**PROFESSOR:** So I'm going to give you a quick example of what I think is a good way to do solutions. Our approach to solutions.

State the problem. I'm going to give you a little formulaic here. Draw figures.

On the first day I said I like to think of dynamics problem. I break them down into three categories. One is to describe the motion. What does describing the motion mean? Well how many-- the number of degrees of freedom in the problem. That implies the number of equations of motion that you're going to need to solve that problem. So you got to identify the number of degrees of freedom.

It also then tells you the number of coordinates you need. So this is all part of describing the motion. It's figuring out how many coordinates, assigning them. So assigning the coordinates.

And then finally, essentially all underneath this you essentially do the kinematics. And that's the velocities, accelerations, so forth. So once you have-- you've explained the motion.

And this I guess is four. Explain the correct physical laws. You know how they apply. f equals ma. Newton's first, second, third law of conservation of momentum or whatever you want-- you think is the appropriate thing. So explain what the physical laws are and apply them.

And finally, do the math.

So if you could break problems down that way it'll give you a nice, logical flow. So I'm going to give you a bit of an example problem. And I'm also going to kind of pose a brain teaser to you at the end of this problem today. I want to give you something to think about.

So I'm going to draw my problem. This is a block on an incline. It's got some scales to measure your weight. And you're standing on this thing. Riding it down the incline. And the first question about this is to find the position as a function of time.

So that's the problem. That's part a. So state the problem, find the position.

So how-- well what do we do? Well draw figures, I've started with that. Next, describe the motion. I need a free-- We have to figure out how many degrees of freedom this problem has. I'm going to just declare no rotation. Going to treat it as a particle. So a particle has how many, generally how many degrees of freedom? How many coordinates to completely describe where it's at?

**AUDIENCE:** Three.

**PROFESSOR:** Three. So I may need as many as three coordinates to describe the motion of this thing. And if I really doing complete equations of motion I need three equations of motion. So this is two, three-- describing the motion I have three degrees of freedom. I'm going to need three coordinates.

So in this case here's my picture I'm going just to set up a Cartesian coordinate system aligned in a helpful way. X, y, z coming out. And this is my fixed inertial reference frame. And here's my center of mass. And basically my coordinates are describing the position of the center of mass.

So that's pretty much the describing the motion, what I need for now. We'll get to the velocities and accelerations when we get to the math part.

Apply the physics. I'm going to use Newton's second law. Sum of the external forces, mass times the acceleration. That's the physical law I'm going to apply.

Draw a free body diagram. And I'm just going to consider the block and the person this is the whole collection, it's one thing. I'm just not keep drawing the person on it, but-- so here's my object, including the weight of the person.

And it's going to have-- here's its center of mass. Obviously in mg, gravitational force, it's going to have a normal force. Going to have a friction force and how do I figure out what the friction-- which direction the friction is? I assume motion down the hill. Friction will oppose it. I draw in the arrows in the direction I expect the forces to act.

Then I'll use the sign convention that the arrows tell me what signs. So since my x-coordinate is down the hill. This is y. This is x, friction x, in the minus x direction. And I haven't-- I've left out a key piece of information.

Got to have the angle of the slope. And once you have the angle of the slope that's theta, this is theta. And I'm going to need to-- this is in a direction of one of my coordinates and so is this, but I need to break the gravitational piece into components lined up with my coordinates. And so you have a theta here as well. And now I can write my equation to motion.

And the nice thing about vectors is that when you have three equations of motion, three coordinates, each that are components of vectors in the x, y and z direction, it gives us three equations immediately.

So for example, the summation of the forces in this problem in the z-direction, the external forces are sums to?

**AUDIENCE:** 0.

**PROFESSOR:** 0. OK so we get a trivial solution out of that. And we don't have to go much further. Summation of the forces in the y-direction gives us some useful information.

