PROFESSOR: OK, what I thought I would do is I'm going to go quickly through the problems that were assigned for practice. There's seven or eight of them. I'm not going to do all of them. I'm going to go through and kind of talk about what some of the key issues are with each problem and maybe make some points about identifying how you do problems, not necessarily specifically focus on the exact question that was asked here.

So here was this first-- this was a quiz from last year. And you were asked to find an equation of motion for this thing. And it can be a bit of a puzzling problem, so how many degrees of freedom, what are the constraints, what equation should you use.

So first of all, I look at a problem like this. Is it planar motion or not? OK, so maximum three degrees of freedom. What are the constraints, though?

STUDENT: So G is [INAUDIBLE].

PROFESSOR: So G is R/2 from the center. So that's true. And so what you're really saying is it's fixed in radial motion, right? And you're right. So that's one constraint. What's another constraint in this problem? How many degrees of freedom do you expect to end up with? How many equations do you need? How many coordinates?

One, right? So how do we articulate the other constraint? So it can't move radially. We figured that out. So this is actually kind of an important point in this problem. Because if you can get this down, then the problem becomes very simple.

So talk to a neighbor. How would you describe the other constraint here? I'm going to hold it a second.

STUDENT: [INAUDIBLE]

PROFESSOR: Pardon?

STUDENT: Gravity is just [INAUDIBLE].

PROFESSOR: Well, there is gravity, yes. But gravity is not a constraint. So this is a pendulum, actually, of sorts. So talk to a neighbor. How would you describe the other constraint?

OK, how would you describe it? What's the other constraint here? So sometimes if something is pinned, we say, well, then it's fixed in x and y. And that's two constraints. It's only left to rotate.

But this one doesn't seem to-- it's kind of hard to understand where this one is pinned. You know this thing is constrained in x and y. Because you want to use theta for the coordinate, right? But how do you say that it's constrained in that other thing? Yeah.

STUDENT: [INAUDIBLE]

PROFESSOR: OK, so he's saying that you can define x and y in terms of r and theta. And that's true. And I'm going to say it slightly differently. So the way I think about this problem is we've said that it's constrained in the radial direction. So that's one. It can move in the tangential direction, right?

This problem is really exactly the same as the skateboard problem in a bowl. You shorten that thing a little bit, and it looks like a skateboard. And we're ignoring the inertia of the wheels. So it's just a stick sliding up and down.

So what's the relationship between the tangential motion and theta? Is it a fixed relationship? So how far does it move in the theta? If you have a little motion, delta theta, how far does it move tangentially? R delta theta is the delta r tangential, the distance it moves.

So this is the way to say the other constraint, is if you know theta, you know how far it's moved in the radial, in the tangential direction. So that's a way of saying that other constraint. OK, then we can say we completely describe this problem by one coordinate. And that's theta.

And the second you can do that, this problem then has a center of rotation right in the center of the circle. And anytime you have a body which rotates around a central point, then you know that you can describe the angular momentum of the body as some I with respect to A times-- and it's planar motion. So then it's just omega z, as long as this is a principal, as long as you have the mass moments of inertia in terms of principal axes here.

And it's nice when you can write it that simply. The second you can identify a fixed point about which something rotates, then the angular momentum simplifies to that. OK, and then this problem, of course the sum of the torques with respect to the center here has got to be d H with respect to A dt plus vAO cross PGO.

But this is-- what's the velocity at point A? 0, so you don't have to worry about this term. And it's just that. And you know that the sum of the external torques in this problem comes from gravity. So you put in the gravity term and compute the torques about the center.

So our object really looks like that. And here's G. And in the problem, the way it was posed, the theta is drawn from this line. So this is theta. So what's the moment arm? It looks like whatever this distance is, which is R/2. It's given. R/2 cosine theta is the length of this side. So the moment that gravity makes is some Mg. And that's going to be equal to some Izz about A theta double dot.

OK, good. Let's take a look at the next problem. And now so my intention here, I'm going to go kind of quickly one by another. And I'm just trying to hit the important concepts. So if you have a question about the concepts, ask. If I've left you wondering, I really want this to be kind of a conversation here. So that's the whole point. I've given you the essence of what makes these problems work.

