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**MICHAEL SHORT:** So I want to do a quick review of what we did last time, because I know I threw-- I think we threw the full six boards of math and physics at you guys. We started off trying to describe this general situation. If you have a small nucleus 1 firing at a large nucleus 2, something happens, and we didn't specify what that was. A potentially different nucleus 3 could come shooting off at angle theta, and a potentially different nucleus 4 goes off at a different angle phi.

Just to warn you guys, before you start copying everything from the board, starting last week I've been taking pictures of the board at the end of class. So if you prefer to look and listen or just take a few notes rather than copy everything else down, I'll be taking pictures of the board at the end of class from now on and posting them to the Stellar site. So up to you how you want to do it.

We started off with just three equations. We conserve mass, energy, and momentum. Mass and energy-- let's see-- come from the same equation.  $c^2 + T_1 + M_2 c^2 + T_2$  has to equal  $M_3 c^2 + T_3 + M_4 c^2 + T_4$ . We started off making one quick assumption, that the nucleus 2, whatever we're firing things at, has no kinetic energy. So we can just forget that.

What we also said is that we have to conserve x and y momentum. So if we say the x momentum of particle 1 would be  $\sqrt{2} M_1 T_1$  plus 0 for particle 2, because if particle 2 is not moving, it has no momentum. Has to equal  $\sqrt{2} M_3 T_3 \cos \theta$ , because it's the x component of the momentum, plus  $\sqrt{2} M_4 T_4 \cos \phi$ .

And the last equation for y momentum-- we'll call this x momentum, call that mass and energy, call this y momentum-- was-- let's say there's no y momentum at the beginning of this equation. So I'll just say 0 plus 0. Equals the y component of particle 3's momentum,  $\sqrt{2} M_3 T_3 \sin \theta$ , minus-- almost did that wrong-- because it's going in the opposite direction,  $\sqrt{2} M_4 T_4 \sin \phi$ .

We did something, and we arrived at the Q equation. I'm trying to make sure we get to

something new today. So the Q equation went something like-- and I want to make sure that I don't miswrite it at all. So when we refer to the Q equation, we're referring to this highly generalized equation relating all of the quantities that we see here. So I'm not going to go through all of the steps from last time, because, again, you have a picture of the board from last time.

But it went  $Q = T_1 \sqrt{M_1} + T_3 \sqrt{M_3} - \sqrt{M_1 M_3} \cos \theta$ . And last time we talked about which of these quantities are we likely to know ahead of time and which ones might we want to find out. Chances are we know all of the masses involved in these particles, because, well, you guys have been calculating that for the last 2 and 1/2 weeks or so. So those would be known quantities.

We'd also know the Q value for the reaction from conservation of mass and energy up there. And we'd probably be controlling the energy of particle 1 as it comes in. Either we know-- if it's a neutron, we know what energy it's born at. Or if it's coming from an accelerator, we crank up the voltage on the accelerator and control that. And that leaves us with just three quantities that we don't know-- the kinetic energy of particle 3 and the angle that it comes off at.

So this was the highly, highly generalized form. Recognize also that this is a quadratic equation in  $\sqrt{T_3}$ , or  $\sqrt{T_3}$ . And we did something else, and we arrived at  $\sqrt{T_3} = \frac{s \pm \sqrt{s^2 + t}}{2}$ , where s and t-- let's see. I believe s is  $\sqrt{M_1 M_3} \cos \theta$  over  $M_3 + M_4$ . And we'll make a little bit more room.

t should be-- damn it, got to look. Let's see. I believe it's  $-\frac{M_4 Q}{M_3 + M_4}$  plus-- oh, I'll just take a quick look. All right, I have it open right here. I don't want to give you a wrong minus sign or something. I did have a wrong minus sign. Good thing I looked.  $\frac{1}{2} \frac{E_1}{M_3 + M_4}$ .

And so we started looking at, well, what are the implications of this solution right here? For exothermic reactions, where Q is greater than 0, any energy  $E_1$  gets this reaction to occur. And all that that says, well, it doesn't really say much. All that it really says is that  $E_3$ -- I'm sorry--  $T_3$ -- and let me make sure that I don't use any sneaky E's in there-- plus  $T_4$  has to be greater than the incoming energy  $T_1$ . That's the only real implication here, is that some of the mass from particles 1 and 2 turned into some kinetic energy in particles 3 and 4. So that one's kind of the simpler case.

For the endothermic case, where  $Q$  is less than 0, there's going to be some threshold energy required to overcome in order to get this reaction to occur. So where did we say? So, first of all, where would we go about deciding what is the most favorable set of conditions that would allow one of these reactions to occur by manipulating parameters in  $s$  and  $t$ ? What's the first one that you'd start to look at?