And then the y-directed forces I have an n and a minus mg and I think it's cosine theta. Which tells me immediately what n is. So from statics I get to-- and from what we know about models of friction then we know that we can model the friction as mu times n for mu mg cosine theta. So from the statics I learn a bunch of things that I need to know for the problem.

So now I get to the real heart of the problem, writing my equation of motion in the x-direction. And the forces in the x-direction mg sine theta down the hill. So it's positive. Minus the friction is up the hill. So I get mg.

I'm mixing up my m's here but there's no other m so-- this basically says, that x double dot, the m's all cancel out. That x double dot is g sine theta minus mu g cosine theta. And that just happens to be a pretty simple to solve, ordinary differential equation. This is an equation in which the acceleration in the problem is constant. The data's not changing. None of these things are changing and so you can just solve this one.

Now we're to the third part, do the math. This one you can just integrate. And so you find out that well, x dot then and I'm just going to call this c some constant. So x dot ct plus an initial velocity, if it had one. And x of t, what you're looking for. And now so v0 and x0 are just your initial conditions. More than likely 0 if you set it up cleverly.

So that's just modeling quickly what I think a good way to lay out a problem is. Describing the motion, explaining the physics, doing the math, drawing good pictures, stating the problem.

All right so now the brain teaser that I want you to think about. So here's the mass of the block plus the scales. And here's the-- here you're standing on it. So mass of the person. You're riding this down the hill. So this is part b.

Think about that when in the shower. If you've got a really simple way to do it, great write it up. It's not terribly hard and it's mostly-- I'll give you a hint. Thinking in terms of free body diagram helps a lot. And we'll come back to this kind of fun problem.

OK, want to go-- that was part one. I want to go onto this recapping the center of mass quickly. We learned a couple of important things. We got-- we talked about the center of mass because we were just talking about Newton's three laws. From looking at the first law, found that it's useful in determining whether or not you're in a inertial frame, we used it.

Second law we've just applied it. We used to do-- get equations of motion.

Third law was about-- we used it when we're thinking about center of mass. We used it to define what the center of mass was. So we said the total mass of a system, I better draw my picture. Here's my system of particles. M1 with position vectors. So a whole mess of particles with their position vectors.

This is ri with respect to O for example. This is my O x, y, z frame. We said that this total mass of the particles somewhere out here there's a center of mass with a position vector rg with respect to O. So the definition of my center of mass is this is equal to the summation of the m, i, r, i, o and these are position vectors. So that's the definition of my center of mass.

If I take two time derivatives of that we arrived at mt rg with the respect to O double dot. Summation over i of my m, i, r, i, o double dots. And then importantly that's the summation of all of the external forces on each of these-- each one of these by itself satisfies Newton's law, second law, which he wrote about particles. Each one has a summation. I've summed these and I'm going to sum these, but each one has a summation of internal forces acting on it. These I call the f, i, j's.

And we learn-- something about the third law tells us about that summation. The third law tells us what? That goes to 0 and that was that's the really powerful piece of the third law that we make great use of. Because this now essentially allows us to say, that the summation of the external forces on an assembly of particles, on a system of particles, is equal to the total mass times the acceleration of the center of mass.

And that-- what that does in one stroke takes you from Newton, who's laws applied to particles and allows you to apply Newton's second law to rigid bodies. Because rigid bodies can be thought up a bunch of particles, which are represented in that simple equation. And that gets you from particles to rigid bodies. And we all know from physics that a summation of the external force on this thing is the mass times the acceleration of the center of mass.

So that's actually quite an important powerful law. It provides this for us. Now I said I wanted to give you a quick, very useful application of thinking about center of mass. I showed you the other day, I had my carbon fiber tube here. Showed you that trick for finding the center of mass just by sliding your fingers along it. And you end up at the center of mass. Well as a practical matter other things-- you really want to be able find center of mass.

This is a glider, a sailplane called an LS8. I happen-- I'm a glider flight instructor. Been flying gliders for 35 years or something like that. And I flew a glider of this type just recently. That machine has a 49 to one let's call it 50 to one to make it easy, glide ratio. Means if you're a mile above the ground in still air, you will go 50 miles before you touch the ground. So they're really amazing high performance machines.

