like that. It's very, very rare that you would just capture the gamma ray as is.

There's going to be a lot of other things that will go into it, which I'm not going to give away yet because we're going to go over photon interactions in like a week or two, but you do have to think about, well, what other X-rays might you see? What if the gamma-- what if this gamma ray hits an electron on the way out, and then you end up with some X-rays? Let's say you might have some K-level X-rays and some L-level X-rays and maybe some Ms? These could all be possible as well. I'll just label those real quick.

Does anyone know how to find these energies? What those X-ray levels are? Anyone ever

heard of the Lyman series? The emission lines from ionized hydrogen or anything like that? That's kind of the simpler case of it. The idea here is if you want to figure out what's the wavelength of light that's going to be emitted, it'll be that's this thing called the Rydberg constant, a more complex formula for which I have in the notes, times 1 over your final shell squared minus 1 over your initial shell squared.

So the idea here is that you can look up these-- this Rydberg constant for any element that you have, and there's actually-- there's what's called an R infinity constant and I think you just multiply by Z-- I forget what power it is, but I will get it for you next time. And then it's just a matter of the squares between the final and the initial shell levels where n can vary from 1 to theoretically infinity. Realistically I've never heard of anything beyond a g orbital, so let's just say it's that. Or something like that.

I'll leave the infinity there. That's technically correct. OK. And so all you need to do is either look up or calculate this constant for your element, and then plug-in the numbers of the shells, and you know what sort of photon energy you're going to get out. And to make that easier for you, I think now is a good time to introduce the NIST X-ray tables. So I want to make sure you can see my screen. And I'm going to show you something that's on the Stellar site which will help you figure this stuff out.

20.2.0.0.1. Good, you can see. Probably will make log in. And all the way at the bottom of the material section, there's the NIST X-ray Transition Energy Database. So for example, you can look at-- I don't know, we were looking at caesium, right? Let's find caesium. And you can start to look at all transitions-- let's look at the simplest one. KL1. An electron going from shell number 2 to shell number 1. Get transitions, and you end up with a table of these energies in electron volts.

So if I were to ask you, let's say, what sort of gamma rays might you see coming off of caesium, that would be the most likely one where you're more likely to eject an inner-shell electron, and it's most likely that a number 2 shell electron will fall down to a number 1 or from the L-shell to the K-shell, whatever you want to call it, or the-- some level orbital to some other-- there seems to me like eight different letters that describe the same thing. I hope you guys get the idea.

If you want to look at all the possible transitions, I have to zoom back out again. Yeah. Let's just scroll through it. But what I want you to notice is that all of the KL transitions are within like

a couple hundred eV from each other. So this is like the first or second or third L-shell electron falling to the K-level. So they call it the KL1, KL2, KL3. They're all from L-shell. They all just might be one of the different electrons occupying that shell, which is why they're not that different.

So if I asked you, draw all of all of the X-rays and gammas coming out, I don't want to see a line for every single level. I'm very happy for you just to say, this line represents the KL series, this line represents the KM series, all the things from shell 3 to shell 1.

Notice also that because it's final minus initial squared, which I covered up, falling from an even further outer shell to the same inner shell should give you a higher energy, which it does, by like 5 keV. And then the KN levels, another 500 eV up. They don't have KO's. Interesting. But they have the K-edge. Anyone know what the K-edge is?

Exactly. It's level infinity, which would mean an electron from somewhere else, right? So this in effect is like the energy it takes to ionize a K-electron or the X-ray that you would get from an electron falling all the way into the K-shell. So this is your kind of level infinity. Notice, it's not that different from level 5. That's why I wrote 6, I erased it for theoretical correctness, but in all practicality, you won't see much else. Does anyone have a question? Thought I saw a hand. OK.

Let's look at the L-series-- oh yes?

AUDIENCE: Sometimes you see L-alpha, L-beta.

MICHAEL SHORT: Yep.

AUDIENCE: Is that the same as the subscript 1, 2, 3, and 4?

MICHAEL SHORT: Yeah, that's another--

AUDIENCE: --corresponding-- it's just a notation.

MICHAEL SHORT: Exactly. Yeah, there's the L-alphas, the L-betas, or you may see L-alpha 1 and 2 in L-beta 1 and 2. So L designates that it's going to shell level 2. Alpha or beta is like LM or LN. Again, I think I've ranted about notations before. Physics is notorious, because whoever-- to describe something, and it gets enough people to infect with the notation that you decide it sticks.

But the main patterns to look at, then, let's say the L1 M whatever. This is from level 3 to level

2, and notice how much lower an energy these are. And the L-edge. 5 keV compared to like 36 keV. And all of these sort of different transitions you can calculate with this formula. And this has just kind of tabulated this formula for you. So whatever you're more comfortable doing is putting this in Excel, look it up on NIST, your choice. Yeah?

