# MASSACHUSETTS INSTITUTE OF TECHNOLOGY
DEPARTMENT OF MECHANICAL ENGINEERING
## 2.06 Fluid Dynamics

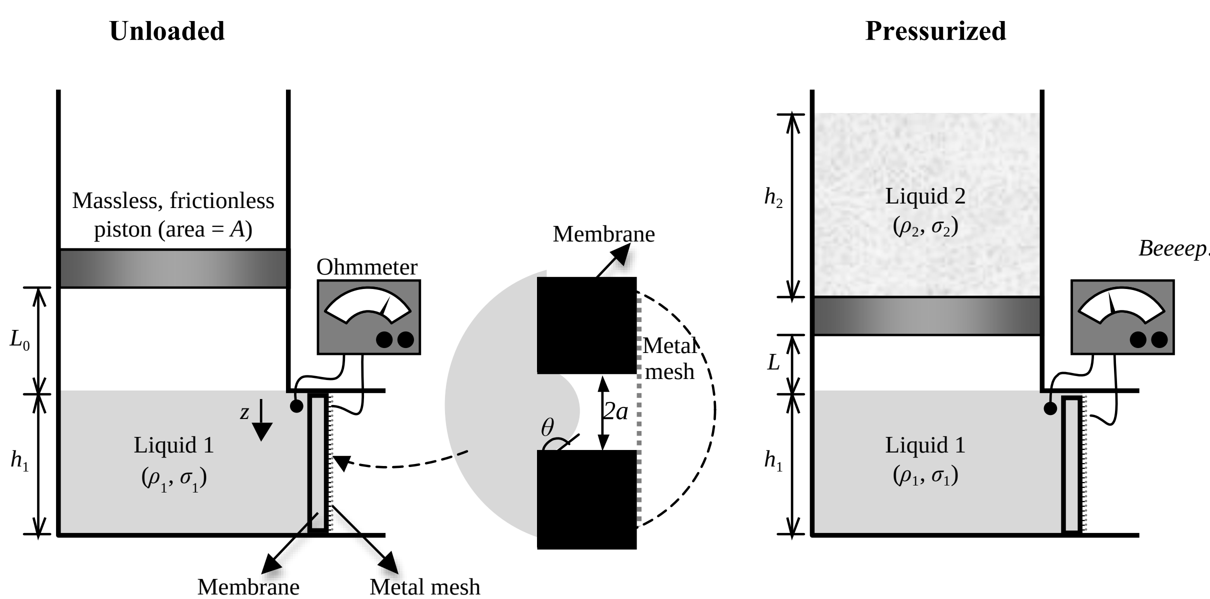
### Practice Problems for Quiz 1, Spring Term 2013

**Problem 1: Membrane Testing**
Membranes are thin, film-like porous structures used in separation and filtration. This problem deals with a piston-cylinder experiment to determine the pore size of the membrane.

Consider a frictionless piston-cylinder set up shown in the Figure. The membrane considered here is a thin disc with uniform through pores of radius *a*. In the "unloaded" state the membrane supports a column of liquid 1 (density $\rho_1$, surface tension $\sigma_1$) of height $h_1$ and a column of air (modeled as an ideal gas with negligible weight) of height $L_o$. The liquid in the pores of the membrane forms a meniscus at the liquid-air interface as shown in the magnified sketch of the pore. The material of the membrane is hydrophobic and the three-phase contact angle of the liquid 1-membrane-air is $\theta$.

The air and liquid 1 are pressurized by adding a second liquid (density $\rho_2$, surface tension $\sigma_2$) atop the piston as shown in Figure b. As the height of the liquid column $h_2$ increases the air pressure increases and ultimately the liquid meniscus in the pores of the membrane becomes unstable and breaks through.

Gravitational acceleration is *g*, the gas constant of air is *R*, and the ambient pressure is $P_o$. The temperature of air may be assumed to be constant throughout the problem. Liquids 1 and 2 are incompressible.




a) Determine the pressure of the air as a function of $P_o$ and the height $h_2$ of liquid 2.

b) Determine the pressure distribution on the membrane as a function of depth $z$. Sketch this pressure distribution.

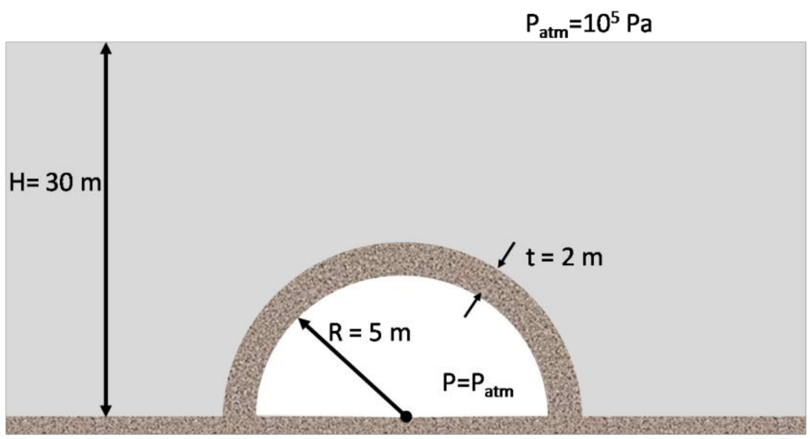
c) Determine the height $h_2$ of liquid 2 at which the liquid meniscus in the pores of the membrane becomes unstable and moves out shorting the electrical contacts. This is known as the break through condition for the membrane and gives a relation between pore size $a$ and height $h_2$. Please express your answer for $h_2$ as a function of the given quantities ($a$, $\rho_1$, $\sigma_1$, $\theta$, $\rho_2$, $h_1$, $g$). (*Hint: At what point on the membrane will the break through happen first?*)

d) Determine the length $L$ of the air column as a function of the height $h_2$ of the column of liquid2

**Problem 2: Tunnel**

As part of a new big dig project, an engineer plans to build a semi-circular tunnel under Massachusetts Bay. For this planning, the tunnel is assumed to lie on a flat seafloor at a depth of 30 m, as shown in the figure. The semi-circular wall has an internal radius of 5 m and a radial thickness of 2 m. The density of seawater is $r_W$ = 1035 kg/m$^3$ and atmospheric pressure is $P_{atm}$ = $10^5$ Pa. The gravitational acceleration g = 10 m/s$^2$

(a) Determine the (i) magnitude and (ii) direction of the total net hydrostatic force acting on the roof of the tunnel per unit length of the tunnel. Clearly justify your answers.



**Problem 3. The Pier**

Consider a floating wood pier moving up and down with the water surface in a lake. As shown on Figure 2.a), it is attached on one end to an anchor system consisting of a nylon rope hooked to a concrete block. Neglect the mass of the nylon rope. The concrete block is resting on the bottom of the lake at a depth $H$ = 10 m. The wood pier has a length $L$ = 10 m, width $W$ = 2 m and thickness $t$ = 0.2 m. The density of the pier is $\rho_p$ = 500 kg/m$^3$. The water density is $\rho_w$ = 1000 kg/m$^3$. The atmospheric pressure is $P_{atm} = 10^5$ Pa. The gravitational acceleration g = 9.8 m/s$^2$.

a) Determine the (i) magnitude and (ii) direction of the total hydrostatic force acting on the wetted shore per unit width (direction into the paper, along the shore).

b) Determine the mass of the wood pier and the height of its submerged portion