One of the things about all aircraft that you actually need to know is, you really need to know where the center of mass of it is. And if the center of mass is in the wrong place the plane will not fly properly. And so you can't just go throw on 50 pounds of lead in the tail of that plane and expect to survive the next flight.

So you have to know where the center of mass is, and in fact, you want the center of mass about 25% of the-- if the wing is this wide from front to back it's called the cord. About 25% back from the leading edge is about where the center of lift of a wing is. And you want your center of mass also called the center of gravity in these situations, you want it to be pretty close to the center of lift so that their balance in the plane flies nicely.

So is there a simple way to find the center of mass of something like a sailplane? So I'm going to drew-- this is exactly how you do it. So I'll draw a quick picture of my sailplane here. You usually have a little skid or a tail wheel on the back.

And to find the center of mass you just set them on scales. You weigh it. You pick a coordinate system. Doesn't matter where it is, as long as it's fixed. The easiest place is right at the nose of the sailplane.

So we'll make this x, make this y. You take it and you can take a tape measure and measure the distance from your reference point to the position where the wheel sits on the scales. We'll call that L1 here. And that's typically about five feet. And back here you have another position to where you have your second set of scales, that's L2. And a typical sailplane that's about 15 feet.

Apply Newton's law. This thing's not going anywhere. Sum of the forces in the vertical direction is equal to 0. Free body diagram. Well you have somewhere about here, where the wing is, this is your center of mass. And you have m total times g downwards there.

You have two weights pushing on the sailplane. A W2 pushing up, holding up the tail. And a W1 holding up the main gear. And so from the sum of the forces in the y-direction, that had better be 0. So you know that W1 plus W2 minus mtg is 0. And so you find out that the total weight of the sailplane is no surprise, the sum of the two weights on the scales.

**AUDIENCE:** [INAUDIBLE].

**PROFESSOR:** Yeah. Let's do this, m, t, g. So the total weight times gravity is just the sum of the two readings on the scales. And the second piece that you need to do to do this problem is, you can have an equation that says, the sum of the external torques with respect to you're-- through a fixed point O, is equal to the mass moment of inertia times the angular acceleration, oftentimes written as alpha. In this case, that's going to be 0, it's going nowhere.

So what are the external torques with respect to this point? Well we have a right-handed coordinate system. You have W1 up, times L1. W2 up, times L2 minus W1 plus W2, which is the total weight of the sailplane times rg, the location of the center of mass. That's this distance that we're looking for.

So we know everything here, except rg. So we can solve for rg with respect to O and it's simply W1 L1 plus W2 L2 over W1 plus W2. And if you run the numbers, typically W1 600 pounds, W2 the numbers I've done here is-- might be 40 pounds.

And I've already said five feet and 15 feet. And you work the answer and you come up with 5.62 feet, which puts this-- here is the wheel and you're a little bit aft of the main gear. And that's where you, basically where you want to be. That's a real practical use of knowing about centers of mass and how to calculate them.

So that's the second item I wanted to talk about today. Essentially a recap of the center of mass. And now I want to move on to talking about a serious introduction to-- we've had the introduction, velocities and accelerations. We have to have a way of writing down the acceleration of a mass, a point, a dog in a rotating, translating, reference frame with the possibility that in addition to that, the dog's moving.

So we want to have equations-- we want to have the ability to write down expressions for the velocity and acceleration of a mass moving in a translating, rotating, reference frame. So we've started this. We did pretty much did velocities to begin with.

So here's my inertial frame. Call it O or O x, y, z. Here's my rigid body out there. It has a point a something else, b might be the dog. And we've described the position of this as the position of this point a, with respect to O.

And at this point we're going to locate a reference frame attached to the rigid body. And so it's going to be called a, and I'll call it x prime, y prime, z prime. It's attached to the rigid body, it rotates with the rigid body and its attached at some fixed point.

