PROFESSOR: All right. Thoughts. Important concepts of the week?

AUDIENCE: Principal axes.

PROFESSOR: Principal axes. Good one. Another one.

AUDIENCE: Dynamic versus static balances.

PROFESSOR: Right. Another one.

AUDIENCE: Symmetry. Symmetry.

PROFESSOR: Symmetry. Right. This is in the context of things like the mass moment of inertia matrix. Yes?

AUDIENCE: Parallel axis theorem.

PROFESSOR: Parallel axis theorem. OK. We haven't done much with it yet but we're going to come back to it. All right. Pretty good list. We only had one lecture.

Dynamic and static balancing. Let's talk about that for a second. Let's talk about static balancing. Tell me what are features of static balancing? How can you-- you have a quiz problem, it's on the final-- you have an object and an axis about which its spinning and you're asked, is this statically balanced or not. What would you look for? Christina.

AUDIENCE: [INAUDIBLE].

PROFESSOR: G needs to be--

AUDIENCE: [INAUDIBLE] axis.

PROFESSOR: So another way of saying that is the axis must-- the axis of rotation must pass through--

AUDIENCE: The center.

PROFESSOR: --G. Is that what you're trying to say? The center of mass? Everybody agree about that? All right. Now, if I have weird shaped object, I know where the center of mass, and I stick the axis of rotation at any angle at all through it, is it statically balance regardless of where I have the x-axis of rotation passing through the center of mass?

AUDIENCE: Yes.

PROFESSOR: I hear a yes. How about the other people?

AUDIENCE: Yeah.

OK. So how do you-- I think that's probably right. But how do you test it? How do you test to see-- what's a simple test you can form to see if something is statically balanced?

AUDIENCE: See if it goes through a low point.

PROFESSOR: Right. So you make the axis that you're rotating about horizontal and see if the thing seeks a low point. Because if it does, it's telling you that what about the center of mass?

AUDIENCE: It's not going through the--

PROFESSOR: It's not going through the axis. To go to a low point, it means the center of mass is somewhere below the axis of spin. Right? But any axis at all, as long as its through G, you are statically balanced.

Dynamic balancing. What are the-- what would you look for on an object to say is this dynamically balanced or not? What kind of properties?

AUDIENCE: There would be no unbalanced torques if you decided to rotate it.

PROFESSOR: So you're looking for now unbalanced torques.

AUDIENCE: Because I'm like if it-- if the object is rotating about the axis that you have it on, you won't need to supply an extra torque to try and keep it balanced.

[INTERPOSING VOICES]

PROFESSOR: --pretty convincing.

AUDIENCE: --the axis it's rotating on.

PROFESSOR: OK. If you put in that last caveat then it's getting pretty correct. Can there be a torque on it that's not balanced and have it be dynamically balanced?

AUDIENCE: Well, yeah.

PROFESSOR: Which one?

AUDIENCE: Within the axis of rotation.

PROFESSOR: If you have a torque that's in the axis of spin and around the axis of rotation and that torque will cause what to happen?

AUDIENCE: Rotation.

PROFESSOR: Cause the rotation rate to?

AUDIENCE: Change.

PROFESSOR: Change, right? That puts energy into the system and accelerates it. And it's a torque that's in the direction of spin that doesn't cause dynamic imbalances. But a torque that is other than in the direction of spin, that's what a dynamic imbalance is. So what are some other ways you can say this in terms of things like evidence of-- let's say I give you H and I give you omega, how can you just tell by looking at those two thing? I give you the angle of momentum and I give you the spin, how can you tell instantly whether or not it is dynamically balanced?

AUDIENCE: If there-- each of the components in the same direction.

PROFESSOR: So you're saying if H and omega are in the--

AUDIENCE: R line.

PROFESSOR: A line. They just have to be parallel. If they're aligned, you do not have any dynamic imbalance. That sound right? Yeah?

AUDIENCE: It doesn't matter if they're parallel or antiparallel, does it?

PROFESSOR: You mean just opposite senses?

AUDIENCE: Yeah.

PROFESSOR: I think that'd be really hard to do. I don't know of any-- I don't think you can make that system that has angular momentum that is negative-- it's actually opposite the direction of the rotation rate. I don't think it-- I don't think our universe supports that physics. But anyway, in any case, they're aligned, you're OK.

