Massachusetts Institute of Technology

Department of Ocean Engineering

Department of Civil and Environmental Engineering

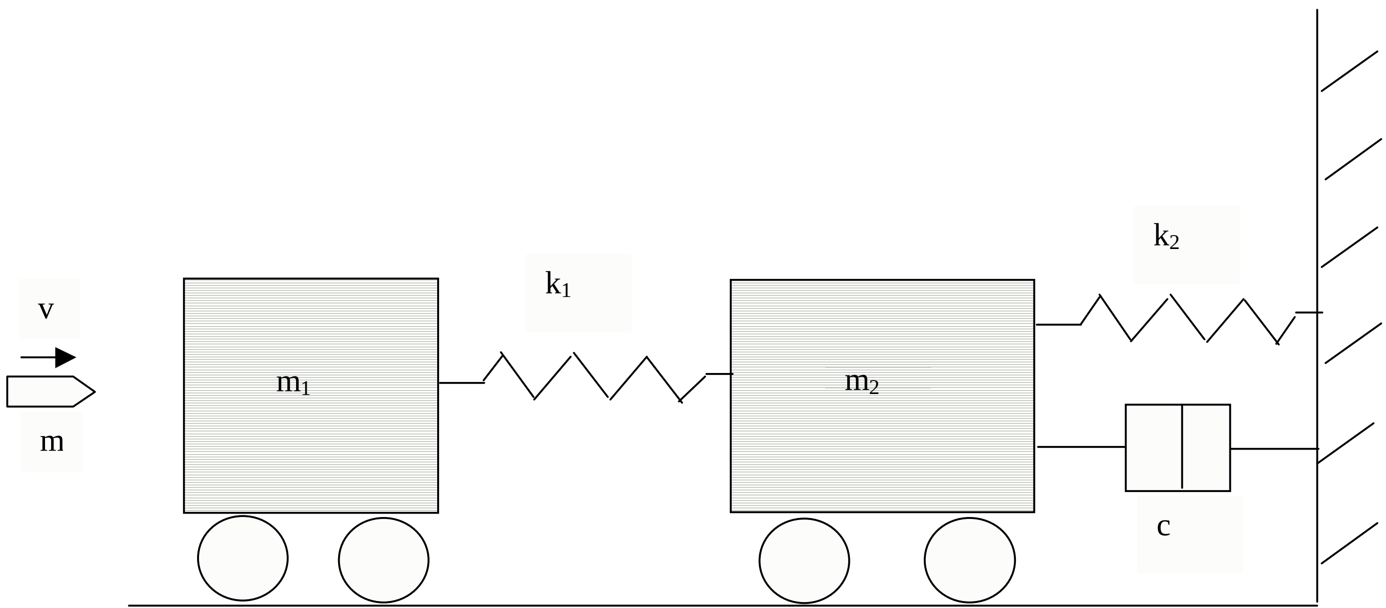
13.013J/1.053J Dynamics and Vibration

Fall 2002

Quiz II

- **"Closed book and notes", two sheets of formulas allowed.**
- **Individual effort.**
- **Read all problems first.**
- **This quiz contains 7 printed pages.**

Figure for Problem 1:



Problem 1: **(25 Points)**

At time $t=0$ a projectile of mass m and velocity v collides with mass m_1 and gets stuck to it. Just before the collision masses m_1 and m_2 are in static equilibrium and the springs k_1 and k_2 are unstretched.

a) Using the direct method find the ordinary differential equations of motion *after* the collision.

(10 Points)

