22.01 Fall 2016, Problem Set 6 Solutions

November 19, 2016

Complete all the assigned problems, and do make sure to show your intermediate work.

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1 Skill-Building Problems (50 points)

1. **Short Answers (5 points each)**

   (a) Explain, using stopping power expressions and cross sections, why the energy loss due to ionization drops off so sharply with increasing energy, while radiation loss increases linearly.
   The complete form of the non-relativistic stopping power expression for any charged particle is as follows:

$$-\frac{dT}{dx} = \frac{4\pi N Z_1^2 Z_2 e_c^4}{m_e v^2} ln\left(\frac{2m_e v^2}{\bar{I}}\right) \quad (1)$$

   Neglecting the constant terms, this takes the following energy-dependent form:

$$-\frac{dT}{dx} \propto \frac{1}{E} ln\left(E\right) \quad (2)$$

   This implies that at lower energies, the $\frac{1}{E}$ term completely dominates, and it drops off very sharply with increasing energy. This is solely due to the charged particle spending less time near each given electron, because it is moving faster. This also relates directly to the differential cross section for energy as follows, from Yip Equation 11.15:

$$-\frac{dT}{dx} = N \int_0^T E \frac{d\sigma}{dE} dE \quad (3)$$

   At much higher energies, the ability of a particle with a given charge to ionize electrons farther and farther away (at higher impact parameters) increases, but not that quickly.

   (b) Explain the quantitative differences in stopping power of electrons as they reach relativistic speeds. What energy cutoff do you consider relativistic, and why?
   As particles reach relativistic speeds, their effective mass increases, meaning that they will be deflected less per unit length for the same applied Coulomb force between them and the electrons in the medium. This gives them greatly increased stopping power. Let's say that we choose an energy cutoff where the stopping power is increased by no more than 5% at its peak from its non-relativistic value. Then we can graph the following ratio as a function of β:

$$Ratio = 1.05 = \frac{\cancel{\frac{4\pi N Z_1^2 Z_2 e_c^4}{m_e v^2}} ln \; \frac{2m_e v^2}{\bar{I}^2(1-\beta^2)} - \beta^2}{\cancel{\frac{4\pi N Z_1^2 Z_2 e_c^4}{m_e v^2}} ln \; \frac{2m_e v^2}{\bar{I}}} \quad (4)$$

where we have used the relativistic stopping power expression from Yip, Equation 11.12. Solving for β:

$$1.05 = \frac{ln\left(\frac{2m_e v^2}{\bar{I}^2(1-\beta^2)} - \beta^2\right)}{ln\left(\frac{2m_e v^2}{\bar{I}}\right)} \tag{5}$$

$$1.05 = ln\left(\frac{2m_e v^2}{\bar{I}^2\left(1-\beta^2\right)} - \beta^2 - \frac{2m_e v^2}{\bar{I}}\right) \tag{6}$$

$$e^{1.05} = \frac{2m_e v^2}{\bar{I}^2\left(1-\beta^2\right)} - \beta^2 - \frac{2m_e v^2}{\bar{I}} \tag{7}$$

and substituting in $v = \beta c$:

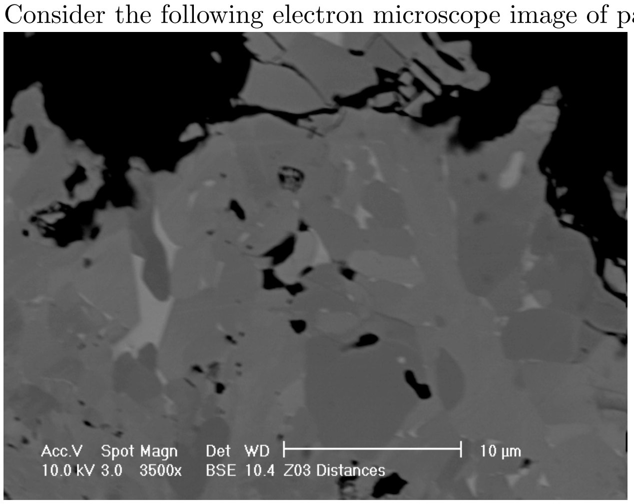
$$0 = \frac{2m_e \beta^2 c^2}{\bar{I}^2\left(1-\beta^2\right)} - \beta^2 - \frac{2m_e \beta^2 c^2}{\bar{I}} - e^{1.05} \tag{8}$$

$$0 = \beta^2\left[\frac{2m_e c^2}{\bar{I}}\left[\frac{1}{\bar{I}\left(1-\beta^2\right)} - 1\right] - 1\right] - e^{1.05} \tag{9}$$

$$0 = \beta^2\left[\frac{1.022\,MeV}{\bar{I}}\left[\frac{1}{\bar{I}\left(1-\beta^2\right)} - 1\right] - 1\right] - e^{1.05} \tag{10}$$

This function was graphed as a function of β, and its intersection with the x-axis will reveal at what value of β the stopping power increases by 5% at its peak. This value was found to be $\beta \approx 0.27$, even over a wide range of $\bar{I}$ ranging from 5-50keV.

(c) Consider the following electron microscope image of palladium diffusion into zirconium carbide:



Where is the palladium in this image, and how do you know, based on your knowledge of electron interaction mechanisms with matter? Back up your answer with a relevant quantitative estimate of electron interactions. Using an image processing program, measure the relative brightnesses of various types of spots in the images. Can you guess the average atomic number of each of the spots? In other words, is brightness linearly proportional to the type of electron interaction(s) that you are interested in?