All right? Let's move on. So today we're going actually just have you work a couple problems. And you really are going to work in groups, and when you finish, one of the groups is going to come up and fill in the blanks of the answer to the problem and we'll talk about it. And we're going to work two problems. And they have to do with things that spin.

And as a quick review just of information-- I guess we can do this. So the problem is a simple one. It's basically this. And I decided to lubricate it and it made the wood swell and so now it won't spin at all. Actually, unless I let the axis spin. So it's just this problem.

The axis passes through the center of mass and it looks just like this. And I've got an x-axis aligned with the axis of the rod. The Z is this way. And the spin is cap omega. And the mass moment of inertia matrix for this problem about G-- and our axis is passing through G-- so mass moment of inertia matrix times omega gives you H. So here's the mass moment of inertia matrix.

I claim is going to be diagonal. And the test of that, can you tell me whether or not I've chosen axes that, just from symmetry, you know will be principal axes? What do you think?

AUDIENCE: Well, the rod is circular now that [INAUDIBLE].

PROFESSOR: Yeah. So one axis-- that's an axis of symmetry is down the shaft. So that's guaranteed to be one. Have I chosen one like that? And any other orthogonal pair after that-- doesn't matter which way I orient them-- will also be. So I've got one up and one into the board. So those are principal axes for this object.

So it's going to be diagonal. And in fact, it looks-- it's MR squared over 2, ML squared over 12, and ML squared over 12 when you work it out. And if you look up in the book, most books for slen-- what they call slender rods, will say that this first term is 0. And that's because they're saying that L is a lot bigger than R-- the radius of this thing. And so MR squared is a pretty small number. So it's energy and rotation spinning this way is not very big for its angular momentum. But its angular momentum spinning like this is much, much larger because you have much greater MR squareds.

So you can leave, for the purpose of this problem, you can't treat this as 0. And in your groups, I want you to come up with the omega vector, H and DHDT. So do get in groups, talk about it. You've got a few minutes to do this. This one's pretty straightforward. And then you can do another one that's harder so warm up and find a group to work in and you're going to work a couple of problems this way.

Got a group that feels pretty good about they're answer?

AUDIENCE: Yes.

PROFESSOR: All right. Write it up. Come up, fill in the omega, H and DHDT.

OK. Can everybody see it? So you have a Z component only for the omega, a Z component k hat only for H, and its i omega-- iZZ omega. And DHDT, the only variable here is omega and you get an omega dot. And there's no-- DKDT is 0 because it doesn't change direction. Any questions about this? Yes?

AUDIENCE: So I was a little bit confused in lecture about how we knew the i hat, j hat, k hat in the H term. Is it just because it's only i hat on the top, j hat in the middle, and k hat on the bottom? Is that--

PROFESSOR: So the convention-- if I understand your question correctly-- the convention when you write out the H vector is it's the result of multiplying the three components of the spin. This is the piece in the i direction, j direction, k direction. And you multiply these out you get three results. Vector times-- and this one is Hx and it is in the i. The second one from this times the middle row gives you Hy and it's in the j. And the third one, this vector times these three in the bottom row, give you Hz in the k.

AUDIENCE: So I thought when-- I don't know, I could be completely wrong-- but I thought when you did [INAUDIBLE] you multiplied this, this, this and then this, this, this and then--

PROFESSOR: So we have a capital A, capital B, capital C terms in the first row?

AUDIENCE: So you do, like, this and--

PROFESSOR: And you have a capital-- little a, little b, little c terms in this one. When you multiply this times this, you get Aa plus Bb plus Cc.

AUDIENCE: Yeah. So that's my question. So if you have the omega x is an i hat, omega y is a j hat, omega z's a k hat, how, if you're multiplying like this, do you get only i hats on the top of the thing.

PROFESSOR: Yeah. Well, it's because this isn't just a matrix. This is actually a tensor. And we haven't gone into messy tensor notation. So you're just being told a convention here. And that is, because this is a vector being multiplied by a tensor, the result will-- even though this is i, j, k, omega xi times this first term in this tensor, it gives you an i back. Omega yj times this one gives you an i back. Omega zk times that gives you an i back.