AUDIENCE: If we were to calculate that by hand--

MICHAEL SHORT: Mm-hmm.

AUDIENCE: --what would you use for like NF? Say, like, 5?

MICHAEL SHORT: Oh, what is the largest NF, you mean?

AUDIENCE: Yeah. What numbers actually go in NF [INAUDIBLE]?

MICHAEL SHORT: For that I'll put my practical thing back there. You'll never see it much higher than 6 unless you're talking about actinides and super heavy elements with even crazier shell levels. But you'll put the integer shell number, regardless of whether they call it L or S or whatever. Just put the number here, and that will give you the-- a pretty good approximation of the energy transition.

Does anyone remember this from high school? I hope they're teaching this now.

AUDIENCE: We learned this in [INAUDIBLE]

MICHAEL SHORT: Oh, they did in 5-111? Oh, that's good to hear. What about 3-091? Anyone take that? No one took 3-091? Wow, OK. Usually it's like half and half or so. Cool. And let's see, how far does it go? All the way to the L3 N's in the L3 edge. So that's the biggest element they have talking about ridiculous transitions. Yeah. So notice also, as you go up in-- that's number 100.

So this is the heaviest one they have, so most likely to have the largest number of levels. So here, the KL1 is like 114 keV, sometimes indistinguishable from some of these smaller nuclear energy level transitions. So remember I said before, chemistry and nuclear differ by about a factor of a million. Well, not so if you're talking about weak gammas versus heavy elements K-shell transitions or their K-edges. Let's see, the largest X-ray you'd expect would be the K-edge at 142 keV. And the technetium-99 gamma ray comes out of 140 keV. How do you know if it's a gamma or an X-ray? You don't. Unless you have really, really good energy resolution and you can tell them apart. Yeah?

AUDIENCE: This formula in this chart is only for calculating energy of X-rays, right?

MICHAEL SHORT: Correct. This--

AUDIENCE: [INAUDIBLE] for gammas.

MICHAEL SHORT: Things get quantum. So the question was, this formula and this chart, yes, this is only for X-rays and electron shells. There are probably equivalent calculations for nuclear energy levels. I will say that's a 22.02 and far beyond topic. For the nuclear energy levels, just use the decay diagrams to find those. Yeah. The table of nuclides and all their different diagrams. Cool. How far do we go here? LN-- yep, there's no O's. So they never talk about anything beyond shell level 4, even for fermium. So ha, I stand corrected.

OK. Cool. So what I wanted to show you quickly is that series of hydrogen emission lines. So how familiar does this look to folks? Where you can have a transition from level 3 to level 2, level 4 to level 2, and you can actually-- this is a kind of neat thing to verify. I don't it as a problem set question because it's not very nuclear, but you can try this on your own and verify that you can actually calculate the expected wavelength of these photons coming off of excited hydrogen.

Also notice here, it goes all the way out to 9 and out to infinity, because this is electronic excitation. You won't usually get the ejection of anything beyond an M or an N electron even in the largest elements from, let's say, from IC-- what is it? From internal conversion. But you can electronically excite them to whatever energy level to the point of even ionizing them. That's where the infinity comes in.

Cool. So it's like two of five of. So I want to open this up to any questions about decay before we move upstairs to talk about activity, half-life, and series radioactive decay, which is what nuclear activation analysis is all about. So anything here? Yep?

AUDIENCE: Just making sure I understand this. So the H-alpha lines transition from N equals to 3 to N equals 2.

MICHAEL SHORT: Mm-hmm.

AUDIENCE: Would you call that-- in our previous notation, would you call that LM transition?

MICHAEL SHORT: Correct. In our pre-- in our other notation, this would be known as an LM electron. And

probably $L1M1$ because there is only one electron in hydrogen. Yep. So don't let the notations trip you up. As long as you-- I'm sure someone's got a chart of L equals 2 equals whatever other Greek letter someone has designated it for. There's just different ways of saying the same thing. So as long as you know the physics, looking up the notation is just kind of a little pain.

Any other questions on radioactive decay or competing mechanisms? Cool. Let's take a 10 minute break. So I'll see you guys upstairs in Room 307 in 10 minutes. There's no projector there, so we'll do it all on the board.

All right. So I want to start off the second half of today's class by posing and answering a question. Who's still mentally having trouble grasping this idea? What did I tell you? Yeah. So whoever I said it to, I said at least half the class is right there with you, it's true. So in a sentence, it's this-- Q is the conversion of mass to energy. That's all. And the whole point of doing this nuclear reaction energetics to find out if things are or aren't allowed, if they're exo or endothermic is to see how much mass is converted to energy or how much energy has to be converted to mass.