Now what would oftentimes would be a smart choice for that fixed point at point A?

**AUDIENCE:** [INAUDIBLE].

**PROFESSOR:** If you're going to write an equation expressing the motion of this. Where would you make point a?

[INTERPOSING VOICES]

**AUDIENCE:** Center of mass.

**PROFESSOR:** Center of mass. So very, very often, especially when objects are free floating around out there you're going to make smart choices and you're going to put this coordinate system right on the center of mass. But it doesn't have to be, but it can be.

So we were interested in knowing things about the motion of this point in our inertial reference frame, in terms of positions of our coordinate system. And then also this vector here rd, with respect to a. Now last time we came up with expressions for the velocity of b with respect to O.

We said in general it's the velocity of your-- where your coordinate system's located. The translating-- the velocity of the translating frame plus the derivative of rba, time derivative of the position as seen from, if you were sitting at a.

And another way to say that, or this is a derivative taken with the rotation rate momentarily set equal to 0. Another way to think of it. Plus a piece that comes from rotation. So the rotation with respect to the fixed frame, these are all vectors, the rotation with respect to the fixed frame, cross product with r, b, a.

And this-- and we said this is actually a general formula for the derivative of-- this piece is the derivative of a vector in a frame, in a fixed frame. You have two pieces, the derivative as seen without rotation plus the contribution that comes from rotation.

When I did this center of mass thing a second ago, I just kind of quickly wrote down two time derivatives of the position vector. There's no omega cross O's in there right? Why could I do that? This is actually kind of an important distinct point. I could do that they didn't say very specifically when I did it was an assumption I was making. Except for perhaps they drew it. This was done in a Cartesian coordinate system. And my coordinates were x, y and z and the unit vectors were i, j, k and do they move? No. What's their time derivative?

[INTERPOSING VOICES]

**PROFESSOR:** When you don't-- when the inner vectors don't have time derivatives you don't get these terms. This is the only term that contributes so I could just write that equation. But we now have a reference frame attached to a body and this reference frame is rotating.

And that means that the direction of the unit vectors attach-- the unit vector attached to x-prime here is moving, it's rotating. And it's going to have a time derivative. So we have to-- and that is given and you take those time derivatives you get this second piece.

I'm going to give you the answer in advance. The acceleration of b with respect to O I'm going to give you the full 3D equation. Then we'll go back and see a bit where it comes from. So here's-- it's the time derivative of that velocity expression with respect to time taken in the inertial frame O, x, y, z. And am I going to have enough room to get this on? It'll be close.

All right this has several pieces. It's got a contribution of the acceleration of a with respect to O. That's just the acceleration of this point. Has nothing to do with rotation, so it's just a straight out acceleration of my translating frame with respect to O. That's the first piece.

The second piece is related-- is the derivative of this guy that comes from the derivative of this. It's the acceleration of b with respect to a as seen in this a frame. If you read the Williams book, it's called the relative acceleration. It's relative to the-- if you were sitting at point A, it's what you would see as the acceleration.

Plus 2 omega cross velocity of b with respect to a as seen from a plus omega dot, the derivative of the rotation rate, cross rba plus omega cross, omega cross r, b, a. Kind of daunting right? A little messy.

Basically one, two, three, four, five different terms. And you're going to-- and they all have names and meanings. And one of the things that will really help you is to get familiar, you really need to be familiar with the meaning of each one of the terms. And it's not terribly difficult.

This one, just the acceleration of the translating frame. So if it's a merry-go-round sitting on a train and the train's heading down the track, its acceleration of the train. Rotating frame is attached to the merry-go-round. And if you've got the dog on the merry-go-round this is then the acceleration of the dog relative to this, the merry-go-round. This position of the coordinate system attached to the merry-go-round has no rotation in it.

This is the velocity of that point, the dog, as seen from the A frame. Again, you have no sense of rotation. Rotation is not a part of this. Cross product with the rotation rate. This is the accelerate. This is the angular acceleration cross product with rba.