OK, this one, we just find a location of the center of mass. I think you're pretty good at that sort of thing. Well, remember just a couple points about center of mass. This one it says, find it. And you're given the two objects. The important point is that you can pick any coordinate at all in order to use it to compute the center of mass.

So I pick S here, the M1. And some distance down here is where the center of mass is that we're looking for. And so this is the G we're looking for. So M1 plus M2 is the total mass times the position of G.

I guess that ought to be a capital-- SG here must be equal to the sum of the parts times their positions, M1 SG1 plus M2 SG2. And you can solve for SG. And that tells you where you're at. So you can pick any coordinate at all to calculate it. And once you find it, you know where it is. So you just solve here for SG. It's that divided by the total mass, obviously.

OK, the next question was, draw principal axes. So does this object have some planes of symmetry? Tell me one.

STUDENT: This one.

PROFESSOR: Slice down through it this way, OK. And that means you have a principal axis where? Perpendicular to every plane of symmetry. So you have a principal axis coming out of the board. That's one. And give me another plane of symmetry.

STUDENT: [INAUDIBLE]

PROFESSOR: Yeah, this one. So that means you've got a plane of axis, principal axis going that way. And so then the third one has to be perpendicular to that. So you have one this way, this way, and this way.

So I'd asked for-- so if this is G, then principal axes with respect to G, one there, one coming out of the board, and one going up like that. So what is it? Let's remind ourselves what it means to be a principal axis. So if you know the principal axes of an object, if you rotate, and you've chosen them to go through G, do they have to go through G? Do all principal axes have to go through G? No, not at all. The principal axes just have to give you, when you work it out, a diagonal inertia matrix.

But what does it mean to have body coordinates that we know are principal axes? Well, one of the things we know, that if you rotate about G, rotate about a principal axis passing through G, if you rotate about any one of them, is it dynamically balanced? Guaranteed. You must rotate about the principal axis, though.

Now, I'll ask you a second question. If you rotate about any axis passing through G, is it statically balanced? Right, if you rotate about any axis passing through G, is it dynamically balanced? Maybe, right? Not necessarily. It might be dynamically balanced if you pass through G, and it is a principal axis, right?

OK, can you have a principal axis not going through G and rotate about that axis and have it be dynamically balanced? Yeah, OK, so we'll talk about an example in a second.

OK, so I think the second question here said, OK, we want to rotate this about this point G2. Now, G2 is right in the middle at this body. That's where its center of mass is. And there's a G1 up here that's the center of mass of the upper body. And there's a composite center of mass right here, which we just computed what it was.

And we want-- there's going to be an axis of rotation passing through G2. And we're rotating this body around it. So first question is, this axis coming through G2, is it a principal axis? Yes. If we rotate about that axis only, do you expect it to be dynamically balanced? Right. Do you expect it to be statically balanced? No.

So we're spinning it around this G2, going like that around G2 at some omega z. We know that this is G2. We know that G for the object is here. That's the center of mass of the whole thing. So there's some distance between those two, which I don't know. But we could figure it out. We'll call it e.

So what's the force required at this pin that its axle is going around? What's the force required to allow this thing to spin around, to hold it, essentially, the force required to hold it in place? And you ought to intuitively have an idea of what it is.

Why is there a force? That thing will spin around this central axle. And there's a force required to keep that thing from moving away. What is it?

STUDENT: Centrifugal force.

PROFESSOR: I hear centrifugal. So you're accelerating the center of mass of that thing. You're making it go in a circle.

Mass times acceleration is a force. And that's the tension in the string when you're swinging the ball around. This is no different from swinging the ball on a string. And you're essentially calculating the tension required to keep that thing from flying off.

And we could probably just guess. We can almost guess what it is, minus the total mass times the acceleration, which is inward. That's where the minus comes from. And what is it?

STUDENT: [INAUDIBLE]

PROFESSOR: Louder.

STUDENT: [INAUDIBLE]

PROFESSOR: Omega squared. I hear an omega z squared. We need something else. Pardon?

STUDENT: [INAUDIBLE]

PROFESSOR: Yeah, the r. And the r is this little e here. It's called the eccentricity, when you have something unbalanced. e omega squared, and it's in the r hat direction. We'll call it inwards. And the inwards is the minus. So that's the acceleration. Mass times acceleration is the force.