And if you have trouble remembering, just go back to the equation that I see on everybody's T-shirts. And like I said on the first day of class, everyone's got it on their shirts and no one quite understands it, not even the nuclear engineers. Because it's very difficult mentally to grasp the idea that energy and matter are two sides of the same coin or two different forms of the same thing.

So for a nuclear reaction where Q is greater than 0 or exothermic, all that means is that energy is spontaneously created from the destruction of mass. That's all. And for a Q less than 0 reaction or endothermic, when you inject energy into the system, it is absorbed and mass is created straight from this equation.

So does this make more sense to the folks that raised their hands, which is almost everybody? Q is nothing but a quantification of the amount of matter and energy that turn from one to the other. And all the balance stuff we've been doing for almost the last month has been to serve, to quantify, and to predict it. So I hope that helps.

I know that MIT students have this gift for being able to hide behind the math, and I know that's true because I used to be one myself. And you could get through the day or get through the class getting the math right without really understanding the physics or the mechanism

behind what's going on. So everything we've done for the last three weeks can be summed up in one sentence-- mass and energy are the same thing. We've just had a lot of math to get there and be able to-- yeah, it still kind of makes your head want to explode, right?

If you think about it, if you make an-- we want to make an endothermic reaction happen, you have to put kinetic energy into one of the particles. And that system in the just barely-allowed state, the nuclei won't really be moving, or at least the-- yeah. The nuclei won't really be moving afterwards, because you'll have turned energy into matter. That kinetic energy is turned into mass energy. Just like you can turn potential into kinetic or thermal into mechanical or vice versa, it's other forms of energy. Kind of mind-blowing.

Well anyway, so now that we finished radioactive decay, I want to get into the concept of activity half-life, and then we're going to start but not finish serial radioactive creation and destruction. In your readings, you'll see an equation that looks something like this where we're going to be at the end of the day today. If you want to show off how much of some isotope N_1 exists as a function of time, you may have seen something that looks like this.

We're going to be able to understand how these equations are created, and because this is MIT, we're going to take it further and add specifically driven mechanisms, like can you create an isotope not just by decay from one isotope to another, but by intentionally making it? Which is exactly how NAA Nuclear Activation Analysis works. You fire neutrons into a material. They turn into something else and start decaying in series. This is what we're going to be working out the math for, but I want to make sure at every step that we get the physics right.

So first, a very quick primer on where-- what is activity and where does half-life come from? So we define the A , the activity of a substance as pretty simple. It depends on the amount of the substance that's there, and it depends on what's called this decay constant. So this amount-- let's just say like atoms. And this decay constant is in units of 1 over second. An activity is in, let's say, atoms destroyed or decays per second. That thing right there is called the decay constant, λ .

And if we want to see, let's say, how much of a substance is decaying at a certain time or what's its activity and get a measure of how quickly does it take to decay away, we can start by saying, well this in effect is a destruction rate of isotope N . So we can make the simplest of differential equations and say, the amount of substance N as a function of time is just minus its activity, which equals minus λn .

I hope I don't have to explain how to solve this differential equation, so I'm going to go through it pretty quickly. What's the method you use for this to get n as a function of t ?

AUDIENCE: 3?

MICHAEL SHORT: Yes. You're all in 18-03 or have finished 18-03, right? This should have been day 1. So we'll just separate variables, divide each side by N , multiply side by dt . So we get dN over N equals negative λdt . You can integrate both sides, and we get the natural log of N equals minus λt plus some integration constant.

We're going to do a little bit of trickery, and to make things in a nice form that we can deal with, let's just call this log of N_0 . They're just numbers, right? We haven't defined what this constant of integration is. Everyone cool with us doing that? So then we can subtract log of N_0 from each side. And we get $\log N$ minus log of N_0 equals minus λt . So this is like saying $\log \frac{N}{N_0}$ equals minus λt . We'll take either the power of both sides and we get $\frac{N}{N_0}$ over and not equals e to the minus λt . And right there you've got your exponential decay equation. This is the easy part.

And what this tells you is it is a larger decay constant. Well let me ask you a question, then. Would a larger decay constant mean a faster or a slower decaying isotope? A larger decay constant-- correct-- means that you've got more of these decays happening per second. So a larger λ means faster decay. And we also can define a quantity called the half-life, which means N at $t = 1/2$ equals $1/2 N_0$. So for that, all you have to do is plug in $t = 1/2$ for t , and $1/2 N_0$ for N , and you actually get this relation where the half-life is just \log of 2 over λ , better known as 0.693. But we'll just leave it as \log of 2 for exactness. And that's all there really is for decay in half-life.

