

18.440 PROBLEM SET FIVE, DUE MARCH 21

A. FROM TEXTBOOK CHAPTER FOUR:

1. Problem 70: At time 0 a coin that comes up heads with probability p is flipped and falls to the ground. Suppose it lands on heads. At times chosen according to a Poisson process with rate λ , the coin is picked up and flipped. (Between these times the coin remains on the ground.) What is the probability that the coin is on its head side at time t ? *Hint*: What would be the conditional probability if there were no additional flips by time t , and what would it be if there were additional flips by time t ?
2. Problem 84: Suppose that 10 balls are put into 5 boxes, with each ball independently being put in box i with probability p_i , $\sum_{i=1}^5 p_i = 1$.
 - (a) Find the expected number of boxes that do not have any balls.
 - (b) Find the expected number of boxes that have exactly 1 ball.
3. Theoretical Exercise 16: Let X be a Poisson random variable with parameter λ . Show that $P\{X = i\}$ increases monotonically and then decreases monotonically as i increases, reaching its maximum when i is the largest integer not exceeding λ . *Hint*: Consider $P\{X = i\}/P\{X = i - 1\}$.
4. Theoretical Exercise 25: Suppose that the number of events that occur in a specified time is a Poisson random variable with parameter λ . If each event is “counted” with probability p , independently of every other event, show that the number of events that are counted is a Poisson random variable with parameter λp . Also, give an intuitive argument as to why this should be so. As an application of the preceding result, suppose that the number of distinct uranium deposits in a given area is a Poisson random variable with parameter $\lambda = 10$. If, in a fixed period of time, each deposit is discovered independently with probability $\frac{1}{50}$, find the probability that (a) exactly 1, (b) at least 1, and (c) at most 1 deposit is discovered during that time.

B. FROM TEXTBOOK CHAPTER FIVE:

1. Problem 8: The lifetime in hours of an electronic tube is a random variable having a probability density function given by

$$f(x) = xe^{-x} \quad x \geq 0$$

Compute the expected lifetime of such a tube.

2. Problem 11: A point is chosen at random on a line segment of length L . Interpret this statement, and find the probability that the ratio of the shorter to the longer segment is less than $1/4$.

C. ANSWER THE FOLLOWING:

1. Compute the expectation of X^n where n is a positive integer and X is a uniform random variable on the interval $[0, 1]$.
2. How does the answer change if the random variable is instead taken to be uniform on $[0, L]$ for some constant L ?

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