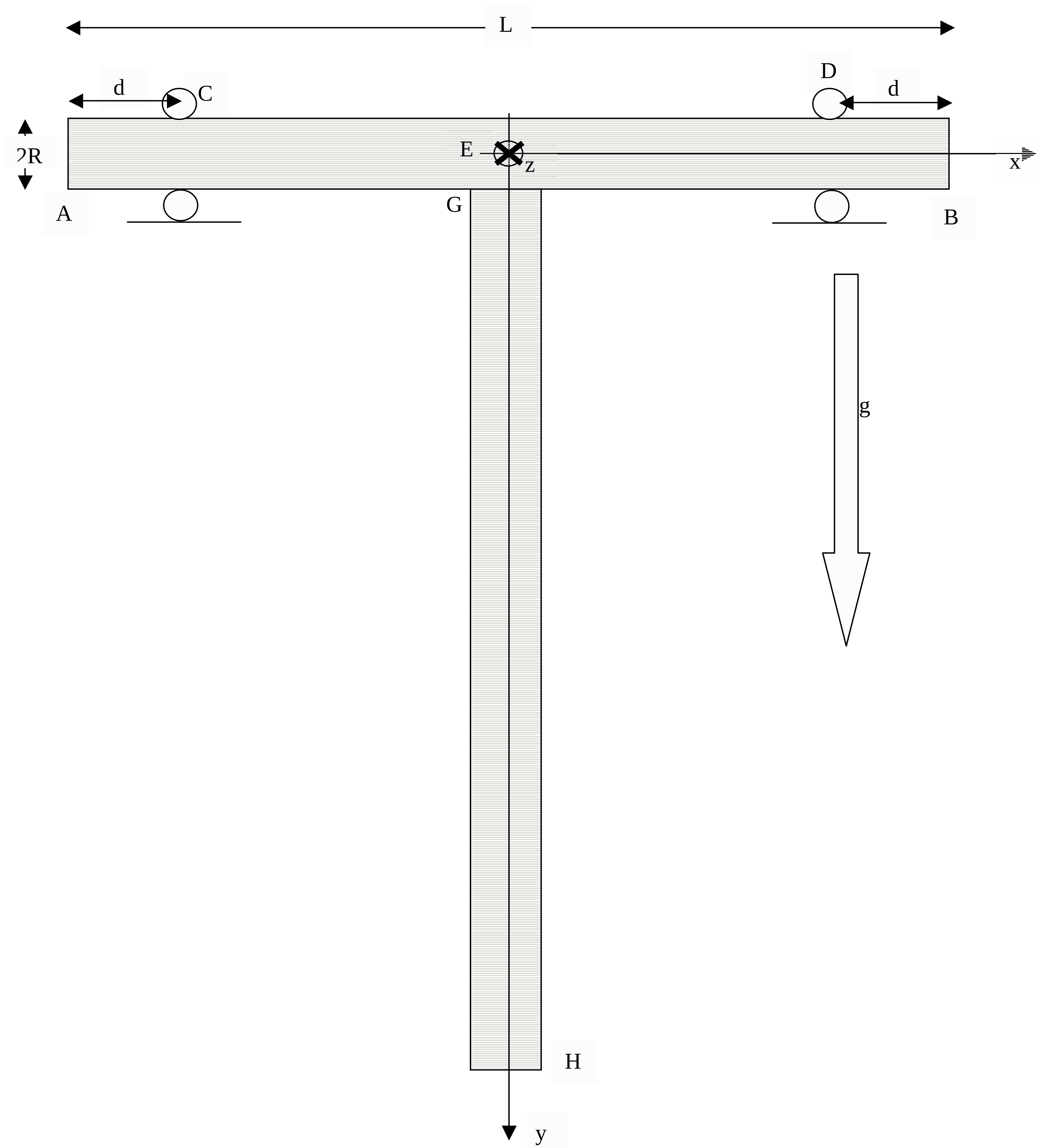
b) Using the indirect method find the ordinary differential equations of motion *after* the collision.

(10 Points)

c) What are the initial conditions for solving the resulting system of ordinary differential equations?

(5 Points)

Figure for Problem 2:



Problem 2: (25 Points)

A circular cylinder *AB* of radius *R*, length *L* and mass *M* is able to rotate about its axis of rotational symmetry, *Ex* which is horizontal, being supported by two frictionless bearings *C* and *D*, at a distance *d* from *A* and *B*.

Another identical circular cylinder *GH* is rigidly attached to the mid point of *AB* so that the axes of the two cylinders are orthogonal.

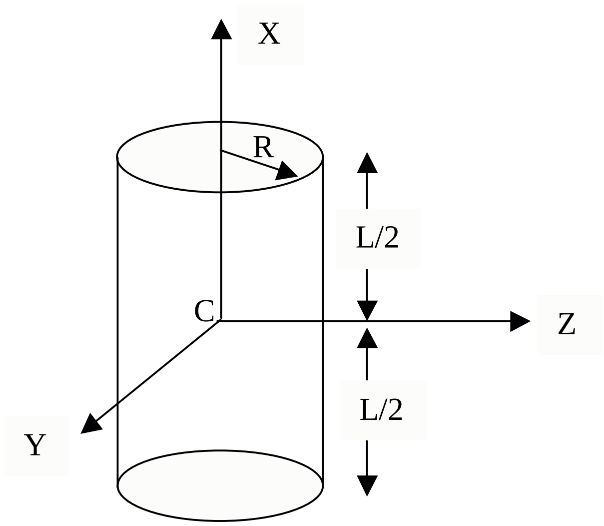
At time $t<0$ the system is at rest as in the figure in a gravity field *g*. Then at $t=0$ the system is suddenly given an initial angular velocity ω_0, about the axis of cylinder *AB,* i.e. about the axis *Ex*.

a) Find the inertia tensor of the system of the two cylinders in the *Exyz* system.
(10 Points)

b) Determine the ordinary differential equation of motion of the system for $t>0$.
(10 Points)

c) What are the initial conditions for solving this ordinary differential equation?
(5 Points)

Hint**:** For a homogeneous cylinder with centroid *C*, mass *M*, radius *R* and length *L*:

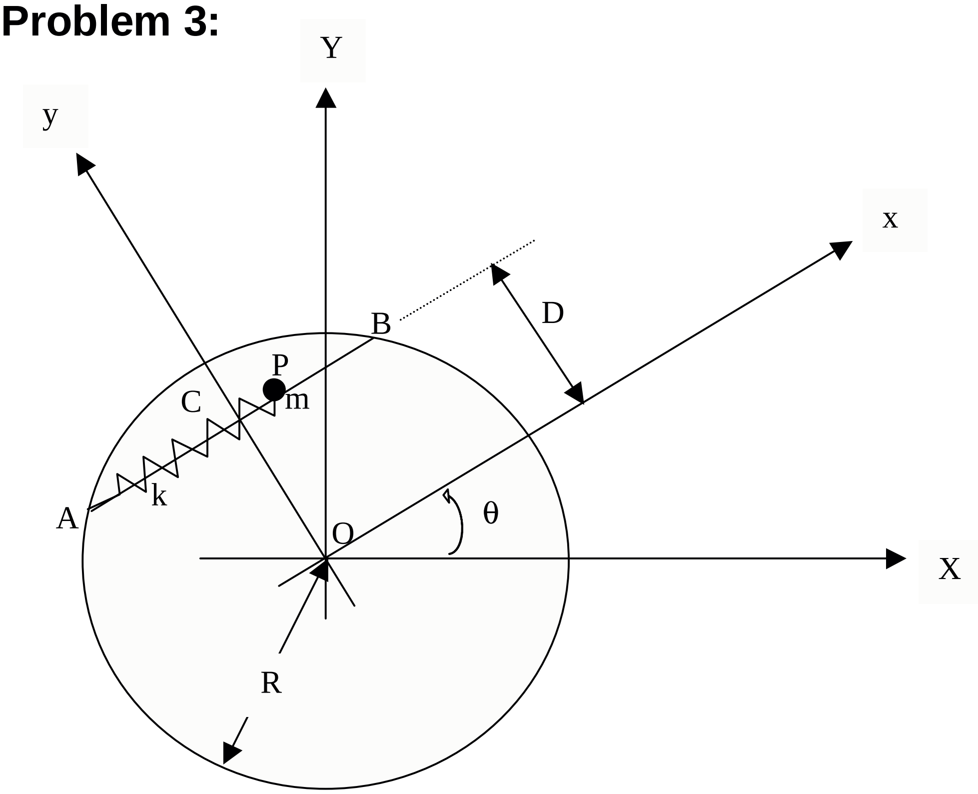


$$I_{xx} = \frac{MR^2}{2}$$

$$I_{yy} = I_{zz} = \frac{M}{12}\left[3R^2 + L^2\right]$$

$$I_{xy} = I_{xz} = I_{yz} = 0$$

Figure for Problem 3:



Problem 3: **(50 Points)**

A circular disk of radius *R*, with mass moment of inertia $I_o = MR^2/2$ about the center *O* is able to rotate about a fixed vertical axis *OZ* without friction. A bead of mass *m* can move along a massless track *AB* at a distance *D* from *O* without friction. The bead is restrained to point *A* on the disk via a massless spring of spring constant *k*. The unstretched length of the spring is *AC*, where *C* is the mid point of *AB*.

Consider a rotating coordinate system *Oxy* attached to the disk, and the angle between the inertial axis *OX* and *Ox* is $\theta(t)$. Further let *CP=x*. Use θ and *x* as generalized coordinates to position the disk and the bead.

a) Using the indirect method derive the following two nonlinear ordinary differential equations of motion of the system:

$$\ddot{\bar{x}} - \ddot{\theta} + [\omega_o^{\,2} - \dot{\theta}^2]\bar{x} = 0 \qquad \text{and} \qquad \ddot{\theta} - \lambda[\ddot{\bar{x}} - \ddot{\theta}\left(1 + \bar{x}^2\right) - 2\dot{\theta}\ \bar{x}\ \dot{\bar{x}}] = 0$$

$$\text{where} \quad \bar{x} = x / D, \quad \omega_o^{\,2} = \frac{k}{m} \quad \text{and} \quad \lambda = 2\frac{m}{M}\left(\frac{D}{R}\right)^2$$

Hint: For questions *(b)* to *(e),* use of the above two nonlinear ordinary differential equations of motion from question *(a)* without solution of question *(a)* is permitted and will carry full credit.

(15 Points)

b) Now assume the disk is forced to rotate at a fixed angular velocity $\dot{\theta} = \omega$=*constant* via an external torque about the axis *OZ*. Derive the ordinary differential equation of motion of the bead in terms of $\bar{x}$.

(10 Points)

c) Under the assumptions of question *(b),* derive an expression for the external torque on the disk, necessary to keep $\dot{\theta} = \omega$ a given constant.

***Hint*:** Determine the force from the bead to the track *AB* normal to the track *AB* and the force from the spring on the disk at point *A*.

(15 Points)

d) Using the differential equation derived in question *(b)*, find a condition under which the static equilibrium $\bar{x}$ *=0* of the bead is *unstable*.

(10 Points)

e) Assume that the condition derived in question *(d)* holds and that at time *t=0* the bead has, $\dot{x}(0) = \mathrm{v_o} > 0,\ x(0) = 0$. Find how much time will elapse until the bead reaches point *B*, distinguishing the two cases:

$$\omega = \pm\sqrt{\frac{k}{m}} \qquad \text{or} \qquad |\omega| > \sqrt{\frac{k}{m}}$$

(Extra: 10 Points)