**Exercises on symmetric matrices and positive definiteness**

**Problem 25.1:** (6.4 #10. *Introduction to Linear Algebra:* Strang) Here is a quick "proof" that the eigenvalues of all real matrices are real:

$$\textbf{False Proof:}\ \ A\mathbf{x} = \lambda\mathbf{x} \text{ gives } \mathbf{x}^T A\mathbf{x} = \lambda\mathbf{x}^T\mathbf{x} \text{ so } \lambda = \frac{\mathbf{x}^T A\mathbf{x}}{\mathbf{x}^T\mathbf{x}} \text{ is real.}$$

There is a hidden assumption in this proof which is not justified. Find the flaw by testing each step on the $90^\circ$ rotation matrix:

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

with $\lambda = i$ and $\mathbf{x} = (i, 1)$.

**Problem 25.2:** (6.5 #32.) A *group* of nonsingular matrices includes $AB$ and $A^{-1}$ if it includes $A$ and $B$. "Products and inverses stay in the group." Which of these are groups?

a) Positive definite symmetric matrices $A$.

b) Orthogonal matrices $Q$.

c) All exponentials $e^{tA}$ of a fixed matrix $A$.

d) Matrices $D$ with determinant 1.