Now that's a term-- what does that mean? I'm swinging a baseball bat and I'm accelerating this thing. Idealize it as just something on a radius accelerating. The acceleration of a point out here is the radius times the angular acceleration. So that's all this term is. And it's called the Euler acceleration. But it's just simply r theta double dot.

This, if you multiply it out and just think about units, this ends up looking like r omega squared. Have you run into that before? What's that? Common language.

**AUDIENCE:** [INAUDIBLE].

**PROFESSOR:** That's as a centrifugal-- centripetal, this is centripetal acceleration. So this is the centripetal acceleration term, that's the Euler acceleration term, this is the local acceleration. This is the acceleration of your frame. This is the strange one. This is the Coriolis acceleration. And we'll get familiar with it too. So that's the full blown 3D acceleration equation. And by the way the vector-- the velocity one is also perfect 3D.

Now in this course we won't do much in the way of 3D dynamics problems. Yes.

**AUDIENCE:** Does the point b on the rigid plane move?

**PROFESSOR:** Does the-- it may. It could be this is the-- an asteroid out there in space and you've got-- this is home base and that's a guy out there in a space suit running. So we want to be able to describe the acceleration of that guy as seen from a fixed reference frame. Now why would we want to know that acceleration? Why do we want to know it in a fixed frame?

Well if you want to calculate the forces on the person. Well how much-- what's he have to do with his feet to brace himself or whatever? What are the actual forces? You have to know the acceleration on the person. But Newton's laws, in order to say f equals ma, Newton's laws have to be applied in inertial reference frames. Is this thing out there doing this an a inertial reference frame? No.

So you can't calculate the forces without having some idea of this inertial frame. So this is the way of getting the acceleration on-- at a location on a moving, rotating body with respect to an inertial frame. And with all the terms present.

Now most discourse has generally has addressed problems which are in most textbooks address only planar motion problems. Planar motion basically means that we can find the translations to a plane.

So imagine an x, and a y, and a z upwards coordinate system attached to the top this table. And I only allow motions that are around the table. And I only allow motions that have a single axis rotation. And that's lined up with z. Those are essentially planar motion problems. And most courses in dynamics, it lists [INAUDIBLE]. That's as far as they get.

And the most of the problems that you'll do will be planar motion problems. But that equation reduces to the planar motion problem as well. We will do a little bit of 3D, because there's a class of problems that I really think it's important for you to understand that just come up all the time that require a little 3D. And as you want to have some things going on out of the plane, but we'll still confine the axis of rotation to a single direction. Yeah.

**AUDIENCE:** [INAUDIBLE] in a planar, but then would you have three to view the freedom? [INAUDIBLE].

**PROFESSOR:** That's a great question she said if you had a dog running on the merry-go-round how many degrees of freedom do you have? So in general rigid bodies, each rigid body, each independent rigid body has-- you have to describe its location of its center of mass and that takes how many coordinates? How many coordinates I'll call them.

**AUDIENCE:** [INAUDIBLE].

**PROFESSOR:** Well in general three. And it can now rotate around three different axes. And that takes three more. So rigid bodies have six degrees of freedom. And any problem when you go to address the problem you essentially for a rigid body you start with six. And you start applying constraints to reduce it down to the number of remaining degrees of freedom.

So if it's confined to a plane and no z-motion is allowed one constraint. If it is on a plane and it's only allowed to rotate about the z-axis that means you've constrained its rotation in around y and x. So that's two more. And so now you're down to three degrees of freedom left, xy and a rotation about the z-axis.

So planar motion problems generally have three degrees of freedom. But instant-- let's just say we're just interested in just something that rotates and doesn't translate. How many degrees of freedom does that have then? Just one. x and y are forced not to-- no motion, two more constraints you're down to one. So lots of problems we do are in fact single degree of freedom problems.

