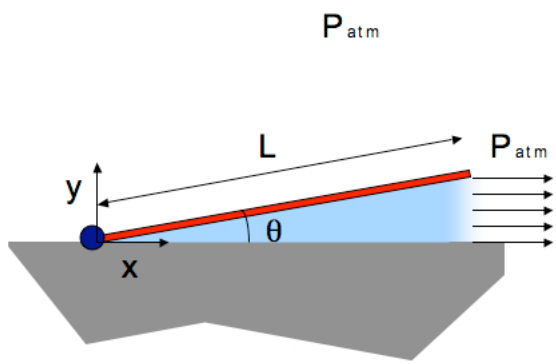
MIT Department of Mechanical Engineering

2.25 Advanced Fluid Mechanics

Problem 5.18

This problem is from "Advanced Fluid Mechanics Problems" by A.H. Shapiro and A.A. Sonin



A flat plate is hinged at one side to the floor, as shown, and held at a small angle θ_0 ($\theta_0 \ll 1$) relative to the floor. The entire system is submerged in a liquid of density ρ. At $t = 0$, a vertical force is applied and adjusted continually so that it produces a constant rate of decrease of the plate angle θ.

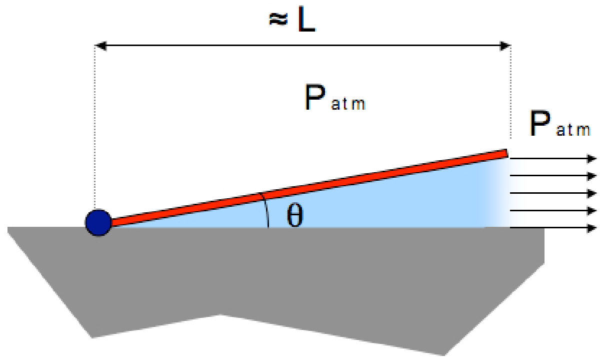
$$-\frac{d\theta}{dt} = \omega = Const, \tag{5.18a}$$

Assuming that the flow is incompressible and inviscid,

(a) Derive an expression for the velocity $u(x, t)$ at point x and time t.

(b) Find the horizontal force $F(t)$ exerted by the hinge on the floor (assume the plate has negligible mass).

Solution:



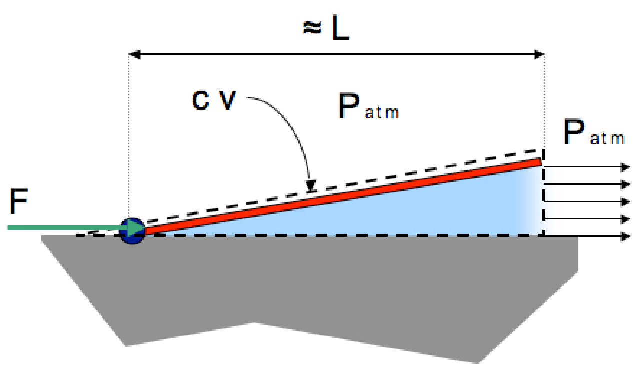
(a) Recall from problem 3.5 (Shapiro and Sonin) that mass conservation gives:

$$u(t) = \frac{x\omega}{2\theta(t)}. \tag{5.18b}$$

Since ω is a constant, we can write $\theta(t) = \theta_0 - \omega t$. Hence,

$$u(t) = \frac{x\omega}{2(\theta_0 - \omega t)}. \tag{5.18c}$$

(b) Now, we can analyze the wedge using form A of the Conservation of Momentum. Let's consider a triangular, deforming C.V. encompassing the moving wedge and the fluid beneath at all the times, as shown in figure (2).



Then, for the horizontal component

$$F_H = \frac{d}{dt}\int_{CV} V_{x\,Fluid} dV + \int_{CS} \rho V_{x\,Fluid}(\underline{V}_{Rel} \cdot \underline{\hat{n}}) dA, \tag{5.18d}$$

$$F_H = \frac{d}{dt}\int_0^L \int_0^b \rho U x tan(\theta) dz dx + \rho\left(\frac{\omega L}{2\theta}^2\right) bL\theta, \tag{5.18e}$$

$$F_H = \frac{d}{dt}\int_0^L \rho \frac{x\omega}{2\theta} x tan(\theta) b dx + \rho\left(\frac{\omega L}{2\theta}^2\right) bL\theta, \tag{5.18f}$$

Expanding the first term, and simplifying using the small angle approximation, $1 \gg \theta$,

$$F_H = \frac{d}{dt}\int_0^L \rho \frac{x\omega}{2\theta} x tan(\theta) b dx = \frac{d}{dt}\int_0^L \rho \frac{x\omega}{2} b dx = 0, \tag{5.18g}$$

Where the derivative of the integral is zero because it does not have any time dependence, all the quantities inside the integral are constant (only θ is time dependent, but was cancelled as shown).

Finally, the sum of forces acting on the x direction are zero because outside the C.V. there is only atmospheric pressure, and we have assumed a massless object compressing the fluid. Therefore,

$$F_H = \rho \frac{\omega^2 L^3}{4\theta} b. \tag{5.18h}$$

Note that we assumed the velocity to be always parallel to the x-axis in this problem. This is a good assumption but not completly true, specially near the plate, where we do have a downward component of the velocity as well. This is partially compensated by having a wall nearly perpendicular to the moving plate.

□

Problem Solution by KM/MC(Updated), Fall 2008

