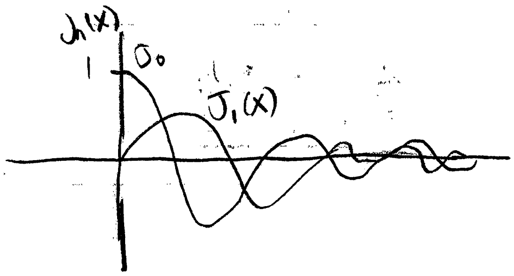
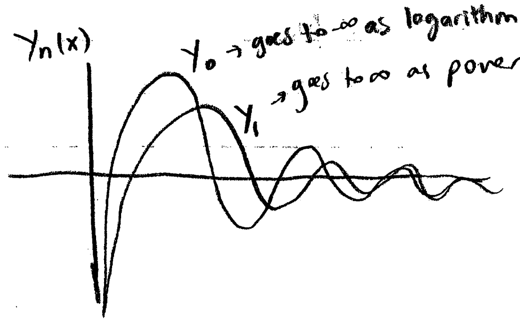
Properties of Bessel Functions:

$x \to 0^+$ $\quad J_p(x) \simeq \frac{1}{2^p p!} x^p$ $\quad$ does not blow up

$J_{-p}(x) \simeq \frac{2^p}{\Gamma(1-p)} x^{-p}$ $\quad$ blows up

$Y_p(x) \simeq \frac{-2^p \Gamma(p)}{\pi} x^{-p}$ $(p \neq 0)$ $\quad \boxed{Y_0(x) \sim \frac{2}{\pi} \ln x \quad p=0}$





$J_p(x)$ does not "blow" up as $x \to 0$. $Y_p(x)$ does blow up as $x \to 0$.

$x \to \infty$ $\quad J_p(x) \simeq \sqrt{\frac{2}{\pi x}} \cos\left(x - \frac{\pi}{4} - \frac{p\pi}{2}\right)$ $\quad$ true for any real p

$Y_p(x) \simeq \sqrt{\frac{2}{\pi x}} \sin\left(x - \frac{\pi}{4} - \frac{p\pi}{2}\right)$

$H_p^{(1)}(x) = J_p \pm i Y_p \simeq \sqrt{\frac{2}{\pi x}}\, e^{\pm i\left(x - \frac{\pi}{4} - p\frac{\pi}{2}\right)}$

Define: $\quad H_p^{(1)} x = J_p(x) + i Y_p(x) \leftarrow$ Hankel function of 1st kind

$H_p^{(2)} x = J_p(x) - i Y_p(x) \leftarrow$ 2nd kind

General solution of Bessel equation: $y(x) = \tilde{C}_1 H_p^{(1)}(x) + \tilde{C}_2 H_p^{(2)}(X)$