So to do planar motion problems we oftentimes use polar coordinates. So I'm going to introduce r theta. And I'm actually going to call it cylindrical coordinates. And cylindrical coordinates then you have an r, a theta and z.

And let's think about well let's see, I have a demo, a little demo here. So here's a problem with a single axis of rotation. And it's a-- there is a mass out here and just this is a rhyme. And so think of this think of this mass out here as being a bug walking out this rod.

And the rod, this thing goes round and round. It's not a merry-go-round but it's a merry-go-round with a gang plank on it. It's going up at an angle and you can walk the gang plank while the merry-go-round's going around. So that's what we got here.

So this is actually allowed to change position of this mass. So how would I describe that with cylindrical coordinates? Let me so it's going to take-- one would be a side view. So you see your vertical axis here and have to have a bearing to hold it in place.

Here's the arm, here's the bug walking out the arm, has some rotation rate. Theta dot this is the z-axis and the position of this point is described by a r vector, in the r hat direction, which I think is the unit vectors you're used to using.

And then this is the z-component in the k hat direction. And this vector here would be r of v with respect to what shall I call it? I'll make this a and over here someplace I have a fixed inertial let's see I got to be careful here. I want that to be z then I have a y going out here x, y, z pointing upwards.

And my-- this fixed inertial system the unit vectors here this would be i hat, j hat and k hat. But these-- this rotating system with its unit vector little k and this vector they're the same, they're parallel.

But looking down on this, this is my polar coordinate system. Now I'm going to look at my top view. I will see a projection. I'll just see the r, this is rr hat, this is my point B. This is theta. And I have a unit vector. So the unit vector r hat is something-- the unit long pointing in this direction.

And the unit vector in this direction is theta hat. And it's perpendicular to that radius. So now I have my three unit vectors. One pointing in the direction of r, here's also my unit vector is just to make sure there's no confusion. This unit vector is in this direction. K is in that direction. Theta is in that direction.

And over here you still have your-- now here's my x, y, z out of the board inertial frame. And this-- my inertial frame this might be r, b, o. So in my inertial frame. I want to know what's going on here. I want to be able to calculate the velocities and the accelerations.

So the notation here gets-- can get a little confusing. The rbo notation that I've been using all along, that's the motion-- that's the position vector describing that point in my inertial frame. And my-- just lowercase r here, no scrub scripts or anything, that's just going to-- that's my polar coordinate r theta and z that happened to be in this case, this is a rotating frame. This is a rotating frame.

It's the center of this coordinate system's at a. But this thing rotates. So this is a pretty simplified version of this general problem. Now because it's simplified, you can actually-- it's a lot easier to use. Also has some real limitations. You can only going just-- there's limited things that you can describe with it.

So let's start by describing velocities in cylindrical coordinates.

Remember this rba is the length of this guy here. And it's made up of rr hat zk hat.

So to express the velocity we have to take a time derivative of this r, b, a and I'm going to express it in terms of r theta nz. And to get acceleration I have to take two time derivatives of this, but this is going to be expressed in my cylindrical coordinates. This is where I'm going.

And lots of problems-- many, many of these problems have fixed axes of rotations and this velocity and this acceleration are zero. You just drop it out. I'm going to do that just to keep this-- make this problem a little simpler. So we can just focus on these terms.

So let's just let the there be no translational of this frame. And that says that Va with respect to O the acceleration of A with respect to O over zero. So I want you to just focus on these terms. I don't lose anything, I can put these back in later if I need them. I just don't want to keep carrying them along.

So I have my-- remember my side view. This is r, r hat zk hat that's my point. And my top view. This is my projection just looking down on it what I see is the length r. And what I see in my unit vector going that way r direction and theta that direction.

And this is x and the i and a y with a j hat vector looking down on it. My rotation rate mega with respect to my inertial frame, is sum theta dot k hat. All right so now let's find the velocity of b with respect to O. Well it's 0, no translation, plus-- and now I need a time derivative of rb with respect to a.