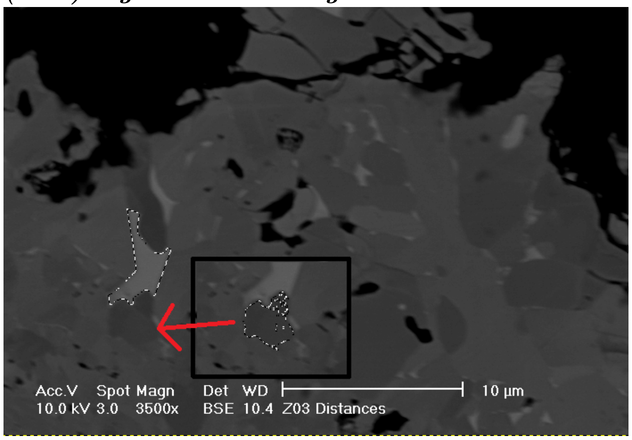
The palladium is the white spots in the image. This was an image made with the BSE (Back-Scattered Electron) detector, where electrons fired from the microscope scatter directly backwards into a detector. The cross section for backscattering is as follows:

$$\sigma_{bs} = \frac{\alpha Z^2}{4\beta} \tag{11}$$

where $\alpha = 1\,barn$, and β is defined as above.

Using the GIMP image manipulation program, regions were automatically selected

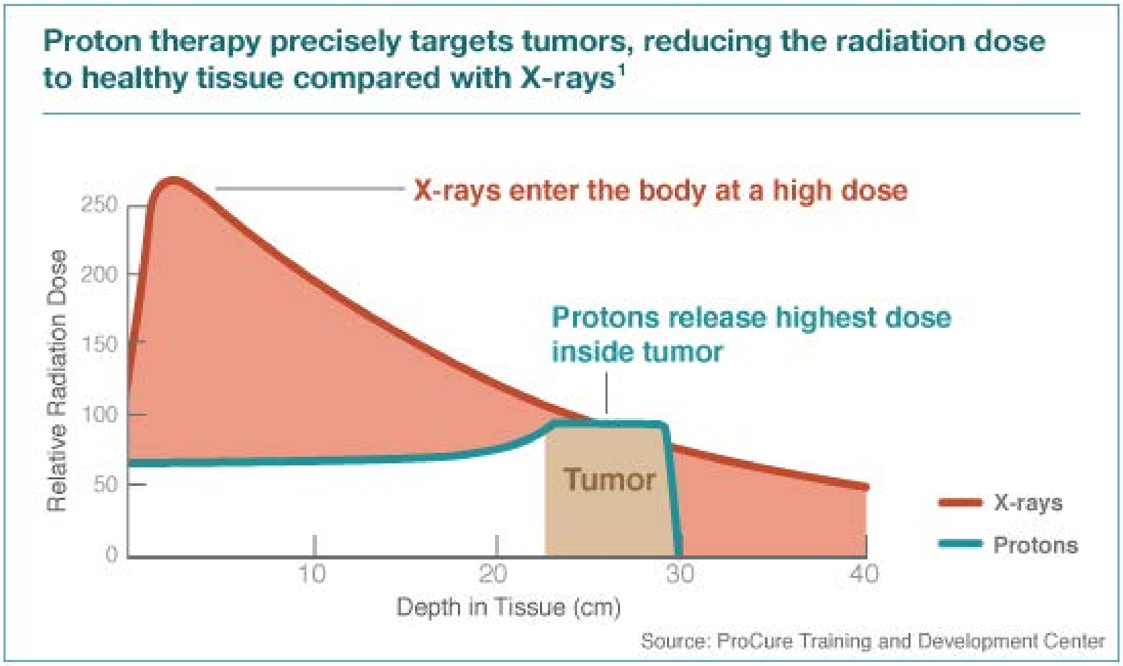
using the "Fuzzy Select Tool" to represent the brightness of bright (Pd) and dark (ZrC) regions. These regions are shown in the image below:



Using the Colors→Info→Histogram menu, the average 8-bit brightnesses of these two regions were found to be 85.7 (bright) and 53.0 (dark), respectively. Using $\mathrm{Z_{Pd}} = 46$ ***and*** $\bar{Z}_{ZrC} = 23$***, this would yield a ratio of exactly*** $\frac{\sigma_{Pd}}{\sigma_{ZrC}} = 4$***. The discrepancy between the expected brightness ratio and the observed one is that in an electron microscope, the user can dynamically change the brightness (offset) and contrast (multiplying factor) of the luminosity of the entire image at once. This was done in this case to enhance contrast in the image, making a direct cross section comparison impossible using just image analysis.***

(d) Explain, using attenuation and stopping power, why protons are far more effective at damaging a localized tumor. Draw any applicable range relations and/or attenuation graphs to make your point.

Because protons are charged particles, they have a finite range in materials, due to their stopping power becoming very large at low energies. This is because as a proton slows down, it spends more time in the vicinity of other electrons/nuclei, transferring more of its energy the slower it gets. Photons on the other hand are uncharged, and only undergo exponential attenuation. This means that most of the radiation damage by photons is done <u>before</u> they reach the tumor, while most of the ionizations by protons happen <u>inside</u> the tumor if emitted at the correct energy. This graph helps to explain the point:



Courtesy of ProCure Proton Therapy Center. Used with permission.