So I'll pose another question to you. Something with a larger decay constant, will it have a larger or a smaller half-life?

AUDIENCE: Smaller?

MICHAEL SHORT: Smaller, because they're inversely related. So I'd say from this quick derivation, these are the two things to note. Is that something with a larger decay constant decays faster and therefore has a shorter half-life. So when we were separating out our isotopes in nuclear activation analysis into what Mike called the shorts and the longs, he was separating them by half-life to say that-- let's say the same amount of activation or the same amount of creation, the ones

with shorter half-lives will be hotter, more radioactive, but for less time.

And so what we're going to be doing is what's called short nuclear activation analysis because we don't want to count for like days or weeks or months. Yep?

AUDIENCE: So is half-life-- is the decay constant just a property of the given substance, given element?

MICHAEL SHORT: The decay constant is a property of the given isotope specific to that type of decay. So if we were to draw that generalized radioactivity diagram-- let's say we have potassium-40, which can either go by beta decay to-- what comes in beta-- what comes after-- I think it's calcium-40. Or it can go by positron or electron capture to argon-40. Each of these processes has a different half-life.

And then in that chain-- remember, let's do the americium-241, and I'm going to channel my one-year-old son in drawing this decay diagram. It looks something like that with all sorts of transitions. I didn't scream like he usually does, but whatever. We're on camera. Each of these decay-- each of these transitions may also have its own half-life. So between isometric transitions, they're usually very, very fast, but once in a while they're not. Like technetium-99 metastable to technetium-99 has a half-life of around six days, which is why it's so useful as a medical isotope.

So when you see something marked M for metastable, all that means is there is some sort of a gamma, also known as an IT transition, with a particularly long half-life. Everyone clear on what all that means? Cool. Well, there'll be some time for it to sink in because with these definitions in hand, I want to pose a problem to you guys.

Let's say I start off with some amount of an isotope N_1 . And it decays by some mechanism-- we don't care what-- to isotope N_2 , and it decays to some isotope N_3 with decay constants λ_1 and λ_2 . This is what we call serial radioactive decay. How do we construct a system of equations to tell us what is N_1 as a function of time, what is N_2 as a function of time, and what is N_3 as a function of time? Where do we begin in a general sense?

OK, so let's start with N_1 . We kind of have an expression for N_1 already, but let's start out with a differential equation. So the form of all of these equations for everything serial radioactive decay and burning isotopes in a reactor and creating isotopes in a reactor is going to take the following form. The general equation is simple. The change equals creation minus destruction. Or the simple thing is let's say the change is a source minus a sink. And we'll have to come up

for every one of these isotopes for a mathematical way to describe what's the source and what's the sink.

So if we want to measure the change in N_1 as a function of time, what are the sources of isotope N_1 ? Yeah?

AUDIENCE: Isn't there no sources?

MICHAEL SHORT: No sources. We're starting off with some fixed quantity of N_1 . Let's just call it $N_{1,0}$. But you're right, there's no continuous source of isotope N_1 . What about its destruction?

AUDIENCE: Decay to N_2 ?

MICHAEL SHORT: Yeah. Decay to N_2 . So we've got the equation for that right there. It depends on the decay constant of number 1 and the amount of number 1. So what we're doing here-- I love how this course is timed with 18-03 because you're learning ordinary differential equations in 18-03, and we're going to be solving everyday ordinary differential equations for a fair bit of this course. So it's one of those rare times you get to like learn math and put it to use at the same time instead of six years later, it's just kind of nice.

So that's easy. Let's go to the more challenging one. What is the source of isotope N_2 ?

AUDIENCE: Decay from N_1 ?

MICHAEL SHORT: Decay from N_1 . So how would I mathematically write that?

AUDIENCE: Just $\lambda_1 N_1$.

MICHAEL SHORT: Just $\lambda_1 N_1$. And what is the destruction of isotope N_2 ? Anyone else?

AUDIENCE: $\lambda_2 N_2$?

MICHAEL SHORT: Takes the same form. It depends on the decay constant of isotope 2 and the amount of isotope 2 that's there. How about isotope 3? Where does that come from?

AUDIENCE: $\lambda_2 N_2$?

MICHAEL SHORT: That's it. The source is $\lambda_2 N_2$. What are the sinks or the destruction?

AUDIENCE: Nothing.

MICHAEL SHORT: Nothing. So we have a very simple set of posed differential equations to describe the production and the destruction of these three isotopes. So let's imagine now that N_1 was, let's say, radium, which exists all throughout the soil and rocks; N_2 is radon, the gas that's produced from radium decay; and then N_3 could be let's say one of the stable daughter products of radon.