But this is then r, b, a is r, r hat plus z k hat. And I need the time derivative of that. So I get an r dot r hat plus an r r hat dot plus a z dot k.

So this is the product of two things. They're both time dependent. So I have to get two pieces, k does not change in direction. So it has no time derivative. So I only have a z dot k. So this is a result of doing this, but I now have to figure out what is the time derivative of the unit vector in the r direction.

So when I told-- when we worked out this formula the other day for the time derivative of a rotating vector, I mostly did it, it was kind of an intuitive argument. So on this one occasion I'm going to give you an example of actually figuring out what the derivative of this rotating vector is. And if you go read that kinematics handout and it does this in kind of full blown form for-- in general.

So I'm just going to do it as one example. So here's our looking down on this, the projection on the xy plane, here's our r-vector. And here's this unit vector and it starts from-- I have a unit vector starting from a it's unit-- it's one long. And this is r hat. And it's of unit length and it's in this particular direction.

Now in a little bit it time delta t, it moves. It moves to here. So this is delta r hat and what direction does it move?

**AUDIENCE:** [INAUDIBLE].

**PROFESSOR:** Yeah, it moves in the-- moves in the theta hat direction. And the amount that it moves is the rotation rate, theta dot, delta t. So delta r hat, if I solve for this, delta t. And this is in the theta hat direction, is theta dot theta hat. So this is the limit as t, delta t goes to 0 you get the derivative of r hat with respect to time. Its direction is in the theta hat direction and its magnitude is theta dot. So that's the time derivative of the unit vector r hat. Yeah.

**AUDIENCE:** How does that work with units?

**PROFESSOR:** How does it work with units? What's left out of here is that this is unit length and has dimensions. It's unit length, one whatever unit system you're working. So that is implicitly in here. It's one meter theta dot and that theta dot, the delta t, the times go away. You're left with one meter times the magnitude of theta dot.

So the distance it actually moves is r theta, the r delta theta, delta theta is theta dot delta t and the radius happens to be 1. So whatever unit system you're working in it's a unit vector. Has unit length. So its units are buried right there. Good question.

OK so now we know what this is. So now we can come back finish our description of the velocity of b with respect to a then is r dot r hat plus z dot k hat plus theta dot times so r times the derivative of the unit vector r, which we just figured out is theta dot theta hat times r. R theta dot theta hat. So that's my velocity of b with respect to a. My velocity of B with respect to O all you have to add in is the velocity of A with perspective to O, which we've let be 0 for now.

So for the moment this is also d with respect to O. But this is the general piece of the velocity of b with respect to a in polar cylindrical coordinates.

Now we could have-- so I've actually worked it out, just shown you, just drew the picture and figured out the derivative. We could have used that magic formula. The formula for the derivative of a vector in a rotating frame.

So I'll just do that quickly to remind you how we could have done this. rba with respect to time as seen in the O frame. Is the partial derivative of rba with respect to time as seen in the rotating frame, plus omega cross r, b, a.

This term is that and that. The derivative of this rba as seen from inside of the rotating frame, is just the change in length, this is rba here from the side. So the change in length of that vector, the derivative of-- the time derivative of it. It's the vector sum of the r dot plus z dot.

So this piece comes from this and this. And this piece should-- this one here, it better be this. Well this is-- let's figure it out. This is omega in the k hat direction, cross and rba is r r hat plus z k hat. k cross k is 0. k cross r theta hat. K cross-- k hat cross r hat is theta hat positive. r omega theta hat, same thing as r theta dot theta hat. So we could have just applied this formula for the derivative of a rotating vector and we would have gotten the same thing.

OK just ran out of boards.

Now a quick little exercise you could do on your own is, we're going to need to be able to calculate the derivative of theta hat. Well just plug it in that little formula. And the first term you'll find out the derivative of the theta hat, the length doesn't change in time, it's a unit vector. So you only have the second piece. So it's sum omega cross theta and you're going to get minus theta dot r hat.

