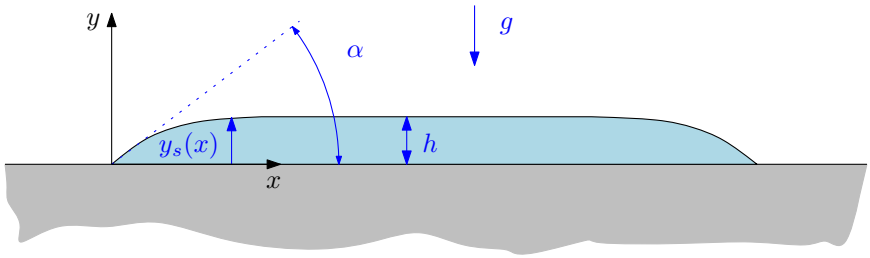
MIT Department of Mechanical Engineering
2.25 Advanced Fluid Mechanics

Problem 2.02
This problem is from "Advanced Fluid Mechanics Problems" by A.H. Shapiro and A.A. Sonin



A liquid of density ρ and surface tension σ has been spilled on a horizontal plate so that it forms a very large puddle whose depth (in the central parts) is h. Consider the region near the edge of the puddle, which can be viewed to good approximation as two-dimensional. If the contact angle is α, derive an expression for the shape of the liquid surface $y_s(x)$.

Assume for simplicity that α is small, so that the radius of curvature of the surface is large compared with h and can be approximated by

$$R = \frac{1}{\left|\dfrac{d^2 y_s}{dx^2}\right|}$$

ans:

$$y_s = h\left[1 - \exp\left(-\sqrt{\rho g/\sigma}\, x\right)\right]$$
$$h = \tan\alpha\sqrt{\sigma/\rho g}$$

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