Also, you could compute this by computing d by dt of P of G with respect to O. And that's then M1 plus M2 times v of G. You have to take the d by dt of it. And that gives you that, which is the acceleration. So the forces you get directly from computing dP/dt, if you need to remember a way to do that.

All right, so conceptually, if I don't know what e is, then I don't want to have to figure it out. So I'm going to describe to you a way in which you could get directly at this answer, which reminds us something about angular momentum and these bodies. This object is made up of two objects. And we're spinning it about the center of one of them.

So let's take them one at a time. How much force is required to keep this spinning about that axle that goes right through its center of mass? None, right? Because there's no r omega squared. There's no acceleration. You're not making the center of mass of this object go in a circle.

So this piece doesn't enter into the solution. The actual force, the mass, this mass here, is M2 in the problem. M2 doesn't actually have anything to do with the answer. Because it's perfectly statically balanced around that point. All of this force making the thing go in a circle is making this piece go in a circle.

So in fact, this force up here is minus M1. And now we need the distance from the axis of rotation to its center of mass. Well, that's some r. We need to know what that r is. And I think that's basically-- this was L/2.

And this was b, so plus b/2. So the distance from this rotation point to that center of mass is L/2 plus b/2, omega z squared. Mass-- this is acceleration, and it's inwards. And that's the total force. All right, let's look at the next problem. Vicente, can you pull up the next one?

OK, this problem-- this problem and the next are rather similar. This is the elevator problem with the pendulum in the elevator. There's a couple subtleties here. The elevator, you're told that it's moving upwards at some particular rate of acceleration, some y double dot. That's given.

So how many degrees of freedom does this problem have? OK, two or one? I hear one. I see a two. How many coordinates is it going to take to completely describe the motion, or how many equations of motion do you think you're going to need? I see ones and twos. So there's a little not-- two, OK.

If we know the acceleration of the thing-- we start with some initial conditions, at time 0, it's sitting on the ground, and y equals 0, and now it takes off from the first floor-- we know for all time its position. So that's actually a given. It's a specified thing. And we don't have to write a separate equation of motion for that that we then will have to solve. So it's actually given.

So the actually only dynamic equation of motion we have to write is about the pendulum. Now, will it involve y double dot? Absolutely. But it's a given number. It's just a number you're given. You don't have to write a separate equation of motion. So actually you've got just one degree of freedom. Yeah.

STUDENT: [INAUDIBLE]

PROFESSOR: OK, so put up the next problem-- that one. So this problem has how many degrees of freedom? This is definitely two. Because you don't know. The motion in the up and down of the mass and the slider with a spring on it is unknown. You're going to have to write an equation of motion, which would have to be solved to get it, so a distinction, a subtlety that can trip you up.

When the motion is actually specified, and you know what it is for all time, you don't need a separate equation for that. So this problem requires two. The behavior of the pendulum part is essentially the same in both. When you write the equation-- well, let's talk about it. How will we go about solving this problem? We need two equations, two coordinates. What would you pick for your coordinates?

STUDENT: [INAUDIBLE]

PROFESSOR: Theta, and then a coordinate. This one I guess is called y, so y for the square block sliding up and down, and a theta for the pendulum, two equations of motion. And to get the y equation of motion, you would use-- we're not using Lagrange here. We're going to do a little review of direct method. How would you write the equation of motion using-- up there I kind of wrote the key to everything that we do with the direct method, some of the forces, some of the torques.

Every rigid body has at max how many degrees of freedom? Six for rigid body. In planar motion, this is reduced to three. But every rigid body in 3D space has six degrees of freedom-- three positions, three rotations.

So you write some of the forces on the body. That is equal to the mass times acceleration. That gives you three equations, one in each vector, component direction. You write the sum of the torques. That's also a vector equation. You can get as many as three equations out of it if you need it.

Then we keep reducing down how many equations we need to get by figuring out the constraints until we get down to the number of actual degrees of freedom. And that's the remaining equations, the number of equations you actually need. So this one requires two. And if you're going to write an equation of motion about the main mass that slides up and down, what law would you use?

STUDENT: [INAUDIBLE]

PROFESSOR: Yeah, sum of the forces equal to the mass times acceleration. Then we need to get another question. What law would you use for the second one?

STUDENT: Torque.

PROFESSOR: Torque about where?

STUDENT: A.