So I really want to get here. The acceleration of b and O. That's the real-- that's the single piece we really need to finish the kinematics. So we can do most any problems. Got to be able to describe the acceleration of a point and translating rotating frame. And that's going to be the acceleration of a with respect to O, plus a time derivative of the velocity of b with respect to O.

We've calculated the velocity, we need to be able to essentially carry out this derivative. Two time derivatives of the r, b, a, or a single time derivative of bva. Well we just computed the velocity in this-- of this rotating frame and this is our final expression. So we need to compute the time derivative of that.

I just-- so it's going to look like r dot r hat plus over here a term r theta dot theta hat and pardon me for doing this I think it'll be cleaner in the end. I'm going to start with my z dot k, keep it over here, plus r dot r hat plus this term. And this is going to take up a lot room. Just spread out this way so it--

So let's just-- I'm going to write down where this comes out, this is a little tedious, but then you'll have seen it once hopefully believe that it really works. So these terms, this first term here just gives you z double dot.

So let's write her down. So just z double dot k hat time derivative of this, plus an r double dot r hat, but now I have to take-- do it, flip it and do the other side of it. So I get my-- how do I want to do this? Yeah, I like this.

So this term kept-- leads to this. This term brings you to here. This term, you get r dot times theta dot theta hat plus an r and now you need to take a time derivative of theta dot theta hat. So that's going to expand. So this brings you to here.

Now we've done this derivative, so we can put it in. So this gives us this term over here, so let's keep adding these up. Notice this gives me an r dot theta dot theta hat. This gives me an r dot theta dot theta hat. Two identical terms.

Now this term gives me an r theta double dot theta hat. And now this-- now we need to take the derivative of theta hat and that gives you minus theta dot r hat. So you get a minus r but multiplied by theta dot again squared r hat. So I think we're about there. We're going to start collecting things together.

Now we have a two r dot theta dot theta hat plus an r theta double dot theta hat, minus r theta dot squared r half. So these things, these derivatives, just all kind of flowed down and led to more terms.

But now if we compare-- that we get one, two, three, four, five, I clump these together. This is the change in length of the r vector. Stretch in the position has a z component and a r component. This is the movement of the coordinate system if it moves. This is the Coriolis term. This is the Euler acceleration and this is the centripetal acceleration.

So this is the-- what happens when you start with that messy vector thing and apply it, restrict it to a cylindrical coordinate problem, which is basically planar motion. But you allow some things in the z direction only. So polar coordinates is a more limited form of that, but it's-- every term comes back, every term is still in it.

So acceleration of the moving coordinate system, change of the length of the position vector in the moving coordinate system, the Coriolis term the Euler acceleration term that angular acceleration speedup and finally the centripetal term. Now the way you go about solving problems, usins-- doing problems in polar coordinates. So now you're asked to find an equation of motion. This is an expression for the acceleration of whatever it is you're trying to describe in an inertial frame.

So that when you say-- you can now say f equals ma. If you know the-- sometimes you're given the forces in problems and your asked to find the accelerations. But what if you're given the acceleration and you're asked to find the force? All right, I give you the simplest problem of this kind.

What's the tension in the string? Well if I know that-- so I just say, the way you do these problems is how many things can you eliminate? Well I wasn't moving, this term goes away. That's zero. Z wasn't involved, it's not changing, it's just constant angular rotation, that term is 0.

What's the change in length of the string while I was doing it? Now if that term's 0 we're getting easier all time. How far-- how fast was the length getting longer? That terms gone away. Was I speeding up, or slowing down, or constant speed? Well we'll say it's constant speed, ooh this problem's getting easier all the time. I'm down to one term. f equals ma. Minus r theta dot squared r hat.

So the force that I must have been applying to the string was in the minus r hat direction and had magnitude mr omega squared. So that's actually all there is to it. We're using polar coordinates and cylindrical coordinates to do second law problems.

So there's a couple of problems that you're doing these kind of things on the homework set that's being put out today. So give them a try.

Have a good weekend.