Exercises given with a numbering are from *Basic Analysis: Introduction to Real Analysis (Vol I)* by J. Lebl.

Reading Sections 2.5, 2.6, 3.1

Exercises

1. Exercise 2.6.2
2. Find all real numbers x so that the series converges.
   (a) $\sum_{n=0}^{\infty} 2^n x^n$
   (b) $\sum_{n=0}^{\infty} n x^n$
   (c) $\sum_{n=0}^{\infty} \frac{1}{(2n)!}(x-10)^n$
   (d) $\sum_{n=0}^{\infty} n! x^n$
3. (Cauchy-Schwarz inequality) Prove that if $\sum |x_n|^2$ and $\sum |y_n|^2$ converge, then the series $\sum x_n y_n$ converges absolutely and
$$\left| \sum_{n=1}^{\infty} x_n y_n \right| \leq \left(\sum_{n=1}^{\infty} |x_n|^2 \right)^{\frac{1}{2}} \left(\sum_{n=1}^{\infty} |y_n|^2 \right)^{\frac{1}{2}}.$$
4. Prove that every real number is a cluster point of the set of irrational numbers.
5. Exercise 3.1.13
6. Let $S \subset \mathbb{R}$, let c be a cluster point of S, and let $f : S \to \mathbb{R}$.
   (a) Assume $\lim_{x \to c} f(x)$ exists. Prove that there exist $B \geq 0$ and $\delta > 0$ such that if $x \in S$ and $0 < |x - c| < \delta$ then $|f(x)| \leq B$.
   (b) Assume that $\lim_{x \to c} f(x) = L > 0$. Prove that there exists $\delta > 0$ such that if $x \in S$ and $0 < |x - c| < \delta$ then $f(x) > 0$.