PROFESSOR: Torque about A. So let's look at that. So that's the second equation up there. Torque about A is the time derivative of the angular momentum with respect to A plus that vA cross momentum term, which is a little annoying to work out. So the simpler version is the third equation.

It's a pretty quick derivation to go from that second equation to the third. And it's much less work, generally, to get that mass times acceleration term, a lot less work actually. And if I've got time, I'll go back and we'll work out a problem like this. And we'll do that part. But let's go on to the other questions. OK, this problem-- two rigid bodies. Is it planar motion?

STUDENT: Yes.

PROFESSOR: Right? How many possible degrees of freedom for each rigid body? Times 2, 6, now how many constraints? So let's take them one rigid body at a time. It's a big roller. Pardon?

STUDENT: No y.

PROFESSOR: No y, she says. OK, that's one constraint in it-- two left, two possible things left.

STUDENT: No slip.

PROFESSOR: Ahh, so I think this one's no slip. So how does a no slip then give you a constraint?

STUDENT: So x dot equals r omega of the wheel turning.

PROFESSOR: So r, there's an r. X equals r theta, or minus r theta depending on how you define the theta, right? So x is not independent of theta. And that's a second constraint. So you only need one equation, one generalized coordinate, to describe the motion of the wheel. And what would you use?

STUDENT: Theta?

PROFESSOR: OK, theta. You could use theta. Or you could use x. All right, and now the other object, how many constraints does it have? And what are they? What are the constraints on the T bar?

STUDENT: Can't translate the y.

PROFESSOR: OK, that's true. Let me give you a little hint about how I process this when I'm looking at it. Remember when you're picking generalized coordinates, they need to be independent. And independent means if you freeze all but one, that last one can still move. OK, so we've picked one. You said theta or x. Let's freeze it. What motion is left of that T bar?

STUDENT: [INAUDIBLE]

PROFESSOR: It can only rotate, right? And that means it has a pin, a point about at which it rotates, which you have just fixed. So it's constrained. That pin constrains it in how many directions?

STUDENT: Two.

PROFESSOR: Two, right off the bat. You have the two constraints right at the pin. But it's not so obvious until you say, let's freeze at other coordinates. And then we see that, ahh, there's only one possible motion left, the rotation.

OK, so we have a rotation and a translation, or two rotations. That's the way we could write equations of motion. If we did translation of the main disk, what equation would you use to write the equation of motion for the main disk?

STUDENT: Force.

PROFESSOR: Force, yeah. Newton's second law, sum of the forces in what direction? It can only move in--

STUDENT: [INAUDIBLE]

PROFESSOR: Horizontal, yeah. So sum of the forces in the x direction, sum of external forces in the x direction, equals mass times acceleration. Now this, though, has some problematic external forces, right? What are they, the difficult ones?

STUDENT: [INAUDIBLE]

PROFESSOR: Well, friction. There's a friction force. And there's also the internal forces at the pin. So you're going to have to sort out how to work your way through that. If you don't want to mess with external forces at the pin, you have to sum rotations at the--

STUDENT: [INAUDIBLE]

PROFESSOR: Yeah, so this problem, there's no escaping something messy. So let's say one equation will be the sum of the forces on that main roller in the horizontal direction. And that's going to force us to deal with a friction force and an internal, two reaction forces at the pin.

Well, you just write them down. Then go and write the same expression, sum of the forces on the other object. And it'll also show up with those two internal forces. You add those two equations together. Those forces cancel. And that gives you the first equation. And it'll involve acceleration of both masses. And you'll have to figure out the acceleration of that second mass around its own.

But you know how to do that. That's just kinematics. And then you've got to do the torque equation. And you'd probably do it about A. And does this involve-- what about the velocity of A in this problem? Is it 0? No, so you may have to deal with that second term. And again, I would go use the third expression. Because you need to find the accelerations anyway. And I would use that. Yeah.

STUDENT: Could you also take the sum of the torques about d?

PROFESSOR: Some of the torques about?

STUDENT: d?

PROFESSOR: Then d is the point of contact. Yeah, and that would give you a way of getting at, I think, an equation of motion that mostly describes the big roller. I don't know. In terms of the other one, I'm not-- summing it, then talking about the second rigid body? I don't think I would do it.

It gets a little complicated. But it's interesting. It might be worth a try. I'd probably do it about A, is the point I would choose, and deal with the fact that you need to know the acceleration of A. But in this problem, that's pretty straightforward.