<u>Image</u>: https://www.procure.com/Proton-Therapy-Basics/what-is-proton-therapy
Photons undergo exponential attenuation:

$$I = I_0 e^{-\left(\frac{\mu}{\rho}\right)\rho x} \tag{12}$$

while protons have a finite range defined as:

$$R = \int_0^{E_i} -\left(\frac{dT}{dx}\right)^{-1} = (Constants) * \int T = (Constants)\, T^2 \tag{13}$$

2. **Calculation Problems (10 points each)**

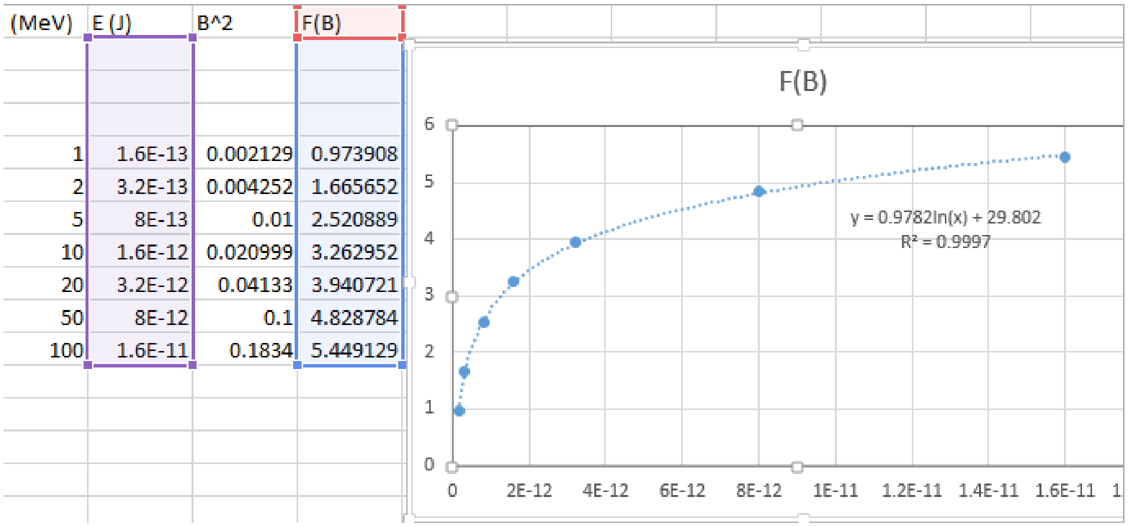
   (a) Analytically develop a graph of the range of protons in lead vs. their energy, over a range between 100 keV and 100 MeV, with at least ten points on the curve. Check your answer by using SRIM (www.srim.org) to simulate the range of protons in lead at each energy.
   For this problem, we take the equation for the range of a charged particle traveling through matter:

$$R = \int_0^{E_i} -\left(\frac{dT}{dx}\right)^{-1} dT = \int_0^{E_i} \left[\frac{4\pi N k_0^2 z^2 Z e^4}{m_e c^2 \beta^2}\left[ln\left(\frac{2m_e c^2 \beta^2}{\bar{I}\left(1-\beta^2\right)}\right) - \beta^2\right]\right]^{-1}; \qquad k_0 = 8.99 \cdot 10^9 \frac{N-m^2}{C^2} \tag{14}$$

   Let's redefine all the logarithm stuff as some simple function to make writing our equations easier:

$$F(\beta) = \left[ln\left(\frac{2m_e c^2 \beta^2}{\bar{I}\left(1-\beta^2\right)}\right) - \beta^2\right] \tag{15}$$

   We use the mean ionization potential of lead as 10*Z=820eV, and we tabulate this function $F(\beta)$ to make it easier to deal with in the integral:



(MeV)	E (J)	B^2	F(B)
1	1.6E-13	0.002129	0.973908
2	3.2E-13	0.004252	1.665652
5	8E-13	0.01	2.520889
10	1.6E-12	0.020999	3.262952
20	3.2E-12	0.04133	3.940721
50	8E-12	0.1	4.828784
100	1.6E-11	0.1834	5.449129

We use this trend line to redefine $F(\beta)$ ***:***

$$F(\beta) = 0.9782\, ln(E) + 29.802 = 0.9782\, ln(29.802\, E) \tag{16}$$

Now we substitute everything in:

$$R = \int_0^{E_i} -\left(\frac{dT}{dx}\right)^{-1} dT = \int_0^{E_i} \left[\frac{4\pi N k_0^2 z^2 Z e^4}{m_e c^2 \beta^2}\right]^{-1} [F(\beta)]^{-1} dT = \int_0^{E_i} \left[\frac{m_e c^2 \beta^2}{4\pi N k_0^2 z^2 Z e^4}\right]\left(\frac{M}{M}\right)[F(\beta)]^{-1} dT \tag{17}$$

$$= \int_0^{E_i} \left[\frac{m_e\left(\frac{1}{2}Mv^2\right)}{2\pi M N k_0^2 z^2 Z e^4}\right][F(\beta)]^{-1} dT = 6.07\cdot 10^{21}\int_0^{E_i} E\,[F(\beta)]^{-1} dT = 6.07\cdot 10^{21}\int_0^{E_i} \frac{E}{0.9782E\, ln(29.802\, E)} dT \tag{18}$$

where we have used quantities in the above formulas are given in SI units as shown below:

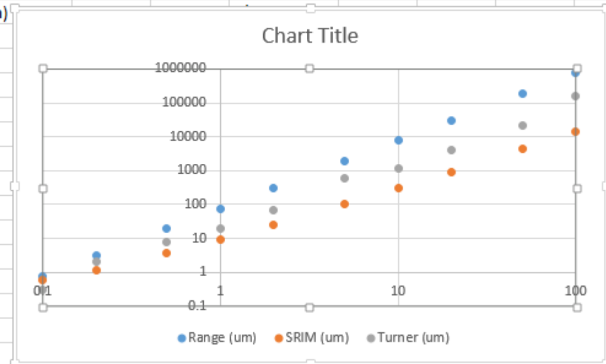
Constant	Meaning	Value	Unit
m_e	Electron mass	$9.11\cdot 10^{-31}$	kg
M	Incoming ion mass	$1.67\cdot 10^{-27}$	kg
N	Lead number density	$\frac{\rho N_A}{MM} = \frac{\left(1{,}134\frac{kg}{m^3}\right)\left(6\cdot 10^{23}\frac{atoms}{mole}\right)}{0.208\frac{kg}{mole}} = 3.27\cdot 10^{27}$	$\frac{atoms}{m^3}$
k_0	Coulomb constant	$8.99\cdot 10^{9}$	$\frac{N-m^2}{C^2}$
z	Proton charge	1	—
Z	Lead nuclear charge	82	—
e	Electron charge	$1.6\cdot 10^{-19}$	C

