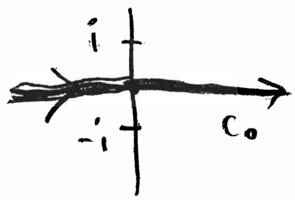
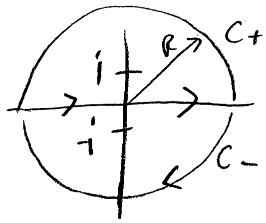
(II) $I = \int_{-\infty}^{\infty} dx\, e^{i\alpha x} \frac{P_m(x)}{Q_N(x)}$ $\alpha > 0$ Apply steps of Case I.

or $\int_{0}^{\infty} dx \begin{cases} \cos(\alpha x) \\ \sin(\alpha x) \end{cases} \cdot \frac{P_m(x)}{Q_N(x)}$

ex $I = \int_{-\infty}^{\infty} dx \frac{e^{i\alpha x}}{x^2+1}$, $\alpha > 0$

(i) $x \to z$ $f(z) = \frac{e^{i\alpha x}}{z^2+1}$; simple poles at $z^2+1=0 \to z = \pm i$



(ii) 

$0 < \theta < \pi$

$\pi < \theta < 2\pi$

$e^{i\alpha z}$, α: real, $z = re^{i\theta} = r(\cos\theta + i\sin\theta)$

$$i\alpha z = i\alpha R(\cos\theta + i\sin\theta) = \underbrace{-\alpha R\sin\theta}_{\text{real}} + \underbrace{i\alpha R\cos\theta}_{\text{imaginary}}$$

$$e^{i\alpha z} = e^{-\alpha r\sin\theta} e^{i\alpha R\cos\theta}$$

$$|e^{i\alpha z}| = |\underbrace{e^{-\alpha R\sin\theta}}_{>0}|\cancel{|e^{i\alpha R\cos\theta}|}^{1} = e^{-\alpha R\sin\theta} \xrightarrow[R\to\infty]{} \begin{cases} 0, & 0<\theta<\pi, \sin\theta>0 \\ +\infty, & \pi<\theta<2\pi, \sin\theta<0 \end{cases} \alpha>0$$

$\alpha > 0$

$$\begin{cases} +\infty & 0<\theta<\pi \\ 0, & \pi<\theta<2\pi \end{cases} \alpha<0$$

Choose C_+ for I. $C = C_0 + C_+$

(iii) Apply residue theorem with path C

$$\oint_C f(z)\,dz = 2\pi i \operatorname*{Res}_{z=i} f(z) = 2\pi i \frac{e^{i\alpha z}}{z^2+1} = 2\pi i \frac{e^{i\alpha i}}{2i} = \pi e^{-\alpha}$$

$$\xrightarrow{R\to\infty} = \left(\oint_{C_0} + \cancel{\oint_{C_+}}^{0}\right) dz f(z) = I$$

$$\boxed{I = \pi e^{-\alpha}}$$

ex

$$I = \int_0^\infty dx \underbrace{\cos(\alpha x)}_{\text{even}} \frac{1}{x^2+1} = \frac{1}{2}\int_{-\infty}^{\infty} dx \cos(\alpha x)\frac{1}{x^2+1} = \frac{1}{2}\int_{-\infty}^{\infty} dx \operatorname{Re}(e^{i\alpha x})\frac{1}{x^2+1}$$

those have to be real

$$= \frac{1}{2}\operatorname{Re}\int_{-\infty}^{\infty} dx \frac{e^{\alpha x}}{x^2+1} = \frac{\pi}{2}e^{-\alpha}$$

Think about $\int_0^\infty dx \sin(\alpha x)\frac{1}{x^2+1}$