AUDIENCE: OK. That's what I was really confused --

PROFESSOR: Right. And that's a great question because it wasn't obvious. We didn't do the full tensor mathematic. We just gave you a result. By definition, this row times that is in the i direction, j direction, k direction. Great question. OK.

OK. So let's go on to harder problem. So you're great at that one. We're going to do this one now. So we just did this problem. Now we want to do this problem. And so the same i matrix is used here. It's this one and you can let that term is 0 because the coordinate system that has been defined on this object is x, z, y into the board. But the direction of spin now is like that. And we'll call it cap omega again.

But it's in that direction. And these are 45 degree angles for the purposes of this problem. So that's sine cosine of 45 root 2 over 2. So now, what's omega? What's H? And what's DHDT? So work in groups and sort that one out. You ready to go?

AUDIENCE: We can be.

PROFESSOR: OK. Go for it. We'll probably do this occasionally. I'll say it again. I have no intention of embarrassing you at the board. So the practice will be, you put your answer up, you sit down and then we talk about it. It's not about you. It's about what's on the board. OK?

AUDIENCE: Should I keep going or should--

PROFESSOR: No. Put the whole thing up. Yep. And if you want to use that big piece of chalk it shows better. Just to your right.

OK. So let's talk about omega first. How do you figure-- we have a minus root 2 over 2 here. Can't see that. Minus root 2 over 2 omega i 0 root 2 over 2 omega k. Are people-- feel good about that? Any differences?

All right. So this thing-- this omega-- that spin clearly has components that are like that and like this. So this is your z piece and your i piece. And it is, indeed, in the minus i hat direction. So that seems OK.

And H would be some i-- the i times omega. And the first term, if you take that upper left term as 0, then you don't get anything. The second term you don't get anything. So how do you feel about the angular momentum? Any differences? OK.

And the DHDT. You take the time derivative of this thing-- and you got an omega dot. OK. That looks OK as far as it goes. But any other thoughts?

AUDIENCE: So k hat dot is not 0.

PROFESSOR: OK. So you're saying k hat is changing with time.

AUDIENCE: Yeah.

PROFESSOR: And let's think about that. So it's-- that's this piece here. This is omega z k hat. And as this thing goes around that unit vector is doing this. Right? So you need a DkDT. So do you want to give-- somebody else give me the second term here?

AUDIENCE: Sure. ML squared times omega squared because you have the changing directions in your k hat principal axes. And that's going to be-- well, times root 2.

PROFESSOR: This is root--

AUDIENCE: Well, omega squared.

PROFESSOR: OK. And then what about the root 2 over 2?

AUDIENCE: [INAUDIBLE].

PROFESSOR: Got to square that too, right?

AUDIENCE: That'd be squared as well.

PROFESSOR: That gives you a half. OK. And--

AUDIENCE: That's over 12 as well.

PROFESSOR: 12. And--

AUDIENCE: That would be in the j hat direction.

PROFESSOR: Positive--

AUDIENCE: Positive. Yes. In this case.

PROFESSOR: Started out with a negative but--

AUDIENCE: Negative [INAUDIBLE].

PROFESSOR: i cross k is minus j times a minus gives you the plus, right? So this looks-- this comes out ML squared omega squared j hat over 24 I think. Like that. That term. Now-- OK. And people good with that now?

So just to remind you, when I was doing these problems-- when I first had to teach this course a few years ago, and I was trying to figure out an easy way to teach this and how to do these problems, I got myself all confused trying to figure out which components of the rotation vector are rotating and what do I have to cross into it to get the answer. And in fact, you don't have to make it anywhere near that hard. And let's take a quick look at something here.

H is ML squared over 12 root 2 over 2-- where did H go-- root 2 over 2 omega k hat. H is a rotating vector. A derivative of a rotating vector in an inertial frame is the derivative of that rotating vector in the rotating frame. Which is the same thing as saying that omega is 0-- that's the change in length of the vector-- plus omega cross H.

Let's test that and see if that gives us the right answer. This derivative gives you-- all it does is gives you the theta, the omega dot back. Right? That's that first term. And the second term should just then look like your omega, which is root 2 minus root 2 over 2 cap omega i plus root 2 over 2 cap omega k cross a bunch of constants times k. Right? So the k cross k terms are?

