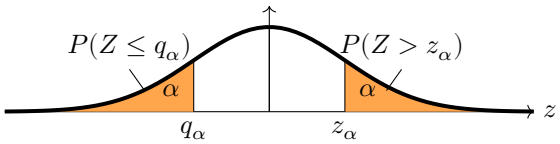
Class 22 in-class problems, 18.05, Spring 2022

Concept questions

Concept question 1. Critical values



1. $z_{0.025} =$

(a) -1.96 (b) -0.95 (c) 0.95 (d) 1.96 (e) 2.87

2. $-z_{0.16} =$

(a) -1.33 (b) -0.99 (c) 0.99 (d) 1.33 (e) 3.52

Board questions

Problem 1. Computing confidence intervals

The data 4, 1, 2, 3 is drawn from $\mathrm{N}(\mu, \sigma^2)$ with μ unknown.

(a) Find a 90% z confidence interval for μ, given that $\sigma = 2$.

For the remaining parts, suppose σ is unknown.

(b) Find a 90% t confidence interval for μ.

(c) Find a 90% χ^2 confidence interval for σ^2.

(d) Find a 90% χ^2 confidence interval for σ.

(e) Given a normal sample with $n = 100$, $\overline{x} = 12$, and $s = 5$,
find the rule-of-thumb 95% confidence interval for μ.

Problem 2. Confidence intervals and non-rejection regions

Suppose $x_1, \dots, x_n \sim \mathrm{N}(\mu, \sigma^2)$ with σ known.

Consider two intervals:

1. The z confidence interval around $\overline{x}$ at confidence level $1 - \alpha$.

2. The z non-rejection region for $H_0 : \mu = \mu_0$ at significance level α.

Compute and sketch these intervals to show that:

$$\mu_0 \text{ is in the first interval} \quad \Leftrightarrow \quad \overline{x} \text{ is in the second interval.}$$

Problem 3. Polling

For a poll to find the proportion θ of people supporting X we know that a $(1-\alpha)$ confidence interval for θ is given by

$$\left[\bar{x} - \frac{z_{\alpha/2}}{2\sqrt{n}},\ \bar{x} + \frac{z_{\alpha/2}}{2\sqrt{n}} \right].$$

(a) How many people would you have to poll to have a margin of error of 0.01 with 95% confidence? (You can do this in your head.)

(b) How many people would you have to poll to have a margin of error of 0.01 with 80% confidence. (You'll want R or other calculator here.)

(c) If $n = 900$, compute the 95% and 80% confidence intervals for θ.

Discussion questions

1. Width of confidence intervals

The quantities n, c = confidence, $\overline{x}$, σ all appear in the z confidence interval for the mean.

How does the width of a confidence interval for the mean change if:

1. We increase n and leave the others unchanged?
2. We increase c and leave the others unchanged?
3. We increase μ and leave the others unchanged?
4. We increase σ and leave the others unchanged?

(A) it gets wider (B) it gets narrower (C) it stays the same.

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