What's the acceleration of point A? And let's say our horizontal coordinate is x. What's the acceleration of point A? x double dot in the I direction-- pretty easy to stick that in this third expression up here. And you know Rg with respect to A is just the length of the distance down to the center of mass of that T. So that term's pretty easy to figure out.

And then the lead term is just everything with respect to g. And that's pretty straightforward. OK, good, how are we doing on time? Not a lot left. And is that the last problem? Yeah.

STUDENT: Quick question. The previous problem, we don't need to worry about friction, do we?

STUDENT: If it was slipping, we would.

PROFESSOR: Yeah, will friction ultimately end up in the answer to this one? No.

STUDENT: [INAUDIBLE]

PROFESSOR: Well, speaking in Lagrange terms, does it do any work?

STUDENT: [INAUDIBLE]

PROFESSOR: I know, but you know about it now, right? Constraints that do no work usually don't end up in the final answer. I don't have a simple way of explaining why it doesn't. But if in the first case, instead of doing Newton's law on the first one, we had done torques about d, it would have given us an equation of motion for the roller. And then you could have done torques about A, a different equation. And we'd have got an equation of motion largely dealing with T. You still have some internal forces and torques that you're going to have to eliminate.

So this is not a simple problem, but straightforward if you know what laws to apply to some of the forces, some of the torques. And be careful with the extra term. If this object were just given an initial deflection, like the T bar is picked up and let go, and it's just there, and it's just doing its thing, what can you say about the center of mass of the system?

STUDENT: [INAUDIBLE]

PROFESSOR: Well, it's not that it doesn't necessarily move. Well, let's put it this way. What if I gave this thing a push to start with, and now it's going to roll along? Huh?

STUDENT: [INAUDIBLE]

PROFESSOR: OK, in that case, you're saying that the center of mass moves with constant velocity. And Newton would agree with you. Why?

STUDENT: [INAUDIBLE]

PROFESSOR: Because once it gets rolling, is there any friction force?

STUDENT: No.

PROFESSOR: So you've got to draw the free body diagram and decide whether or not there are any forces on the system. If there are no external forces on the system, Newton says no acceleration, no change in momentum. So once you get it rolling, there's actually no friction force. Because it isn't trying to either speed up or slow down the rolling.

So it goes actually to 0. Is there slip? No, it still isn't slipping. But it's just happily rolling along. There are no external forces acting on it. It means the mass times the acceleration of the center of mass of the system is zero. So the center of mass does not accelerate. That means it has constant velocity.

Now, if I contrive to have the initial velocity of the center of mass be 0, just cause that thing to swing back and forth, what would you see the center of mass do? It has no linear momentum. The linear momentum of the system is 0.

So the center of mass sits still. Its velocity now is 0. There's still no acceleration. It's just this T bar is rocking back and forth. So what must the roller be doing?

STUDENT: [INAUDIBLE]

PROFESSOR: So the roller has to move to the left when the T bar is going to the right. So the two things are going like this. In the center of mass, wherever it is, it's just sitting still.

All right, is there a last problem? This forces you to go back to the fundamental definition of angular momentum. It's R cross, this little summation of each RI cross PI. And add them up, and you get the angular momentum with respect to the rotational point, 1 degree of freedom, has a center of mass. Where is the center of mass of the system? Can you describe it?

STUDENT: Right in between--

PROFESSOR: Somewhere in between the two masses. There are two point masses, center mass has to be someplace in between. So once you figure out where that is, then you can concentrate all the mass there and go from there.

Does this have mass moment of inertia? Can you write the equation of motion? This is a pendulum. Can you write I about A theta double dot equals minus Mg something sine theta?

This is a pendulum. The restoring torque on it comes from gravity. And you'll end up with an expression that looks like some I with respect to the point of rotation. So basically, you just need to figure out what is I with respect to A. And they're particles. So each particle has an I with respect to its center of mass. It's equal to, for a particle, concentrated point mass?

STUDENT: Mr squared?

PROFESSOR: Yeah, but there's no r. So I for a particle about its center mass is 0. So this only has parallel axis theorem components. It has M1 times this distance squared from the center plus M2 times this distance squared from the center. And that's the total Izz. So then you can write it out. OK, so I think we've run out of time.