

Exercises given with a numbering are from *Basic Analysis: Introduction to Real Analysis (Vol I)* by J. Lebl.

**Reading** Sections 2.5, 2.6, 3.1

**Exercises**

1. Exercise 2.6.2
2. Find all real numbers  $x$  so that the series converges.
  - (a)  $\sum_{n=0}^{\infty} 2^n x^n$
  - (b)  $\sum_{n=0}^{\infty} n x^n$
  - (c)  $\sum_{n=0}^{\infty} \frac{1}{(2n)!} (x - 10)^n$
  - (d)  $\sum_{n=0}^{\infty} n! x^n$
3. (Cauchy-Schwarz inequality) Prove that if  $\sum |x_n|^2$  and  $\sum |y_n|^2$  converge, then the series  $\sum x_n y_n$  converges absolutely and

$$\left| \sum_{n=1}^{\infty} x_n y_n \right| \leq \left( \sum_{n=1}^{\infty} |x_n|^2 \right)^{\frac{1}{2}} \left( \sum_{n=1}^{\infty} |y_n|^2 \right)^{\frac{1}{2}}.$$

4. Prove that every real number is a cluster point of the set of irrational numbers.
5. Exercise 3.1.13
6. Let  $S \subset \mathbb{R}$ , let  $c$  be a cluster point of  $S$ , and let  $f : S \rightarrow \mathbb{R}$ .
  - (a) Assume  $\lim_{x \rightarrow c} f(x)$  exists. Prove that there exist  $B \geq 0$  and  $\delta > 0$  such that if  $x \in S$  and  $0 < |x - c| < \delta$  then  $|f(x)| \leq B$ .
  - (b) Assume that  $\lim_{x \rightarrow c} f(x) = L > 0$ . Prove that there exists  $\delta > 0$  such that if  $x \in S$  and  $0 < |x - c| < \delta$  then  $f(x) > 0$ .

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