Well, let's start by picking the angle. Let's say if there was a-- if we had what's called forward scattering, then this cosine of theta equals 1. And that probably gives us the highest likelihood of a reaction happening, or the most energy gone into, let's say, just moving the center of mass and not the particles going off in different directions. Let's see.

Ah, so what it really comes down to is a balance in making sure that this term right here, well, it can't go negative. If it goes negative, then the solution is imaginary and you don't have anything going on. So what this implies is that  $s$  squared plus  $t$  has to be greater or equal than 0 in order for this to occur. Otherwise, you would have, like I said, a complex solution to an energy. And energy is not going to be complex. That means the reaction won't occur. So this is where we got to last time. Yes.

**AUDIENCE:** When you say  $s$  squared plus  $t$  is greater than 0 or greater than or equal to 0, it's endothermic, wouldn't it also be greater than or equal to 0 for an exothermic?

**MICHAEL SHORT:** It would. But there's a condition here that-- let's see. In this case, for exothermic,  $Q$  is greater than 0 and that condition is always satisfied. For an endothermic reaction,  $Q$  is negative. So that's a good point.

So if endothermic, then  $Q$  is less than 0, and it's all about making sure that that sum,  $s$  squared plus  $t$ , is not negative. What that means is in order to balance out the fact that you've got a negative  $Q$  here, you have to increase  $T_1$  in order to make that sum greater than or equal to 0. Yes.

**AUDIENCE:** So that condition,  $s$  squared plus  $t$  is greater than or equal to 0, is that basically a condition for the endothermic reaction to occur?

**MICHAEL SHORT:** That's correct. If  $s$  squared plus  $t$  is smaller than 0, which is to say that this whole sum right here doesn't help you balance out the negative  $Q$ , then the reaction is not going to happen. And something else might happen. So let's say you were looking at a case of inelastic scattering where a neutron would get absorbed by a nucleus and be re-emitted at a different

energy level. If the energy is too small for that to occur, then the neutron is not going to get absorbed. Instead it might bounce off and undergo elastic scattering.

And as a quick flash forward, I'll show you a quick plot of elastic and inelastic cross-sections that kind of hammers this home. You'll be looking at a lot of these plots, that are going to be logarithmic in energy space and probably logarithmic in microscopic cross-section, to bring back that variable from before.

If you remember, the cross-section is like the probability that a certain reaction is going to occur. The larger the cross-section, the higher the reaction rate for a given flux of particles. And let's say we'll split this into two things. We'll call it  $\sigma$  elastic and  $\sigma$  inelastic. And we'll give them-- that will be-- oh, we have colors. Let's just use those. Even better. Where's my second color? Under the paper. Awesome.

So let's say the elastic cross-section is in red, and the inelastic cross-section is in green. And for white, we'll plot  $\sigma$  total. Usually, one of these cross-sections for any old interaction-- I'm not even being specific on which one. Let's just say neutrons hitting something big-- would tend to look like this. There will be some insanity here, that we'll discuss, and it might start to increase a little bit as it goes to high energies. And this is definitely not to scale. Just for the purposes of illustration.

The elastic cross-section is going to look something like exactly this. See how closely I can draw on top of it. So at low energies, when a neutron can't be absorbed and re-emitted at a different energy, the inelastic scattering process can't happen. Which is to say that the incoming kinetic energy-- so this log of  $E$  right here, better known as  $T_1$  in the symbols up there-- it's not high enough to allow inelastic scattering to occur.

So if we want to graph the inelastic scattering cross-section, it will typically look like that, where once you reach your threshold energy, determined by that condition there, then the inelastic scattering turns on and it's actually able to proceed. So this is why we're getting into these threshold reactions, because it helps you understand why do some of the cross-sections that we study have the shapes that they do.

And this holds true for pretty much every inelastic cross-section I've seen, is they all-- almost all of them, if you're starting at the ground state, require some initial energy input to get going. Whereas elastic scattering can happen at any energy. So let's write that last condition right here. Elastic scattering, which means things just bounce off like billiard balls, you have  $Q$

equals 0. No energy changes hands, so to speak. You just get some kinetic energy from 1 being imparted to nucleus 2, but you don't turn any mass into energy. So any questions here before we go into-- yes.

**AUDIENCE:** If theta isn't 0 for cosine theta, how would you plug it in? If you don't know what the angle is, that it's not [INAUDIBLE]?

**MICHAEL SHORT:** So the question is, if you don't know the angle, what do you do about it, right? If you don't-- so in this case, we've said, what is the bare minimum threshold for this reaction to occur, and the best way for that to happen is for theta to equal 0. If theta is larger, that will actually mean that the reaction is not allowed to proceed unless you get to an even higher energy. So this condition still holds. But if cosine-- so let's say if cosine theta is less than 1, then the value of s goes down, and that makes this condition harder to satisfy. So that's a good question.