AUDIENCE: 0.

PROFESSOR: 0. The i cross k--

AUDIENCE: Minus j.

PROFESSOR: --minus j. And so you get root 2 over 2 times root 2 over 2. You get what's in H here multiplied by root 2 over 2 omega hat. And you get back exactly these two terms. So it's that easy. It's just the derivative of a rotating vector. Just doing omega cross H is the easiest way to deal with that derivative of the rotating piece. Any rotating vector, you can take this time derivative that way. All right.

We've got a few minutes. I want to-- actually, any other questions about this kind of problem? In physics, most of the problems you've worked before, that involved rotation, are planar-- what we call planar motion problems. The axis of spin was always perpendicular to the plane and the rest the problem was confined to the plane. So hockey pucks sliding along and stuff like that.

But it was always assumed that the axis of rotation was perpendicular to the plane and that the angular momentum was parallel. So this is actually-- this is 3D problem. This is a 3D dynamics problem. As soon as that H and the omega are different directions, you can come up with torques in all three directions. Right? All right. So any other questions about this? And if not, I want to talk about something that was on the quiz. OK.

So here's-- this is the one problem on the quiz that gave more conceptual difficulty than any other single problem. People made mistakes on other problems, but got into conceptual trouble with this problem.

Remember, you had a vehicle driving up a bridge and the bridge could be changing at some angular rate and it has some angular acceleration. And we said to keep the problem simple, that you could treat this as a particle. And that really means you didn't have to deal with angular momentum. i omega for the object. You could just treat it like a particle.

And one of the-- where the confusion came from was in figuring out free body diagrams. You're told in the problem the language is something like, the action of the tires on the road result in a net force up the incline called t. We didn't call it friction, we didn't say anything, but people-- a number of people got confused about what's that have to do with friction. How does friction come into this?

So let's do the free body diagram for this thing. So tell me what's-- here's my car, tell me what to put on the free body diagram.

AUDIENCE: Weight.

PROFESSOR: OK. Mg?

AUDIENCE: Yep.

PROFESSOR: All right. So you've got an Mg. Next.

AUDIENCE: [INAUDIBLE].

PROFESSOR: A normal force? OK. That's going this way we'll call it.

AUDIENCE: T.

PROFESSOR: T. OK. Now, what about the tires on the road? What about friction?

AUDIENCE: [INAUDIBLE].

PROFESSOR: Pardon?

AUDIENCE: That is T.

PROFESSOR: That is T. So you think of the free body diagram, you just say, what are all the possible sources of external force on this thing? And they come from gravity-- a body force-- and then other things that are in contact with it. And the wheels are in contact with the road. And through the wheels you get the normal force. And through the wheels you get any active friction. So that is the-- that's total net friction force.

And then once you got that far, you were asked to come up with an equation of motion in this direction. In the direction up the bridge. And how do you-- so when you go to get an equation of motion, you say, the sum of the external forces is equal to?

AUDIENCE: [INAUDIBLE].

AUDIENCE: [INAUDIBLE].

PROFESSOR: And we'll make these in the x direction here. Is the mass times the acceleration in the x direction. And how would-- what terms appear?

AUDIENCE: [INAUDIBLE].

PROFESSOR: Pardon?

AUDIENCE: T.

PROFESSOR: Well, those are the forces. They're on the other side. I want you to come up with the accelerations. You need to come up with the acceleration terms.

AUDIENCE: [INAUDIBLE].

PROFESSOR: Pardon?

AUDIENCE: [INAUDIBLE].

PROFESSOR: So this problem most easily to think through, I think, in terms of r and theta and polar coordinates because you've worked dozens of times with a complete acceleration of something in a rotating system. And this is a rotating system. Polar coordinates is a pretty good way to do this problem. So what are the accelerations in this direction?

AUDIENCE: [INAUDIBLE] r and theta?

PROFESSOR: Well, I do it in terms of-- well, I do it in terms of r and theta. Given a theta dot and a theta double dot and x and r mount up to the same thing. But I-- if you call it r, you'll recognize the terms in your acceleration equation. So what are the accelerations in the direction up to bridge?