Now we note that the integral above would be absolutely hideous, as the integral of $E/ln(E)$ ***is a special, complex function called the 'exponential integral.' We don't want to deal with this, and we note that*** $F(\beta)$ ***doesn't vary much over the full energy range. Let's just forget it for now, and see how we do.***

$$= 6.07\cdot 10^{21}\int_0^{E_i} E dT = 3.03\cdot 10^{21} E^2\, [m] \tag{19}$$

We have just re-derived the approximation for range as $R \propto T^2$***. Now we can just calculate the stopping power at one energy and scale accordingly. Using this formula, which takes in the energy of the proton in Joules, we arrive at the following table of data and graph, having only calculated the stopping power at 100keV and scaled other energies accordingly:***

E (MeV)	E (J)	Range (m)	Range (un	SRIM (um	Turner (um)
0.1	1.6E-14	7.7568E-07	0.77568	0.605	0.3
0.2	3.2E-14	3.1027E-06	3.10272	1.2	2
0.5	8E-14	1.9392E-05	19.392	3.57	8
1	1.6E-13	7.7568E-05	77.568	9.27	20
2	3.2E-13	0.00031027	310.272	25.4	70
5	8E-13	0.0019392	1939.2	101	600
10	1.6E-12	0.0077568	7756.8	301	1180
20	3.2E-12	0.0310272	31027.2	937	4180
50	8E-12	0.19392	193920	4370	21800
100	1.6E-11	0.77568	775680	14200	155000



Clearly just calculating the range from ionization stopping power is insufficient except for very low energies. This neglects the total stopping power used in SRIM, which includes both nuclear and radiative terms. It's perfectly OK if your calculated results don't line up with those in SRIM, provided you calculated ranges with only the ionization stopping power, and actually checked your answers using SRIM. Note that we have also provided the tabulated values from Turner p. 127, note how all three differ! In practice one would use codes like SRIM to compute full stopping power/range values.

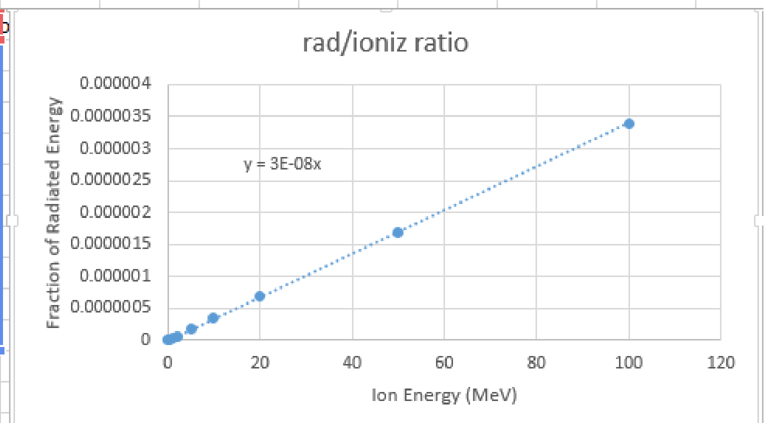
(b) Analytically develop a graph showing the ratio of radiated bremsstrahlung energy to the incoming ion energy for the cases above.

For this problem, we can assume that the relative strength of the bremsstrahlung stopping power compared to the ionization stopping power captures this value. So, we just set up a calculation as follows, taking our formula from the blackboard on October 25th:

$$\left(\frac{\left(\frac{dT}{dx}\right)_{rad}}{\left(\frac{dT}{dx}\right)_{ioniz}}\right) = Z\left(\frac{m_e}{M}\right)^2\left(\frac{T}{1400m_ec^2}\right) = 212,196T \tag{20}$$

We simply plug in the values of T in Joules for each energy, note that Z=82 for lead, and M=1amu for protons:

E (MeV)	E (J)	rad/ioniz rati
0.1	1.6E-14	3.39514E-09
0.2	3.2E-14	6.79027E-09
0.5	8E-14	1.69757E-08
1	1.6E-13	3.39514E-08
2	3.2E-13	6.79027E-08
5	8E-13	1.69757E-07
10	1.6E-12	3.39514E-07
20	3.2E-12	6.79027E-07
50	8E-12	1.69757E-06
100	1.6E-11	3.39514E-06



We note that this relation is linear, and that basically no energy is lost to bremsstrahlung by protons in lead up to 100MeV. This is because the protons are far too massive to be considerably deflected at high energies, so almost all energy is lost by ionization stopping power.

(c) Analytically develop a graph of the range of 1 MeV ions in lead vs. their atomic mass, with at least ten points on the curve. Check your answer by using SRIM (www.srim.org) to simulate the range of protons in lead at each energy. What other variable do you have to constrain to make a uniform comparison?

