18.100C Lecture 23 Summary

General discussion of the Taylor series of an (arbitrarily differentiable) function. This can be quite badly behaved - it may not converge; and even if it does, it may not converge to the original function.

Reminder: definition of $\exp(x)$, $\sin(x)$, $\cos(x)$. How to derive their derivatives. Euler's formula $\exp(it) = \cos(t) + i\,\sin(t)$.

Lemma 23.1. *There is a smallest number $\pi > 0$ such that $\cos(\pi/2) = 0$. Moreover, for that number $\sin(\pi/2) = 1$.*

Lemma 23.2. $\exp(x + 2\pi i) = \exp(x)$*, for all complex numbers x.*

Definition of log as inverse function of exp.

Lemma 23.3. $\log(x)$ *is differentiable for all $x > 0$, and its derivative is $1/x$.*

Theorem 23.4. *The series $x - x^2/2 + x^3/3 - x^4/4 + \cdots$ converges to $\log(1+x)$ inside its radius of convergence (which means for $|x| < 1$).*

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