What that actually means is that for certain nuclear reactions very close to the threshold energy, only certain angles are allowed. I'm not going to get into the nitty-gritty of which angles are allowed. I think it's-- I'll call it minutia for the scope of this class, but it is in the Yip reading, which I'll be posting pretty soon.

But suffice to say that the only time you can-- let's say if  $s^2 + t = 0$ . The only time that can happen is when theta equals 0 or cosine equals 1. And that means that the nucleus can only recoil in a very, very narrow cone forward. As that energy increases, the allowable angles start to increase further. Does that answer your question?

**AUDIENCE:** Well, yes. So you're just saying [INAUDIBLE] 0 is the minimum [INAUDIBLE].

**MICHAEL SHORT:** Or you'd say-- let's say if cosine equals 180. Then-- I'm sorry. If theta equals 180, cosine would be negative 1. And that would be, let's say, the least favorable condition. Yes.

**AUDIENCE:** Why did you put the sigma total under sigma elastic?

**MICHAEL SHORT:** Oh, I'm saying-- so sigma elastic plus-- sorry-- sigma inelastic would, let's say, give you the total scattering cross-section.

**AUDIENCE:** And then what was the green line?

**MICHAEL SHORT:** The green line-- oh, it is a little hard to see. The green line is the inelastic cross-section. Yes. I can imagine from back there the green and the white might look a little similar, yes. OK, cool.

Yes. Like one of these, they give they give me an almost black one. That's about as invisible as it gets. So make sure to use visible colors. OK.

So now let's take the case of elastic neutron scattering. And can anyone tell me how can we simplify the general Q equation for the case of neutrons hitting some random nucleus? What can we start plugging in for some of those values to make it simpler? What about the masses? What's M1 in atomic mass units?

**AUDIENCE:** 1.

**MICHAEL SHORT:** M1 is just 1 to a pretty good approximation. It's actually 1.0087, which we're going to say is 1. And what about M3?

**AUDIENCE:** 1.

**MICHAEL SHORT:** Is also-- yes, also 1. If this is elastic neutron scattering, the same neutron goes in and goes out. So M1 and M3 are the same thing. What about M2 and M4? Have we specified what this nucleus is? So what mass would we give it if it's a general nucleus with N neutrons and Z protons?

**AUDIENCE:** A.

**MICHAEL SHORT:** A, sure. So A, that's again A for the mass number. Cool. So with those in mind, and then the last thing is we only have T1 and T3. Just for clarity, let's call T1  $T_{in}$ , like the neutron energy going in. And T3, we'll call  $T_{out}$ .

So let's rewrite the Q equation, the Q-eq, with these symbols in there. And the last thing to note, what's Q for elastic scattering? 0, yes. Because no mass is turned into energy or vice versa.

So to rewrite the whole Q equation, we'll get 0 equals  $T_{in}$  times  $M_1$  over  $M_4$ , which is just 1 over A, minus 1 plus  $T_{out}$  times 1 plus  $M_3$  over  $M_4$ .  $M_3$  is also 1,  $M_4$  is also A. And minus 2 root  $M_1 M_3$ . They're both 1.  $T_{in} T_{out} \cos \theta$ .

So this looks a whole lot simpler. I'm going to do one quick thing right here and take the minus sign that's hiding in here outside this equation. It's going to make the form a lot simpler. So I'm just multiplying the inside and the outside of this term by negative 1, but hopefully you can see that it's the same thing. It will just make the form a lot nicer in the end.

And so now we want to start asking, what is the maximum and minimum energy that the neutron can lose? So let's start with the easy one. What is the minimum amount of energy that the neutron could lose? Anyone? I hear some whispers.

**AUDIENCE:** 0.

**MICHAEL SHORT:** 0. And if the neutron comes in-- if theta equals 0, then you end up with actually  $T_{in}$  will equal  $T_{out}$ . And, that way, let's say  $\Delta T$  or  $\Delta T$  neutron could equal 0. So a neutron can lose at least none of its energy in an elastic collision. Hopefully that makes intuitive sense because we would call that a miss.

Now let's take the other case. At what angle would you think the neutron would transfer as much energy as possible to the recoil nucleus? So if we have a big nucleus of mass  $A$  and we have a little neutron firing at it, at which angle does it transfer the most energy? Yes.

**AUDIENCE:** Pi? If it's like--

**MICHAEL SHORT:** Exactly.

**AUDIENCE:** [INAUDIBLE].

**MICHAEL SHORT:** At theta equals pi, which means this-- we call this backscattering. So, yes, good one. I'll correct your statement, though. You said if the neutron just stopped and the nucleus moved forward. Does not happen in every case.