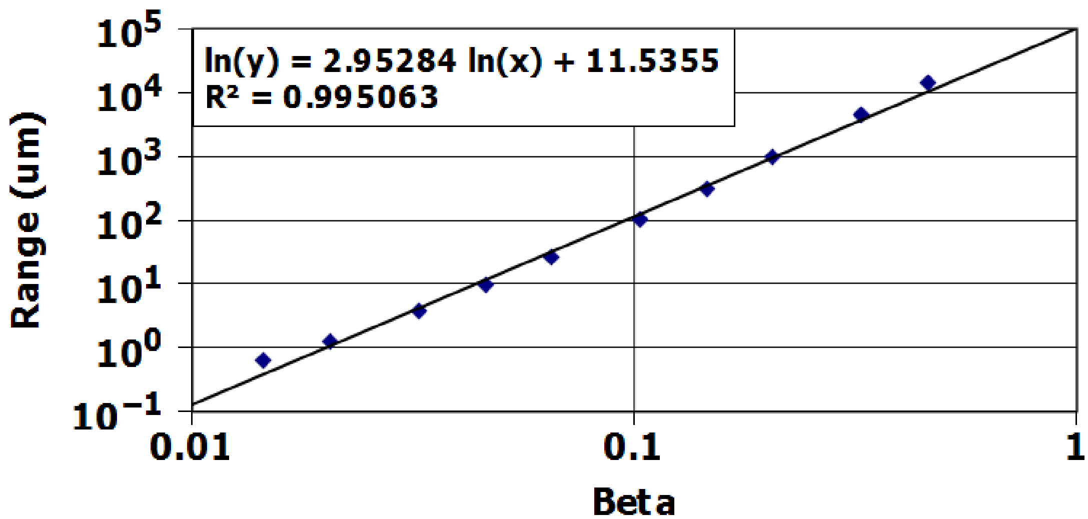
For this problem, we refer to Turner pp. 126-128 for a range-energy relation between different ions of similar speed v (or β):

$$R(\beta) = \frac{M}{z^2} R_p(\beta) \tag{21}$$

where R_p is the range of a proton of the same speed β. Let's choose the light atomic numbers, so we choose from M=1 amu to M=10 amu in increments of 1 amu, and we calculate the speed expressed as β for each 1 MeV ion:

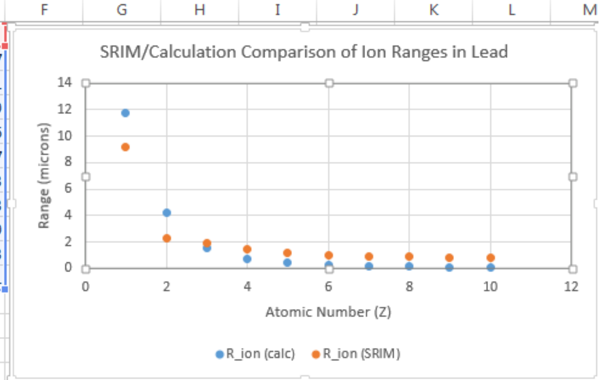
$$\beta = \frac{v}{c} = \frac{1}{c}\sqrt{\frac{2\left(\frac{1}{2}Mv^2\right)}{M}}; \qquad \frac{1}{2}Mv^2 = E_i = 1\,MeV \tag{22}$$

We now tabulate these values for each incoming ion, look up the corresponding proton range from our previous calculations in Problem 2a, and compute the expected range from Equation 20 above. We don't have enough low-β values from our previous calculations, so let's fit a function to the tabulated values and interpolate. We use the SRIM values, to see how well our range-energy relation compares to an exact calculation:



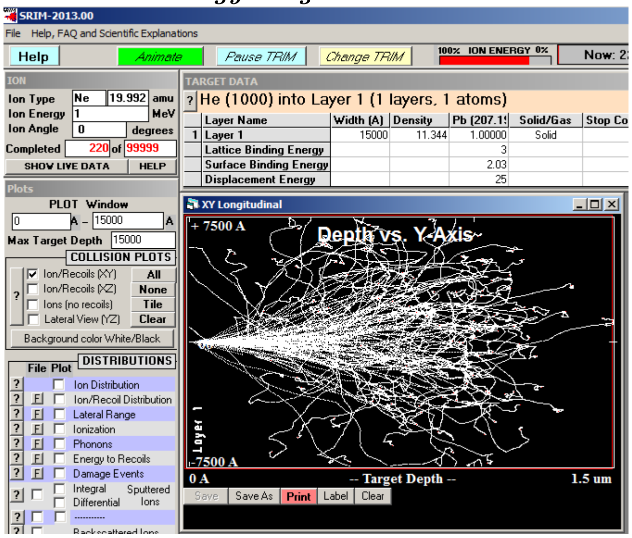
This graph appears to fit pretty well, so we can just plug in values of beta for each ion, use Equation 20, and estimate the range of each ion. We arrive at the following tabulated data and graph:

Z	beta	Rp (SRIM, um)	R_ion (calc)	R_ion (SRIM)
1	0.046280676	11.72041037	11.72041037	9.17
2	0.03272538	4.212075156	4.212075156	2.31
3	0.026720161	2.31478924	1.543192827	1.89
4	0.023140338	1.513733441	0.756866721	1.46
5	0.020697347	1.088853682	0.435541473	1.17
6	0.018894007	0.831887788	0.277295929	1.03
7	0.017492451	0.662556882	0.189301966	0.93
8	0.01636269	0.544004759	0.13600119	0.89
9	0.015426892	0.457172493	0.101593887	0.83
10	0.014635235	0.391311686	0.078262337	0.81



Here we have chosen the light ions, and assumed that their atomic mass is equal to double the atomic number ($M = 2Z$). Note how much larger the SRIM range is, that is likely because we don't account for the decreased stopping power at very low energies due to ion charge neutralization. From this screenshot of SRIM, note how much

the ions deviate from their original directions, indicating that our physical model of ionization stopping power from small-angle Coulombic scattering is not correct at such a low energy range:



Courtesy of James F. Ziegler. Used with permission.

2 Noodle Scratchers (50 points)

1. One way of ensuring a uniform dose to a tumor of finite size (not small) is called intensity modulated radiation therapy (IMRT), where the proton beam is modulated in energy and/or angle to shift the Bragg peak to different specific locations. The goal is to maximize dose to the whole tumor, while minimizing the dose to surrounding tissue. For the following questions, assume we are trying to treat a tumor 1cm in diameter, surrounded by 5cm of healthy tissue.