So these are the sorts of calculations that are done all the time in real life to see-- if you know how much radium is in the rock, how much radon do you expect to breathe in? Because at the same time you're producing radon from radium decay. And the radon is decaying itself. So you can't just say, oh, the activity of the radium equals the amount of radon because it's being created and destroyed all at the same time, and it depends on how much there is around.

Same thing with nuclear activation analysis. You'll, let's say, you'd start off with sodium-21, you can create sodium-22. Sodium-22 will decay by positron emission. What comes before sodium? Probably neon, I think, 22. That's my guess. Yeah. So if you want to say how much sodium-22 is there, well you're both creating it from sodium-21 from neutron absorption, and you're decaying it naturally by positron emission among other processes.

We're going to get back into how do we deal with the neutrons thing probably on Tuesday, But for now, let's work on solving this system of equations. So I think N_1 is pretty easy because we already have the solution for it. So I'll just write it. $1t$. The harder one is N_2 . So what can we start by doing? We've got an ordinary differential equation with-- it's just this first order, but there's two variables. So how do we deal?

AUDIENCE: Pick another equation?

MICHAEL SHORT: We do it-- well, we have other equations, so-- actually, we've got this one. Yes. Substitute N_1 in here so we get everything in terms of N_2 and constants and type. So we'll rewrite this equation as dN_2/dt , which I'm also just going to write as N_2 prime. And when you use this a little bit of notation that I'll leave up there on the board because it's going to be a lot faster in writing. Which equals λ_1 and $10e$ to the minus $\lambda_1 t$ minus $\lambda_2 N_2$.

Can we separate variables here? I don't see an easy way. So, 18-03 experts, how do we solve this type of first order differential equation? And I'll give you-- I won't give you a hint, I'll give you a little bit of consolation. No one from last year knew how to approach this. So if you don't know, I won't be disappointed. Yeah?

AUDIENCE: You've got N_2 dot plus $\lambda_2 N_2$ equals $\lambda_1 N_1$ dot--

MICHAEL SHORT: Mm-hmm.

AUDIENCE: So you could put-- add $\lambda_2 N_2$ to both sides.

MICHAEL SHORT: Add $\lambda_2 N_2$ to the both sides. I think you're on to what I'm thinking about. Also, if you can't read something I write, please stop me and let me know and I'll be happy to erase it. I don't think I've said that yet, but it takes a lot of control for me to get my handwriting legible on the board let alone on a piece of paper. So if you can't read, please let me know.

OK. So now we've got the N_2 's separate from the N_1 's, what do we do next?

AUDIENCE: -- N_2 has the form A times e to the negative $\lambda_1 t$?

MICHAEL SHORT: OK, let's try this. Assume N_2 has the form e to the what?

AUDIENCE: Negative $\lambda_1 t$?

MICHAEL SHORT: Negative-- has the form e to the λ_1 negative t .

AUDIENCE: With that A in front--

MICHAEL SHORT: With an A , some constant in front. What makes you say that?

AUDIENCE: Like if you take the derivative with respect to time, then the next term will still have a e to the negative $\lambda_1 t$?

MICHAEL SHORT: Mm-hmm.

AUDIENCE: Cancel all those out and solve for A .

MICHAEL SHORT: Cool. So that is one way to do it. It's going to get a little messy, though. There's another method specifically for equations of the form. Let's call it y prime plus-- I'm going to use notation they may have used in 18-03. I hear a couple of aha. Anything look familiar about this type of equation? OK, what is it?

AUDIENCE: Not following.

[LAUGHTER]

MICHAEL SHORT: It's not that hard. Anyone remember the word integrating factor? And it was probably done horribly and on like six math boards or whatever. So I'm going to show you the simpler way to do this. The idea here is we want to multiply everything by something-- by some function μ . Put a μ there, put a μ there, put a μ there. I think that's the notation that's usually used in differential equations, such that this thing right here is shrinkable through the product rule.

Or the product rule-- I'm not assuming everyone remembers-- says that let's say you have some function a of t , b of t prime is like a prime times b plus a times b prime. So we're kind of getting around to the method that Luke was talking about, but we're going to do it by a little bit of a cleaner way. We multiply every term by some function μ such that this part is one of these perfect product rules at which point we can shrink and integrate the expression.

Without going through the derivation of how integrating factors are done, I'll just let you know that this function μ ends up being e to the integral of p . That is our integrating factor. So that's the end result of what was probably six boards of 18-03. Am I right or am I mistaken? Well, things haven't changed in 15 years. Cool.

OK, so what is μ for this equation? Luckily, p of t is pretty simple. Which part of this equation right here is our p of t -like term?