For example, if you were to-- and I'd say don't try this at home, kids-- put on a nice helmet and run charging at a truck, can you actually just stop cold? And we're not assuming any bones breaking or anything. Chances are you'd bounce right back off. Yes. That's the analogy I like to give for what happens when a neutron scatters off uranium. It's like running at a truck with a helmet on. It will just bounce back.

So in the case of theta equals pi-- so we're going to substitute in theta equals pi. Therefore, cosine theta equals negative 1, and we have an even easier equation.  $0 = T_{in} - T_{out}$ , let's just say,  $T_{out} = T_{in}$  times  $1 + 1$  over  $A$ . I'm going to arrange these terms in order of their exponent for  $T_{out}$  since that's our variable again. And if this stuff is negative 1, then the 2 minus signs cancel conveniently. And we have  $2 \sqrt{T_{in} T_{out}}$ . Let's see. That's it. And we have minus  $T_{in}$  times  $1$  over  $1 - A$ .

Ideally, we'd like to try to simplify this as much as possible. So let's combine. Let's try to get everything in some sort of a common denominator, because that would make things a lot easier. If we multiply each of these 1's by  $A$  over  $A$ -- so let's put that as a step, because we can totally do that-- we get  $0$  over  $T$  times  $A$  over  $A$  plus  $1$  over  $A$  plus  $2$  root  $T$  in  $T$  minus  $T$  in times  $A$  over  $A$  minus  $1$  over  $A$ .

At this point, we can-- well, everything's in common terms, right? We can just extend that fraction sign and put the sign in here. Extend the fraction sign, put the sign in here. And we'll just say that's  $A$ . We'll say that's  $A$ .

Last step that we'll do is try and isolate  $T$  so at least one of our quadratic factors is going to be simple, like  $1$ . So next step, divide by  $A$  plus  $1$ . And then we get  $0$  equals just  $T$  plus  $2$  over  $A$  plus one root  $T$  in  $T$  minus  $T$  in times  $A$  minus  $1$  over  $A$  plus  $1$ . Now we've got a simple-looking quadratic equation, even though it's quadratic in the square root of  $T$ . Yes.

**AUDIENCE:** What happened to the  $A$  from the denominator?

**MICHAEL SHORT:** Let's see.

**AUDIENCE:** Could it be  $T$  over  $A$ ?

**MICHAEL SHORT:** What did I do? Did I miss an  $A$  or dividing by  $A$ ?

**AUDIENCE:** The last two equations.

**MICHAEL SHORT:** It's from back here?

**AUDIENCE:** No, no, no. It's probably the step you just did.

**MICHAEL SHORT:** Just these steps.

**AUDIENCE:** So you divide by  $A$  plus  $1$ .

**MICHAEL SHORT:** Ah, I see.

**AUDIENCE:** Should it be  $T$  over  $A$ ?

[INTERPOSING VOICES]

**MICHAEL SHORT:** Yes, you're right. So I want to make sure I didn't skip a step in dividing an  $A$ . Let me just check something real quick.



**AUDIENCE:** There should've been an A in the minus 2 square root.

**MICHAEL SHORT:** Oh, you're right. If we go back to our Q equation-- let's see. There's an M4 missing, isn't there? That's it. Hah. See, this is what happens when you don't look at your notes.

I'll go back and correct those, because then there should have been an over A. There should have been an over A. There should have been an over A. Thank you for pointing that out. And there should have been another-- oh, in this case we can just cancel all of the A's. I knew it came out nice and clean. OK, cool.

So at this point, this is a quadratic in root Tout, where we have-- what are our a, b, and c terms for this quadratic formula? So what's a first of all if it's quadratic in root Tout?

**AUDIENCE:** 1?

**MICHAEL SHORT:** Just 1. That was part of the goal of this manipulation, is to make at least one of these things pretty simple. What's b?

**AUDIENCE:** 2 over A plus 1 times radical Tin?

**MICHAEL SHORT:** Yes. 2 root Tin over A plus 1. And c is just that whole term right there. I'll do this up here. So then we can say root Tout equals negative b plus or minus the square root of b squared. So that's 4 Tin over A plus 1 squared. Minus 4 times a times c, so just minus 4 times c. So minus 4 times negative Tin A minus 1 over A plus 1.

So let's see what cancels. So, first of all, those minus signs cancel. And everything has-- oh, and over 2a. Don't want to forget that. Over 2a, which is just 2.

First thing we note is that everything here has a 2 in it, either directly as a 2 or hiding as a square root of 4. So we can cancel all of those. 4, 4, 4, 4. Let me make sure that minus sign is nice and visible.

