MIT Department of Mechanical Engineering

2.25 Advanced Fluid Mechanics

Particle Kinematics

Lagrangian and Eulerian Frames - Material Derivatives

The stream function — which will be discussed in more detail later in the course — in cylindrical co-ordinates $(r,\,\theta)$ for flow past a circular cylinder of radius a with clockwise circulation Γ is given by

$$\psi(r,\theta) = U\left(r - \frac{a^2}{r}\right)\sin\theta + \frac{\Gamma}{2\pi}\ln\left(\frac{r}{a}\right) \qquad (1)$$

a) Write the stream function $\psi(x,y)$ in Cartesian co-ordinates, and find the components of the velocity u_x and u_y in the x and y directions.

Hint: The stream function is defined in terms of the velocity components as

$$u_x = \frac{\partial\psi(x,y)}{\partial y}$$

$$u_y = -\frac{\partial\psi(x,y)}{\partial x}$$

b) Derive the ordinary differential equations that govern the particle path lines.

c) Find the equation for the particle trajectory passing through the point $r = 2a$, $\theta = 0$ (or equivalently, $x = 2a$, $y = 0$).

d) Show that a particle on the surface of the cylinder always stays on the cylinder. Find the tangential velocity component of such a particle, and determine the stagnation points.

Hint: In cylindrical co-ordinates,

$$u_r = \frac{1}{r}\frac{\partial\psi(r,\theta)}{\partial\theta}$$

$$u_\theta = -\frac{\partial\psi(r,\theta)}{\partial r}$$

e) Sketch the stream lines for the case $\Gamma > 4\pi aU$. What happens as $r \to \infty$?