AUDIENCE: $\lambda^2 N^2$.

MICHAEL SHORT: Actually, just λ^2 . Because we've got our variable right here that right there is our p of t . So we'll just say that μ equals e to the integral of-- uh, yep-- of $\lambda^2 dt$, which is just e to the $\lambda^2 t$. That's our integrating factor right there. So we'll multiply every term right here by that.

So we'll say e to the $\lambda^2 t$ times N^2 prime plus $\lambda^2 e$ to the $\lambda^2 t$. Anyone see what is going on here? There's the product rule thing going on-- times N^2 plus e to the $\lambda^2 t$ times $\lambda^2 N^2$ equals 0. And we have successfully created something here that can be shrunk up with a product rule. Yeah?

AUDIENCE: Should that be a minus e to the $\lambda^2 t$ $\lambda^2 N^2$ -- when you moved it over? It's positive--

MICHAEL SHORT: Oh yeah. There's an equal sign there, isn't there? That's what tripped me up. Thank you. So there is indeed a minus sign there because I skipped the step putting everything on one side of the equation. Yep?

AUDIENCE: Could we do this with variational parameters instead?

MICHAEL SHORT: Where you replace one variable with-- or replace a couple of variables with another one?

AUDIENCE: Well yeah, just like the homogeneous solution, and then you'll find like a factor--

MICHAEL SHORT: Yeah. There are lots of ways of solving a first order ODE like this. Sure. So this would work with various parameters. It would work with what Luke's talking about. It works with this one. This just happened to be a particularly simple one because the integrating factor's so simple. So let's cram this up right here. So this is like saying N_2 times e to the $\lambda_2 t$ prime minus-- and we've got two e to the somethings that we can combine right here. So I'll just say $\lambda_2 - \lambda_1$ and $10e$ to the $\lambda_2 t$ minus $\lambda_1 t$ equals 0.

So now I will put this term back on the other side by doing that. And now we just integrate both sides. And we get $N_2 e^{\lambda_2 t}$ equals, let's see. Becomes $\frac{1}{\lambda_2 - \lambda_1} 10 e^{\lambda_2 t} + C$. And in this case, our initial condition, well, how much of isotope N_2 did we start with? Have we specified that yet? No? Let's make it simple.

Let's assume that the initial amount of isotope N_2 equals 0. We put some isotope in the reactor or start it off with some amount of isotope like radium. Didn't start off with-- it didn't start off with any radon, and just kept going. So then all we have to do is divide each side by either the $e^{\lambda_2 t}$, and that cancels those, that cancels that and that, and we end up with N_2 as a function of time equals $\frac{1}{\lambda_2 - \lambda_1} 10 e^{(\lambda_2 - \lambda_1)t}$. OK. And we've got an expression for N_2 .

How about N_3 ? Do we even have to solve this one? I see a couple of people shaking their heads no, why is that?

AUDIENCE: Basically you already solved for N_1 , they're just not minus signs.

MICHAEL SHORT: Well, not quite. Because we-- well yeah, I guess we've kind of solved it for N_1 , but now we take this expression for N_2 , stick it in here, and then solve that, it's going to get messy. So I'm going to show you something mathematically now that I'll show you graphically later. There's a conservation equation that we're missing here. If we sum up isotopes N_1 , N_2 , N_3 , equals what? Conserving total number of atoms.

AUDIENCE: N1,0?

MICHAEL SHORT: Exactly. N1,0. In this situation where we started off with some known quantity of isotope 1 only, you can't change the number of atoms here, you can only change the type of atoms. So we don't have to solve for E3-- oh sorry, for N3, because N3 is just N1,0 minus N1 minus N2. And that takes like an extra 10 minutes out of today's lecture.

So later on when we have a projector on Tuesday, I will show you these equations graphed out where I've-- actually, I'll share this with you. Did I tell you guys about the Desmos graphical calculator? Or have I shown this to you yet? Go here for all of your graphing needs. It's free, and the best part that I like is that anytime you define some parameter, it automatically makes a slider bar so you can play with the equations. And you can just-- say, like, well what if L1 and L-- lambda 1 and L2 are equal? What if they were way different? And it just graphs the solutions for you. It's pretty useful. So I'll show you some of that on Tuesday when we actually have a screen.

Let me see what time it is. Oh sweet, we've got plenty of time. So now I want to pose the following questions to you guys. I'm going to erase stuff from here because we still have some space. How do we model nuclear activation analysis using this kind of equation? We'll start off with the same equation. So let's say we'll have a minus lambda 1 N1, we'll have N2, minus lambda 2 N2 minus something. N3 equals something-- let's see, there's a lambda 1 N1, there's a lambda 2 N2 minus something.