What else is common to everything here? Well, I'll tell you what. I'll write it all out simpler without all the crossed-out stuff. Minus root Tin over A plus 1 plus or minus root Tin over A plus 1 squared plus Tin times A minus 1 over A plus 1. So with that written a little simpler, what's also common and can be factored out of everything?

**AUDIENCE:** Square root of Ti?

**MICHAEL SHORT:** That's right. Square root of  $T_{in}$ . Because there's a  $\sqrt{T_{in}}$  here, and then you can-- everything's got a  $T_{in}$  inside the square roots. You can pull that out. So we have a direct relation between  $\sqrt{T_{out}}$  and  $\sqrt{T_{in}}$ .  $\frac{-1 \pm \sqrt{1 + A(T_{in} - 1)}}{A + 1}$ . What do we do here to simplify all the junk in the square root?

**AUDIENCE:** Multiply the right side by  $\frac{A + 1}{A + 1}$ .

**MICHAEL SHORT:** That's right. You can always multiply by something, better known as 1. And that gets everything here-- just like there was a 2 or a  $\sqrt{4}$  everywhere in the equation, or there was a  $\sqrt{T_{in}}$  and a  $\sqrt{T_{in}}$  everywhere else in the equation, we'll do the same thing to get the  $A + 1$  out of there.

So we'll multiply this by  $\frac{A + 1}{A + 1}$ . I'll stick it over there. OK. And we get  $\sqrt{T_{out}} = \frac{\sqrt{T_{in}}}{A + 1} \sqrt{1 + A(T_{in} - 1)}$ . Starting to get a lot simpler. Let's see how much-- if I run out of space for this one.

So this stuff right here is just  $A^2 - A + A - 1$ . The minus  $A$  and the plus  $A$  cancel out. And then the plus 1 and the minus 1 cancel out. And all that's inside the square root is  $A^2$ . So the only hopefully nonlinear board technique, I'm going to move to the left. And we end up with  $\sqrt{T_{out}} = \frac{\sqrt{T_{in}}}{A + 1} A$ . And all that's left there is  $A - 1$  if we take the positive root.

Almost done. Just square both sides. And we should arrive at a result that might look familiar to some of you.  $T_{out} = T_{in} \frac{A - 1}{A + 1}$ . And we've gotten to the point now where we can determine how much energy the neutron can possibly lose or the recoil nucleus can possibly gain in an elastic collision. It's this factor right here. I'll use the red since it's more visible.

This is usually referred to in nuclear textbooks as  $\alpha$ . It's sort of the maximum amount of energy a neutron can lose or a recoil nucleus can gain. So what we've arrived at is a pretty important result, that, let's say, the energy, the kinetic energy of a neutron, has to be between its initial kinetic energy and  $\alpha$  times its initial kinetic energy.

This right here is one of the ways in which you choose a moderator or a slowing down medium for neutrons in reactors. So it's this  $\alpha$  factor right here that really distinguishes what we call

a thermal-- or what is it? Like a light water reactor or a thermal spectrum reactor from a fast spectrum reactor. Let's look at a couple of limiting cases to see why. Let's see. Anyone mind if I hide this board here? Or you have a question?

**AUDIENCE:** Yes. Can you explain why you ended up dropping the negative case?

**MICHAEL SHORT:** Let's see. If we took the negative case, we'd end up with minus 1 minus A. You just have an A plus 1 on the top. Yes. So in that case, you just have-- let's see. You just have root Tin, right? Let me see.

**AUDIENCE:** Negative root Tin actually.

**MICHAEL SHORT:** Oh, yes. So that wouldn't make very much sense, right? Yes. So in that case, well, you don't want to have a negative energy. So that case doesn't make physical sense. Thanks for making sure we explained that. And did I see another question? Yes.

**AUDIENCE:** Yes. What happened to the coefficients you had before Tin? You had 4. You needed 4 or 2, but [INAUDIBLE].

**MICHAEL SHORT:** Ah. OK. So what I did is I took the square root of 4 out of every term inside the square root and said, OK, they're all 2's. Just like in the next step, I said, all right, there's all of these A plus 1's, including all of these A plus 1's squares inside the square root, and took that out. Or I think even over here. Yes, so the whole thing here has been combine and destroy. Any other questions on what we did here before I go on to some of the implications of what we got? Cool. Let's look at a couple limiting cases.

I'll rewrite that inequality right there because that's the important one of the day. So what is alpha for typical materials? Let's say for hydrogen. Alpha equals-- well, it's always A minus 1 over A plus 1 squared. And for hydrogen, A equals 1, equals 1. And then we have 1 minus 1 in the numerator. Alpha equals 0.