   (a) (30 points) Derive a relationship between the required energy and the atomic mass of a singly charged ion required to reach the center of the tumor. This will tell you how big of an accelerator one needs to use each type of ion. You may want to use SRIM to roughly check your calculations. Use the following formulas of stopping power in your calculations:

$$-\left(\frac{dT}{dx}\right)_{ioniz.} = \frac{4\pi N k_0^2 z^2 Z e^4}{m_e c^2 \beta^2}\left[ln\left(\frac{2m_e c^2\beta^2}{\bar{I}\left(1-\beta^2\right)}\right) - \beta^2\right]; \qquad k_0 = 8.99\cdot 10^9 \frac{N-m^2}{C^2} \tag{23}$$

$$\frac{-\left(\frac{dT}{dx}\right)_{ioniz.}}{-\left(\frac{dT}{dx}\right)_{nucl.}} = \frac{2M}{m_e Z}\frac{ln\left(\frac{\gamma_e E_i}{\bar{I}}\right)}{ln\left(\frac{\gamma E_i}{E_d}\right)} \tag{24}$$

$$\frac{-\left(\frac{dT}{dx}\right)_{rad.}}{-\left(\frac{dT}{dx}\right)_{ioniz.}} = \left(\frac{m_e}{M}\right)^2\left(\frac{ZE_i}{1400 m_e c^2}\right) \tag{25}$$

The most important concept in this problem is that all three mechanisms of stopping power are active, so the total stopping power (which ultimately determines the range) must add all three mechanisms. Because the other two types of stopping power are given as ratios of the ionization stopping power, we can write an expression for the total stopping power:

$$\left(\frac{dT}{dx}\right)_{total} = \left(\frac{dT}{dx}\right)_{ioniz.} + \left(\frac{dT}{dx}\right)_{nucl.} + \left(\frac{dT}{dx}\right)_{rad.} \tag{26}$$

We already found in Problem 2b that protons have essentially no radiative stopping power at these energies (even more true for particles heavier than protons), so we

can neglect it:

$$\left(\frac{dT}{dx}\right)_{total} = \left(\frac{dT}{dx}\right)_{ioniz.} + \left(\frac{dT}{dx}\right)_{nucl.} + \left(\frac{dT}{dx}\right)_{rad.} \to 0 \tag{27}$$

This yields the following formula for total stopping power:

$$\left(\frac{dT}{dx}\right)_{total} = \left(\frac{dT}{dx}\right)_{ioniz.} \left(1 + \left[\frac{-\left(\frac{dT}{dx}\right)_{ioniz.}}{-\left(\frac{dT}{dx}\right)_{nucl.}}\right]^{-1}\right) \tag{28}$$

$$= \frac{4\pi N k_0^2 z^2 Z e^4}{m_e c^2 \beta^2} \left[ln\left(\frac{2m_e c^2 \beta^2}{\bar{I}\left(1-\beta^2\right)}\right) - \beta^2\right] \left(1 + \frac{m_e Z\, ln\left(\frac{\gamma E_i}{E_d}\right)}{2M\, ln\left(\frac{\gamma_e E_i}{\bar{I}}\right)}\right) \tag{29}$$

Now we turn our attention to figuring out how much the nuclear stopping power matters. For the case of 80 MeV protons, the quantity $\frac{m_e Z\, ln\left(\frac{\gamma E_i}{E_d}\right)}{2M\, ln\left(\frac{\gamma_e E_i}{\bar{I}}\right)}$ ***evaluates to roughly 0.0019, so we can neglect that also:***

$$\left(\frac{dT}{dx}\right)_{total} = \frac{4\pi N k_0^2 z^2 Z e^4}{m_e c^2 \beta^2} \left[ln\left(\frac{2m_e c^2 \beta^2}{\bar{I}\left(1-\beta^2\right)}\right) - \beta^2\right] \left(1 + \frac{m_e Z\, ln\left(\frac{\gamma E_i}{E_d}\right)}{2M\, ln\left(\frac{\gamma_e E_i}{\bar{I}}\right)}\right) \to 1 \tag{30}$$

We also note that $\beta = \frac{v}{c}$ ***and*** $\beta^2 = \frac{v^2}{c^2} = \frac{2E}{Mc^2}$***, so we substitute this into the formula above:***