I've left some trailing minus signs to indicate that we don't have complete equations for this yet. So for the case of nuclear activation analysis where we have some imposed flux-- flux-- of neutrons. So anyone remember from some of our previous flash-forwards how do we turn these into creation and destruction rates of these different isotopes?

AUDIENCE: Could you repeat that?

MICHAEL SHORT: Yep. So let's say we've now stuck N1 in the reactor. And we're now using the reactor to create different isotopes like N2 and N3, but at the same time they're in the reactor, they're getting cooked as well by some imposed flux of neutrons. How do we set up and not solve the system of equations to describe this?

AUDIENCE: Are they-- are N2 and N3 both getting like--

MICHAEL SHORT: They're getting destroyed and whatnot?

AUDIENCE: Are they just like decaying or are they also getting like added stuff from neutrons--

MICHAEL SHORT: Well--

AUDIENCE: Because that depends on the isotope.

MICHAEL SHORT: That depends on the isotope. So let's define what the system-- this system is. Let's say we stuck in some other isotope N_0 , and we put it in, and we're going to have to say it's-- if we have N_0 prime, some minus some creation term. And in this case, N_0 can absorb a neutron to become N_1 . N_1 is decaying to N_2 , N_2 is decaying to N_3 . But also, N_1 can be burned by neutrons N_2 can be burned by neutrons and N_3 can be burned by neutrons.

So here I've given you kind of a simplistic situation that doesn't usually exist. Here I've given you a situation that you could replicate in the reactor. How do we model this nuclear activation analysis process? Well first of all, what's the creation rate of N_0 , the stuff we put in the reactor? Are we creating any? No? Luke?

AUDIENCE: It's all going to be created if N_2 absorbs the neutron and [INAUDIBLE]?

MICHAEL SHORT: It could be. So if we had this for the following nuclear reaction, now it's getting crazy. We can model that, too. Let's do it. OK, I was going to say no, but let's do it. We can-- so what I'm trying to do here is give you the mathematical tools to model any real physical situation. Usually in this class like when I took it, the discussion stopped here and we got to start looking at different graphs of secular versus transient equilibrium like in the reading. But I want you guys to have the intuition to say, all right, let's take any crazy decay diagram, right? And N_3 becomes N_1 , let's just go nuts.

How do we set up the differential equations for this assuming that computers can solve them? In every case-- where's my long pointer? Go back to this here. The change is the creation minus the destruction. So what are all the creation sources in our new scenario for N_0 ? Well I pose you a simpler question. If there is no isotope N_2 , can you create any N_0 ? No. Because the only way to make N_0 is to start with N_2 .

So we know its creation term is going to have an N_2 in it. What else does it depend on?

AUDIENCE: Cross-section.

MICHAEL SHORT: I heard both of the pieces of the answer correct at the same time. It depends on the flux of neutrons, and it depends on the cross-section. This macroscopic cross-section right here. OK? Or I'm sorry, no, no. It depends on the microscopic cross-section because we have an amount of N_2 . But it does depend on how many neutrons you throw at it and what is the probability of each of those neutrons make some N_0 .

So what we've got right here is a reaction rate. So who remembers from the like second or third lecture, we said a reaction rate can be expressed like macroscopic cross-section times of flux, which is the same as a microscopic cross-section times number density times flux? But this is better known as a macroscopic cross-section.

Remember, I kind of showed this to you very briefly when we talk about cross-sections, now is where we actually use them. So the cross-section's like the probability that a neutron coming in to atom N is going to react with it. The macroscopic cross-section in units of, let's say, 1 over centimeters is the total-- let's say the total probability accounting for how many are there, and the flux is in neutrons per centimeter squared per second. Combine these together, and you get a reaction rate in atoms per centimeter cubed per second, a volumetric reaction rate. There we go.

So N_1 can be created. Let's give this cross-section a designation from N_2 to N_0 . So let's call it cross-section Σ_0 . How can N not be destroyed? I'll give you a hint, it looks very similar to this term. Anyone want to take a guess? Yeah?

AUDIENCE: Could it undergo fission?

MICHAEL SHORT: We haven't specified that. I'm going to cut the craziness at there, I think. But just look at the reactions right here. N_0 can absorb a neutron and become N_1 . So how do we mathematically write that?

AUDIENCE: Minus the cross-section N_0 --

MICHAEL SHORT: Yep. Minus the cross-section of?

AUDIENCE: N_1 .

MICHAEL SHORT: Let's say 0 going to 1. Let's just call it that. Times the neutron flux. Times what?

AUDIENCE: Times N_0 ?