What this means is that for the case of hydrogen, you can lose all of the neutron energy in a single collision. That doesn't mean that you lose all energy in every collision with a hydrogen atom if you're a neutron, but it means that you can lose up to all of your energy in one single collision. And this is what makes hydrogen such a good moderator or a slower down of neutrons, is when it undergoes elastic scattering, especially at energies below an MeV or so, which is where most of the neutrons in the reactor are, it just bounces around. And the more it

hits hydrogens, the more it imparts energy to those hydrogens and slows down.

Why do we want to slow the neutrons down in the first place? Well, that has to do with another cross-section, that I'm going to draw if I can find some chalk. Like I think I've mentioned before, every nuclear reaction has its own cross-section. And this time I'm going to introduce a new one called sigma fission, the probability, if a nucleus absorbs a neutron, that it undergoes fission and creates more neutrons and like 200 MeV of recoil energy.

So in this case, I'll draw it for U235, since this is the one I pretty much remember from memory. And it looks something like that. So what you want is for the neutrons to be at low energies. So this would be around the thermal energy, better known as about 0.025 eV. Your goal is that the more neutrons that you have in this energy region-- oh, that chalk erases other chalk.

The more neutrons you have in that energy region, the higher probability you have a fission. So this is the basis behind thermal reactors, is the neutrons all start here. They're born at around 1 to 10 MeV. They don't undergo fission very well at 1 to 10 MeV. So your goal as a thermal reactor designer is to slow them down as efficiently as possible.

What's the most efficient way to slow down neutrons? Cram the reactor full of hydrogen. What's the cheapest and most hydrogenous substance we know? Water. This is why water makes such a good reactor moderator. It's pretty cool. There's also lots of other reasons that we use water. It's everywhere, which is another way of saying cheap. It's pretty chemically inert. There are corrosion problems in reactors, but it doesn't just spontaneously combust when you see air, like sodium does, another reactor coolant.

It takes a lot of energy to heat it up. So it's specific heat capacity, the CP of water, if you remember-- I think it's-- was it 4.184 joules per gram? That's a point. One of the highest substances that we know of. So you can put a lot of that recoil energy or a lot of heat energy into this water without raising its temperature as much as a comparative substance. Metals can have heat capacity's like three or four times lower. So you wouldn't necessarily want to use a metal coolant. Or would you?

In what cases would you want to use a metal coolant for a reactor? Has anyone ever heard of liquid metal reactors before? Just a couple. Good. I get to be the first one to tell you. I did my whole PhD on alloys for the liquid lead reactor. So let's take a look at lead, which has an A of about-- let's call it 200. I think there's some isotopes, like 203 or so. There's probably an

isotope called lead 200.

What would alpha be for lead? Well, let's just plug in the numbers. A minus 1 is 199. A plus 1 is 201. Square that. Almost 1. Almost. This means that when neutrons hit something like lead, basically don't slow down. They can lose at least none and at most almost none of their energy.

And this is the basis behind what's called fast reactors if you want to use a coolant that keeps the neutrons very fast. Because for uranium 238, there's what's called a-- well, what you do is you want to capture neutrons with uranium 238, make plutonium 239, and then breed that. Or uranium 238 has got its fast fission cross-section. I don't think I want to get into bringing it on the screen today since we're almost five of five of.

What I will say is there's lots of other reactor coolants besides water. And it sounds to me like almost no one had heard of a liquid metal reactor. Why would you want to use a liquid metal as a coolant besides keeping the neutrons at high energy? Anyone have any ideas? What sort of properties do you want out of a coolant? Not even for a reactor but for anything.

**AUDIENCE:** Heat transfer.

**MICHAEL SHORT:** Good heat transfer. Metals are extremely thermally conductive. So if you want to get the heat out of the fuel rods and into the coolant and then out to make steam for a turbine, liquid metals are a pretty awesome coolant to use because they conduct heat extremely well. What else? Let's try and think now. If you were a reactor designer, you don't just have to make the reactor work but you want to make it avoid accidents. What sort of thermodynamic properties about metals could prevent accidents from happening?

**AUDIENCE:** They solidify.

**MICHAEL SHORT:** Yes. So there's one problem. They could solidify. So coolants that have been chosen for reactors have been things like sodium, which melts just below 100 Celsius, liquid lead-bismuth, which melts at 123 Celsius. And I know because I've it in a frying pan before. So I did four years of research on liquid lead-bismuth, and if there's anyone that's gotten enough exposure to that, it's me. It does not seem to have affected my brain too much because I only made like two major mistakes on today's board. Good enough.

Yes. So we've got hundreds of pounds of lead-bismuth sitting around. It's pretty inert stuff. It's

really dense, so it can store a lot of heat. The other thing is the boiling point. Anyone know what temperature metals boil at?

