Exercises given with a numbering are from *Basic Analysis: Introduction to Real Analysis (Vol I)* by J. Lebl.

**Reading** Sections 4.1, 4.2 (**Lebl**)

**Exercises**

1. Let $I$ be an interval. A function $f : I \to \mathbb{R}$ is said to satisfy a *Hölder condition with exponent* $\alpha > 0$ if there exists a constant $C > 0$ such that if $x, y \in I$ then
$$|f(x) - f(y)| \leq C|x - y|^\alpha.$$
   (a) Prove that if $f : I \to \mathbb{R}$ satisfies a Hölder condition with exponent $\alpha > 0$, then $f$ is uniformly continuous on $I$.
   (b) Prove that if $f : I \to \mathbb{R}$ satisfies a Hölder condition with exponent $\alpha > 1$, then $f$ is constant.
2. Exercise 4.1.11
3. Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function. Prove that $f$ is Lipschitz continuous if and only if $f'$ is a bounded function.
4. Exercise 4.2.9
5. Exercise 4.2.13

# 18.100A / 18.1001 Real Analysis

Fall 2020