MICHAEL SHORT: Times the amount that's there, N_0 . So this is your destruction term. Using this pattern, we can fill in all the remaining terms for all the remaining isotopes. So what are the creation mechanisms for isotope N_1 ? Well, just follow the arrows.

AUDIENCE: --same term--

MICHAEL SHORT: Mm-hmm.

AUDIENCE: --on the first equation for N --

MICHAEL SHORT: Yep. There's this one right here. We can have--

AUDIENCE: --the sign.

MICHAEL SHORT: But flip the sign because it's creation. So $\sigma_{0,1} \text{ flux } N_0$. And what else can create N_1 because we're just going crazy today? N_3 can create N_1 because we said so. But-- yeah, because we said so. So now we'll say also we'll have this cross-section for 3 turning into 1 times flux times N_3 . Minus the decay of N_1 using our activity expression, minus what else?

AUDIENCE: [INAUDIBLE] cross-section of the [INAUDIBLE]?

MICHAEL SHORT: I heard some-- yep, that will be the cross-section of N_1 . Let's call it going to some isotope we don't care about times the flux times N_1 . So as long as you can draw a like arrow decay and destruction production diagram, we can put this to math. That's the crazy thing. Let's finish it up. How about N_2 ? We know that N_1 can decay to N_2 . What are the other production mechanisms for N_2 ? Follow the arrows.

AUDIENCE: [INAUDIBLE]

MICHAEL SHORT: That's it. Because we're not changing anything at this point. What are all the destruction mechanisms for N_2 ?

AUDIENCE: Decay or [INAUDIBLE]

MICHAEL SHORT: Yep. It can decay, it can be absorbed by a neutron to become something we don't care about. Let's see, let's call it cross-section 2 null times flux times N_2 minus the cross-section from 2 to 0, just this term with a minus sign. How about N_3 ? What are all the ways we can make N_3 ? Could you say it a little louder?

AUDIENCE: Only from decay.

MICHAEL SHORT: Only from decay. Again, just see which arrows are pointing at it. And what about the destruction mechanism for N3?

AUDIENCE: [INAUDIBLE]

MICHAEL SHORT: Yep. So there's some probability it decays, let's say, cross-section 3 null times flux times N3 minus this arrow going back to N1. Running out of space, too many units. From 3 to 1 flux N3. So the point of accepting the escalation of this problem into something crazy is that it doesn't matter how crazy it gets. As long as you have like an arrow-based diagram or a flow chart to say which isotopes become which other isotopes by which other means, you can pose and correctly write the set of equations that defines them. This is when I would bring in MATLAB or Mathematica.

I could make you do this analytically, but this isn't a course 18 class and-- yeah, we don't want to go there. Cool. So I think-- I guess it's probably getting towards 10 of 10 of. Close enough. It's like three of 10 of. So I'd like to open it up to any questions because we let this-- I let this escalate freely to prove the point that as long as you know what decays into what or what creates or destroys what, you can set up the equations correctly. What we'll be doing on Tuesday is graphing this. Where we can pose an arbitrarily complex set of equations and you can start looking at, well, the change in one depends on the amount of the other, and you can almost graphically solve this on paper. Forget Mathematica in MATLAB.

If you look at last year's exam, I actually posed a more complex set of these and said draw the solution. And I'm going to-- we're going to show you how to do that. But any question on how we formed these? Yep?

AUDIENCE: What is sigma again? I'm sorry.

MICHAEL SHORT: So we just said let's say N3 decays to some isotope we don't care about. That's how I initially had it, and then I think Luke said, well can N2 become N0? And I said, yeah, sure. So there can be a cross-section for every type of reaction. So in reality, you might have any reaction under the sun, right? One isotope could absorb a neutron and decay by like any three or four different mechanisms into something else. You may have different probabilities for each of these. Yeah?

AUDIENCE: Do those decay constants [INAUDIBLE]

MICHAEL SHORT: The only time I'd expect you to find these decay constants is if I told you what these isotopes were. Those are also listed on the table of nuclides. As in they give you the half-life, and you know from this half-life relation what the decay constant is. If I didn't tell you what these isotopes were, I would just have you keep these symbols as λ_1 and λ_2 .

And I might pose a question which will solve graphically on Tuesday, like let's say, solve this set of equations for λ_2 is much greater than λ_1 . I don't care what the numbers are, let's just look at that general relation. And all the graphs for that sort of situation will follow the same pattern. Cool.

Any other questions on how we constructed this set of differential equations? And know that I'll never ask you to solve them numerically or analytically. Yeah. That's why we have computers and this is the future. These I might expect you to know how to derive, but this is the simplest possible case.

All right, if there's no questions, then let's take another 10-minute break and I'll be here for recitation to go over whatever problems you guys would like to do.