[INTERPOSING VOICES]

**MICHAEL SHORT:** Extremely high, yes. Sodium boils at approximately exactly 893 degrees Celsius. Liquid lead-bismuth boils at approximately exactly 1,670 Celsius. You'll actually melt the steel that the reactor is made of before you boil off your coolant. And if you boil your coolant, you have no way of cooling the reactor, and that's something you want to avoid.

Water boils at approximately 325-- let's say 288 to 340 Celsius depending on the pressure that's used in the reactors. And that does get reached sometimes, especially in accident conditions. So if you want to make something relatively Fukushima-proof, then you don't want the coolant to boil. So use a liquid metal, which introduces other problems.

It also introduces some other problems. I'm going to flash forward a bit to neutrons and reactor design. Because it does take time for this scattering to happen. These collisions, they happen pretty quickly but they do take a finite amount of time. And in the meantime, you can have what's called feedback coefficients, natural bits of physics that help your reactor stay stable or they don't, depending on whether it's called negative or positive feedback. So we can have either negative or positive feedback.

I'll give you one simple example that I'll just introduce conceptually, and we'll actually explain it a little mathematically later in the course. Let's talk about coolant density. If you were to heat up your reactor and the coolant were to get less dense, what do you think would happen to the reaction rate of, well, anything-- scattering, fission, absorption?

**AUDIENCE:** Go down.

**MICHAEL SHORT:** It should go down. Why do you think that is?

**AUDIENCE:** Because there's not as many particles that's close together, so [INAUDIBLE].

**MICHAEL SHORT:** Exactly. Yes. To reintroduce a bit of the cross-section stuff I mentioned last time, the microscopic cross-section is the probability that, let's say, one nucleus hits one other nucleus. If you then multiply by the number density or how many of them are there, you end up with the macroscopic cross-section. So I'll label this as micro, label this as macro. And then the macroscopic cross-section times your neutron flux gives you your reaction rate.

So if you want to get less moderating happening, or less fission, or less absorption, the simplest way is-- well, that's a property of the material. That's whatever your reactor is doing. You can decrease the number density by heating things up and decreasing the density. So this is one of those cases where you can use the reactor to quickly respond with physics before you could respond with human intervention.

If you want, let's say, a extra power transient or a sudden increase in heat to slow down the nuclear reaction and not speed it up, you'd pick a moderator that behaves in this way. So in this case, water would get less dense, it would moderate less well, and put fewer neutrons in the high-probability fission region.

Then let's think about what happens if you're depending on your neutrons to stay fast or at high energy. Let's say you were to have a really bad day and boil your liquid sodium. All of a sudden, what little moderating power exists in that sodium disappears or gets even lower, and that would cause your reactor power to increase.

So one of the dangers of fast spectrum reactors is positive-- what's called positive void coefficient. Or if you make a bubble of gaseous sodium, your power increases rather than decreases. That would increase the heat. That would cause the power to go up. That would increase the heat. That would cause the power to go up. Luckily, there are many, many other negative feedback mechanisms that could be built in to make sure the overall feedback coefficients are still negative.

This also lets us understand a little bit about what went wrong at Chernobyl. And I'll give you a 1-minute flashforward. Because the control rods that were made of-- let's see. I don't remember what the composition of the control rods is, but they were neutron absorbers. The control rods in Chernobyl looked something like this, where there was the absorber here. And this was-- they were graphite tipped. And as you lower that graphite down into the reactor, you're all of a sudden introducing something.

Well, that's an OK moderator. For carbon, let's say  $A$  equals 12. So our alpha will be  $A$  minus 1 over  $A$  plus 1. I'm not going to write almost equal to 1 because that isn't quite almost equal to 1. What does this actually equal? Let's see. Let's just say definitely less than 1. There is some moderating power to graphite.

It's also a very bad absorber. And what this meant, that as you lowered those control rods into

the reactor, you suddenly introduced a little more moderation when things were already going bad, and that caused the power level to increase further. There were other problems, like it was designed so that if you boiled some of the coolant, you would have positive feedback.

And that is the sort of 1-minute synopsis as to what all went haywire at Chernobyl. But we'll be doing a second by second or, in some cases, millisecond by millisecond play of what went wrong with Chernobyl. And we could probably do the same for Fukushima now, now that we understand what happened, based on the physics you'll be learning in this course.

And this is actually a perfect stopping point, because next up we're going to be looking at the different processes of radioactive decay, many of which are a simplification of this Q equation, and I think some of which are probably familiar to you guys, because radiation decay is part of the normal lexicon, especially nowadays. So since it's five of five of, do you guys have any questions on what we've covered so far? Yes.