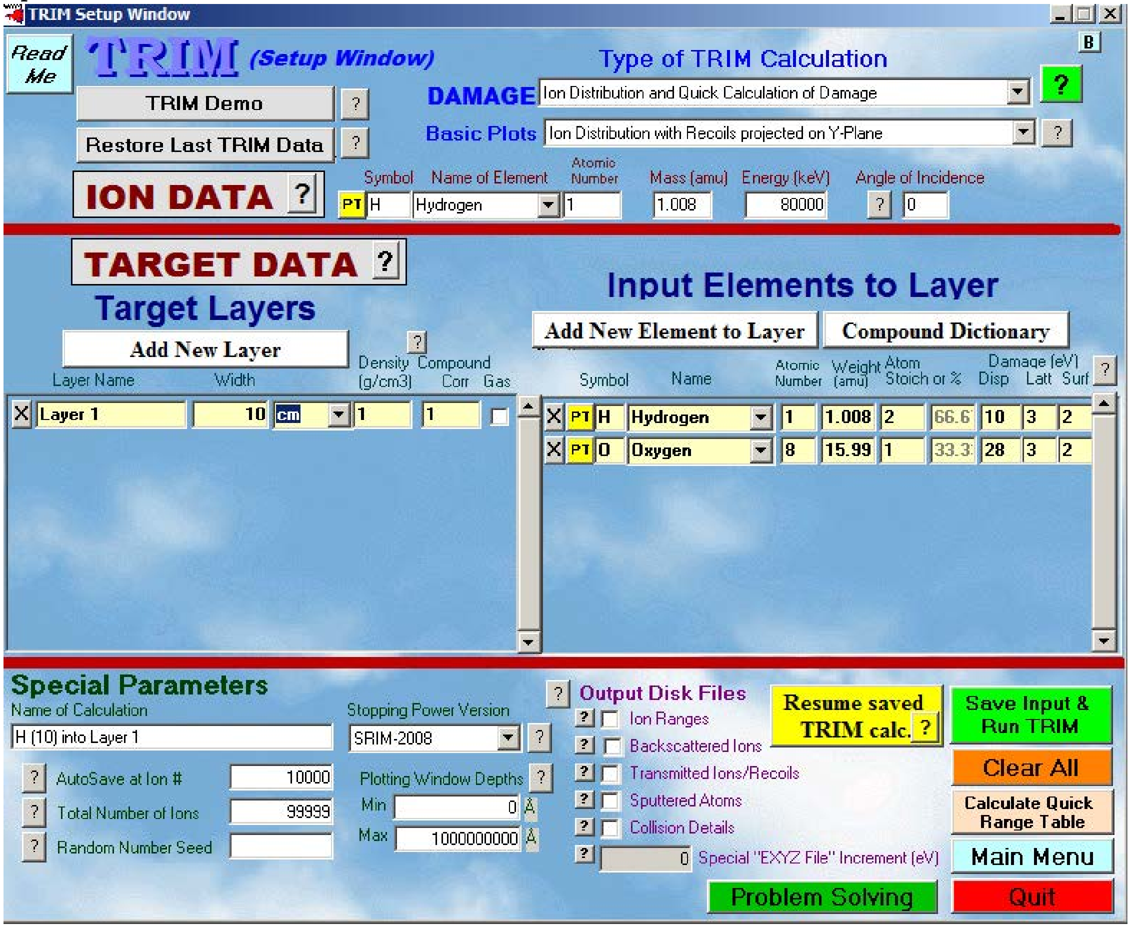
$$\left(\frac{dT}{dx}\right)_{total} = \frac{4\pi N k_0^2 z^2 Z e^4}{m_e \cancel{c^2} \frac{2E}{M\cancel{c^2}}} \left[ln\left(\frac{2m_e \cancel{c^2} \frac{2E}{M\cancel{c^2}}}{\bar{I}\left(1-\frac{2E}{Mc^2}\right)}\right) - \frac{2E}{Mc^2}\right] \tag{31}$$

$$\left(\frac{dT}{dx}\right)_{total} = \frac{2\pi M N k_0^2 z^2 Z e^4}{m_e E} \left[ln\left(\frac{4m_e E}{M\bar{I}\left(1-\frac{2E}{Mc^2}\right)}\right) - \frac{2E}{Mc^2}\right] \tag{32}$$

We can look up from Table 5.3 in the Turner book (p. 127) that the energy of protons required to reach 5.08cm through water of a density of 1 $\frac{g}{cm^3}$ ***is 80 MeV. We then try to estimate how this stopping power will vary with M, the mass of the incoming ion, and what implications that will have on the required energy. For all ions*** β ***is very small, so the term*** $\frac{2E}{Mc^2}$ ***is also very small, since kinetic energies much less than the ion's rest mass are required to get it 5cm into soft tissue (approximated as water).***
We therefore have a relation for total stopping power which varies roughly with M (and therefore A), plus a logarithmic term that has a value of between 4-7 in the ˜100MeV range. Therefore, let's take that as a factor of five. Finally, we note that most of any ion's range is determined by its stopping power at relatively high energies, where it travels the farthest. All ion's stopping powers increase greatly at very low energies. This roughly 5*M proportionality would yield required energies for protons, carbon ions, and iron ions of 80 MeV, 4,800 MeV, and 22,400 MeV, respectively.
Using SRIM to check these values for hydrogen (Z=1, A=1), carbon (Z=6, A=12), and iron (Z=26, A=56), we get required energies of 80 MeV, 1,750 MeV, and 18,500 MeV respectively. Energies of the incoming ions were iteratively tuned until the correct range (or very close to it) was reached. Guesses were made by using the Range-Energy relation in Turner, p. 128 (Equation 5.43), as the Table 5.3 was actually given for ranges of protons in water! We just had to calculate the required energy (in the form of its speed β***) for each heavier ion, and we used those as first guesses in our SRIM calculations:***

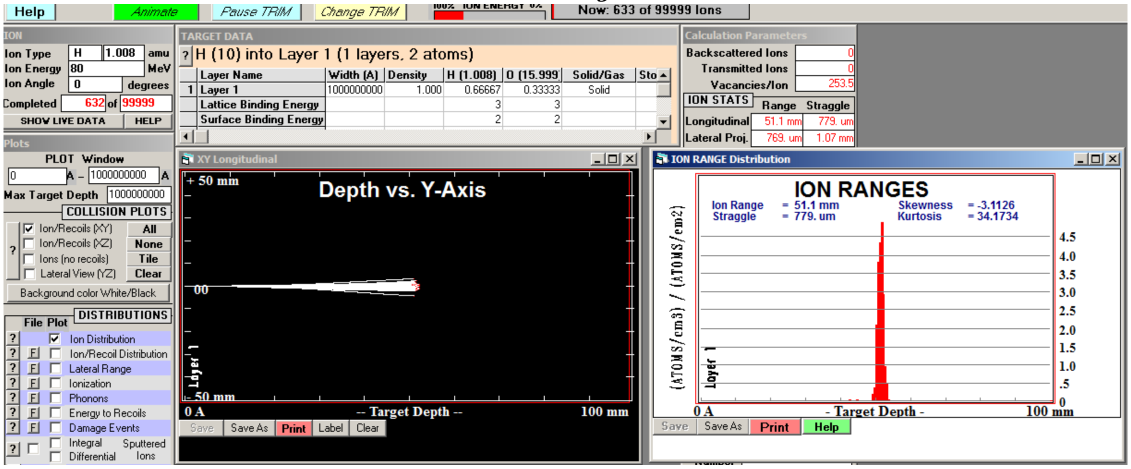
$$5\,cm = R\left(\beta\right) = \frac{M}{z^2} R_p\left(\beta\right) \qquad \frac{5z^2}{M} = R_p\left(\beta\right) \tag{33}$$

Note that in SRIM, you must explicitly set the density of the material to the correct value. SRIM simply calculates it as an average of the elemental number densities, which is very wrong for H_2O. *See the following screenshot for the correct way to set up this calculation:*



Courtesy of James F. Ziegler. Used with permission.

