LATEX EXERCISE 1, 18.100C

Exercise 1. Create and fill in a truth table for the logical formula

$$(1) \qquad ((P \lor Q) \implies R) \implies ((P \implies R) \land (Q \implies R)).$$

You probably will want to use the `tabular` or `array` environment, as well as the `table` environment (to add a caption, for example). Your truth table should have at least four columns (for the propositions P, Q, R and the large formula (1)) and nine rows (one for each possible combination of truth values of P, Q and R plus a top row labeling the columns). *Warning*: Formula (1) may take up too much space if you include it as a column label in your table. Think carefully about how to resolve this problem.

Exercise 2. Typeset each of the following expressions. Then translate them into informal (English) language.

(a) $\exists s \in \mathbb{R} \; \forall a \in A \; (a \leq s \text{ and } \forall t \in \mathbb{R} \; ((\forall b \in A \; b \leq t) \implies s \leq t))$.
(b) $\forall \epsilon > 0 \; \exists a \in A \; (a \leq \sup A \text{ and } a > \sup A - \epsilon)$.

Exercise 3. Consider the following classic induction argument.

Here is a proof that all cows are the same colour. We will use induction on the size of a herd of cows. Consider any herd of just 1 cow. Clearly, every cow in this herd has the same colour. Now suppose we have shown that any herd of n cows is monochromatic. Consider a herd of n + 1 cows. Take one of the cows out of the herd and put it out to pasture, leaving a herd of n cows; by the inductive hypothesis, all n have the same colour (for the sake of argument, let's suppose this colour is purple). Now take one of these purple cows out, and bring back the cow from the pasture; the herd again has n cows, and so all n must be purple (again by the inductive hypothesis). This means that the originally removed cow must be purple, and so all n + 1 cows are purple. This shows that any group of n + 1 cows is monochromatic, and concludes the proof.

(a) Find and discuss, in detail, the flaw in this argument.
(b) Find an image of a purple cow on the web and include it as a labeled figure with your solution. You should acknowledge the source of the figure (this may be accomplished by giving the web address followed by "visited on [date].")

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