**AUDIENCE:** [INAUDIBLE] if you have water as your coolant and it gets too hot, [INAUDIBLE].

**MICHAEL SHORT:** Yes.

**AUDIENCE:** Right. And that will decrease the reaction rate.

**MICHAEL SHORT:** Um-hm.

**AUDIENCE:** And that's how like the [INAUDIBLE].

**MICHAEL SHORT:** It's actually-- are you asking about it the water feedback is part of the backup? That's your primary line of defense. Your backup is human intervention, because, compared to physics, humans are really, really slow, like many orders of magnitude slower. It takes microseconds for things to thermally expand. It definitely takes more than seconds for a human to respond.

Anyone ever done those tests where you have a light blinking and you have to hit a button the second you see the light? What's the fastest any of you guys have ever responded? Anyone remember? Anyone beat a second?

**AUDIENCE:** Maybe.

**MICHAEL SHORT:** Maybe. A tenth of a second?

[INTERPOSING VOICES]



**MICHAEL SHORT:** And there all you have to do is hit a button. All you have to do is hit the only button when you see the only light. What if you're piloting something that's about as complicated as the space shuttle but more likely to explode? What do you think your reaction time will be? Probably long.

You'll probably have to pull out the manual. And probably you'll have to RTFM for a little while. And maybe you'll find out what you have to do and maybe you won't. So it's actually operator error that has caused most of the near misses or actual misses in nuclear reactors. The physics, except for the Russian RBMK design that was for Chernobyl, usually it's human error that's the downfall of these things. So by understanding the physics here, we can rely on it to keep things safe. Yes.

**AUDIENCE:** When you have a lead reactor-- or I don't know [INAUDIBLE] one of these, but what is like-- how do you cool the lead once it starts getting hot?

**MICHAEL SHORT:** Ah, good question. How do you cool the liquid lead? You can't send liquid led through a turbine, right? So at some point you've got to make steam and use that to drive a turbine. You can use what's called a heat exchanger. At its simplest, you can think of it like a couple of tubes where the lead is going through here and the steam is going through here. And they have a very thin barrier between them, so you have all this heat moving from the lead, which is hotter, to the steam, which is colder. They actually have built a bunch of these led reactors.

**AUDIENCE:** Is that real?

**MICHAEL SHORT:** Yes. The Russian fast attack subs, the alpha class subs, were powered by and are powered by liquid led reactors. They're the only reactor that can outrun a torpedo. So when you have a liquid lead reactor powering and you've got a panic button that says, forget the safety systems. Outrun a torpedo. You have a choice between maybe dying in a reactor explosion and definitely getting shot out of the water with a torpedo. You do whatever you can.

And these subs only run two or three not slower than a torpedo. So just like that old algebra problem, if this guy leaves Pittsburgh at 8 AM traveling 40 miles an hour, and I'm trying to get to Boston, 30 miles an hour, if a torpedo leaves one sub moving this velocity and the alpha attack sub senses it from this distance and starts moving at a similar velocity, chances are the torpedo runs out of steam before it reaches the alpha sub. And that's only because they can have an extremely compact liquid lead nuclear reactor at the power source.

**AUDIENCE:** So can you [INAUDIBLE]? How do you move [INAUDIBLE]?

**MICHAEL SHORT:** Good question. How do you use the-- how do you move the liquid lead? You can move it by natural convection but that's extremely slow. So there are multiple ways of moving it. One of the cool ones is called an EM or Electromagnetic pump.

It induces eddy currents in the liquid lead because it's also a conductor. And those eddy currents couple with the EM field from the EM pump and cause the lead to just start moving on its own. So it's a no moving parts pump. The only problem is it's like 1% or 2% efficient. Yes.

So they only use those on the subs, but you can use EM pumps to move conductive coolants. So I think it's pretty awesome. And there have there have been land submarines. In fact, there's a company called AKME Engineering in Russia that's trying to commercialize a small modular liquid lead reactor.

The other nice thing about these liquid metal coolants is you can make the reactors much, much smaller and denser than in a light water reactor. In a light water reactor, you're relying on a lot of water to cool things and a lot of water to be there to moderate your neutrons. In a liquid metal reactor, where you don't need moderation, well, you don't need-- all you need is enough coolant to keep things cool. So you can tighten stuff up and make it more compact.

So that's one of the nuclear startups coming out nowadays. This is an awesome time to be in nuclear. When I started nuclear, there were approximately exactly zero nuclear startups. Like TerraPower didn't even exist yet. Now there's something like 52 in the US and others around the world. So like this is the time to be in nuclear if you're up for startups and not just working in academia, or a lab, or a big corporation. There's a lot of little companies now doing some crazy things based on some pretty good physics. So maybe time for one more question before I let you guys go. If not, then I'll see you guys on Tuesday when we start radioactive decay.