This is what it should look like while running the calculation:



Courtesy of James F. Ziegler. Used with permission.

(b) (15 points) Now derive a relationship between the amount of ionization of each of these ions at their starting energy and within the Bragg peak. This gives ratio of the amount of damage to the tumor compared to the surrounding tissue directly from the ions themselves.

The number of ion pairs (i) *produced per unit path is directly proportional to the*

ionization stopping power, and is given as follows:

$$i = \frac{1}{W}\left(\frac{dT}{dx}\right)_{ioniz} \tag{34}$$

where W is the energy required to create an ion. Therefore the larger the ratio of the maximum stopping power to the stopping power at the ion's maximum energy gives this ratio of "damage" produced by each ion.
We already found in part (a) that the ionization stopping power is really all that matters here, so let's just stick with that:

$$\left(\frac{dT}{dx}\right)_{total} = \frac{2\pi M N k_0^2 z^2 Z e^4}{m_e E}\left[ln\left(\frac{4m_e E}{M\bar{I}\left(1-\cancelto{0}{\frac{2E}{Mc^2}}\right)}\right) - \cancelto{0}{\frac{2E}{Mc^2}}\right] \tag{35}$$

$$\left(\frac{dT}{dx}\right)_{total} = \frac{2\pi M N k_0^2 z^2 Z e^4}{m_e E}\left[ln\left(\frac{4m_e E}{M\bar{I}}\right)\right] \tag{36}$$

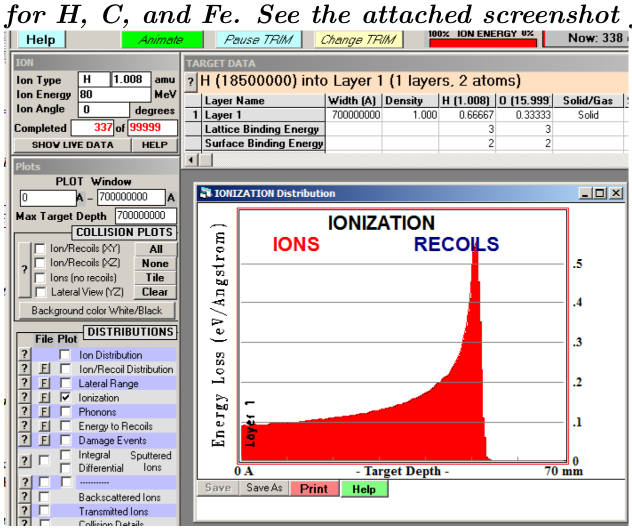
Now we express the ratio between ion pairs created as the stopping powers at E=500$\bar{I}$ and E=E_i:

$$\frac{\left(\frac{dT}{dx}\right)_{max}}{\left(\frac{dT}{dx}\right)_{incident}} = \frac{\cancel{\frac{2\pi M N k_0^2 z^2 Z e^4}{m_e}}\frac{1}{(500\bar{I})}\left[ln\left(\frac{4m_e(500\cancel{\bar{I}})}{M\cancel{\bar{I}}}\right)\right]}{\cancel{\frac{2\pi M N k_0^2 z^2 Z e^4}{m_e}}\frac{1}{E_i}\left[ln\left(\frac{4m_e E_i}{M\bar{I}}\right)\right]} = \frac{E_i\left[ln\left(\frac{2000m_e}{M}\right)\right]}{500\bar{I}\left[ln\left(\frac{4m_e E_i}{M\bar{I}}\right)\right]} \tag{37}$$

Note that all those crazy constants didn't matter, because they all cancel out in a ratio! Finally, to make our calculations even easier, we note that the ratio of the mass of an electron to the mass of a nucleon is roughly $\frac{m_p}{m_e} = 1,836$***. Substituting this in, we get:***

$$\frac{\left(\frac{dT}{dx}\right)_{max}}{\left(\frac{dT}{dx}\right)_{incident}} = \frac{E_i\left[ln\left(\frac{2000}{1836A}\right)\right]}{500\bar{I}\left[ln\left(\frac{4E_i}{1836A\bar{I}}\right)\right]} \tag{38}$$

Using SRIM to compute the number of ionizations per unit length for each of the three ions at their respective incident energies, we get ratios of 6, 9, and 9 respectively for H, C, and Fe. See the attached screenshot for how to get this ratio from SRIM:



(c) (5 points) Consider the cases of electrons, protons, carbon ions, and iron ions. Which type of ion is most suitable for use in IMRT, and why? Hint: Consider other mechanisms of ion energy loss

in tissue, and quantitatively compare how intense they would be in a relative sense.
Protons are by far the most suitable for IMRT, which is probably why it's used in cancer therapy today. Electrons have a far lower mass, and therefore far higher radiative stopping powers. This would present a danger to the patient, plus the fact that an electron can be more deflected from a narrow beam from interacting with another electron or nucleus. Carbon and iron ions have a far higher stopping power due to the z^2 ***dependence in both the ionization and nuclear stopping powers, so they would have to be accelerated to far, far higher energies to reach the center of a tumor 5cm into a person. Therefore, protons represent the best balance.***

22.01 Introduction to Nuclear Engineering and Ionizing Radiation
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