AUDIENCE: r double dot?

AUDIENCE: r double dot.

AUDIENCE: Minus r theta dot.

PROFESSOR: And those are all in the-- you get all of that in the i direction. And then if you looked at that, obviously, you could replace r double dot with x double dot. r with x if you make your origin here. Which you probably would. That's the point of rotation.

So that is your-- that's the mass times the acceleration in the direction of travel. And it's got to be equal to T and minus Mg sine theta probably. Right? And that's one equation of motion. And you're going to do another equation of motion in the theta dot-- theta hat direction. Because what other-- what accelerations are present in this direction? In the y j hat?

AUDIENCE: Theta double dot?

PROFESSOR: Theta double dot. You're [INAUDIBLE] one. What else? So the sum of the forces in the y-- so they are, in terms of polar coordinate terms?

AUDIENCE: 2 omega-- 2 omega dot.

PROFESSOR: Somebody said this one first. Plus 2 theta dot is your--

AUDIENCE: It's [INAUDIBLE].

PROFESSOR: Right. 2 theta dot--

AUDIENCE: r dot.

PROFESSOR: r dot. And that's in the j direction. And now you've got-- now you can work the problem. So how would you go about doing this problem where you can't treat it as a particle any longer?

We've graduated to that because we've been doing-- we've been doing angular momentum stuff with mass moment of inertia matrices. So now you want to do this more as a full fledged dynamics problem, taking into consideration that. So how do you-- what is the mass-- what do you have to modify-- do to modify our free body diagram? Here's my simplified vehicle. What are the forces on it that you need to deal with?

AUDIENCE: Well, there are two normal forces. There are two normal forces.

PROFESSOR: Yeah. So you can add-- need two because you've got two wheels on this thing. And so there's axle one here- so you get one here and one here. I call this one N1, N2. What else?

AUDIENCE: [INAUDIBLE]. The same T and Mg.

PROFESSOR: So you've got a g somewhere. And so you still have an Mg term. What other external forces?

AUDIENCE: T?

PROFESSOR: T. But now T's the problem. So as I gave you before, T is the total net force. But that's not adequate in this problem any longer.

AUDIENCE: [INAUDIBLE]

PROFESSOR: Yeah. So we're going to end up having a friction force that's supplied here-- f1-- and another one applied here-- f2. And f1 plus f2 would be T. OK. So how many unknowns do we have?

AUDIENCE: Four?

PROFESSOR: Well, it looks like four are here. Plus any motion stuff that you have to solve for. So how many equations of motion can you write? Remember, you get one equation for every vector component direction. So x, y, z gives you three components and you could conceivably write three equations. So tell me the relevant equations of motion that we could come up with for this problem.

AUDIENCE: Sum forces in x, sum forces in y.

PROFESSOR: OK. I argue-- sum of the forces in x. That's still true. Yep. Sum of the forces in y you said. Is this 0? No. Because that bridge is moving. OK. What else?

AUDIENCE: Torque.

PROFESSOR: OK. And how do you do that? Sum of the torques in what direction?

AUDIENCE: Theta hat.

PROFESSOR: Well, no. Yeah. Well, you could. Excuse me. I shouldn't say that. Do you have more than one torque term that you can work with here? More than one torque equation?

AUDIENCE: Yes?

AUDIENCE: Yes.

PROFESSOR: How's that? So let's just think through these things. You could have a torque-- you could have a torque in, this is x, y, z out of the board. You could have a z torque making this thing trying to pitch up and down. You could have an x torque making it trying to rollover. But we don't have any information in that direction and its constrained. And we could have a y torque making it try to go like that. And we don't have much information there.

So it seems like we've got a z torque equation that we could write. And that's one-- seems like we got three equations. Sum of the forces x, sum of the forces y, and a torque in the z. And doesn't seem like quite enough.

And that's as far as I've gotten with it. I just thought of this on the way over here. So I'd sit down and start working this out and trying to figure out, where is my fourth equation going to come from? And I'm not exactly sure quite yet.

But, generally, I'd go about setting it up like that and start working those things out. OK. Any other questions or questions about the quiz? We're done early.